Perfect Half Space Games

Thomas Colcombet, Marcin Jurdziński, Ranko Lazić, and Sylvain Schmitz

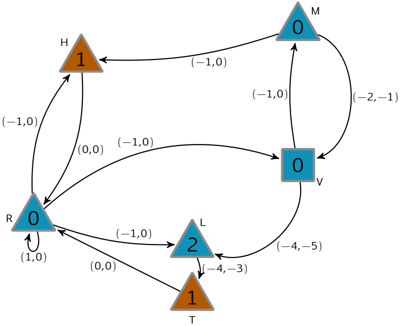
LSV, ENS Paris-Saclay & CNRS & Inria

Highlights 2017, September 13, 2017

Multi-Dimensional Energy Parity Games

Player 1 wins a play if both

- energy objective: no component goes negative
- parity objective: the maximal priority is odd



Example

$$R(0,0) \xrightarrow{(1,0)} R(1,0) \xrightarrow{(1,0)} R(2,0) \xrightarrow{(-1,0)} H(1,0) \xrightarrow{(0,0)} R(1,0) \to \cdots$$

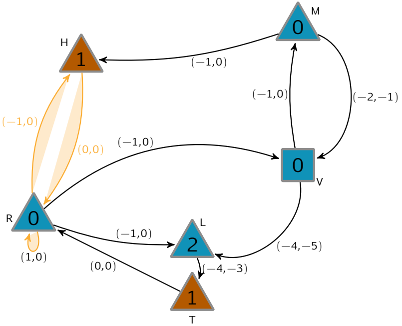
Decision problems: Does Player 1 have a winning strategy

- given initial credit as part of the input
- existential: for some initial credit

Multi-Dimensional Energy Parity Games

Player 1 wins a play if both

- energy objective: no component goes negative
- parity objective: the maximal priority is odd



Example

$$R(0,0) \xrightarrow{(1,0)} R(1,0) \xrightarrow{(1,0)} R(2,0) \xrightarrow{(-1,0)} H(1,0) \xrightarrow{(0,0)} R(1,0) \rightarrow \cdots$$

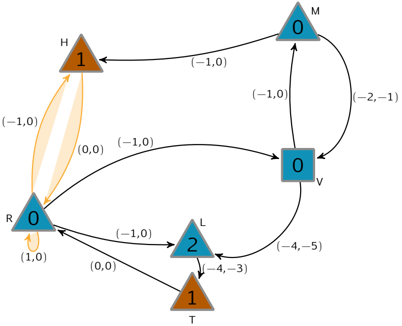
Decision problems: Does Player 1 have a winning strategy

- given initial credit as part of the input
- existential: for some initial credit

Multi-Dimensional Energy Parity Games

Player 1 wins a play if both

- energy objective: no component goes negative
- parity objective: the maximal priority is odd



Example

$$R(0,0) \xrightarrow{(1,0)} R(1,0) \xrightarrow{(1,0)} R(2,0) \xrightarrow{(-1,0)} H(1,0) \xrightarrow{(0,0)} R(1,0) \to \cdots$$

Decision problems: Does Player 1 have a winning strategy

- given initial credit as part of the input
- existential: for some initial credit

Reductions and Strategy Transfers

multi-dimensional energy parity games

↓ (Jančar, RP ’15)

extended multi-dimensional energy games (Brázdil et al., ICALP ’10)

↓

bounding games (Jurdziński et al., ICALP ’15)

↓

perfect half space games (this paper)

↓

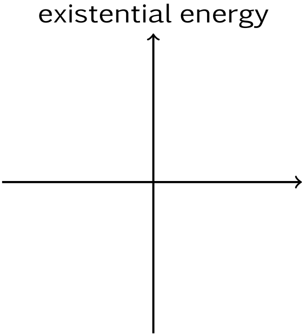
lexicographic energy games (Colcombet and Niwiński)

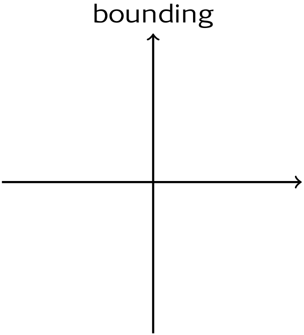
↓

mean-payoff games (Comin and Rizzi, Algorithmica ’16)

Energy and Bounding Games

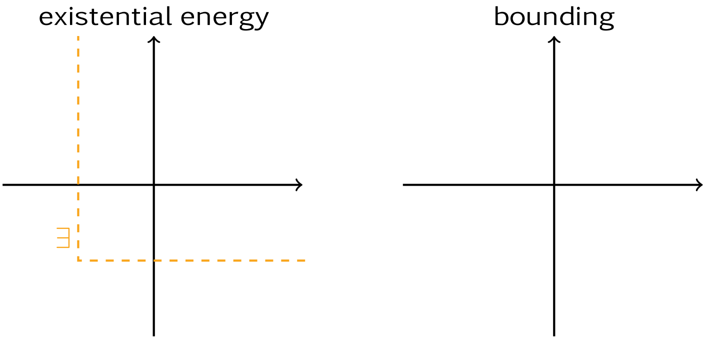
Player 1's Objective





Energy and Bounding Games

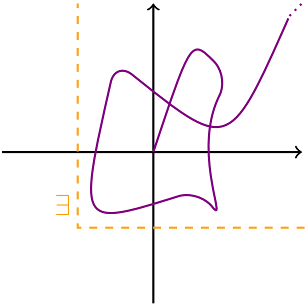
Player 1's Objective



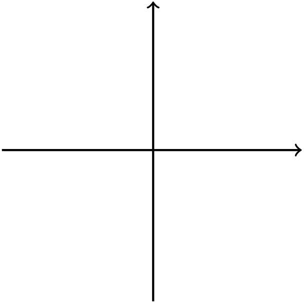
Energy and Bounding Games

Player 1's Objective

existential energy



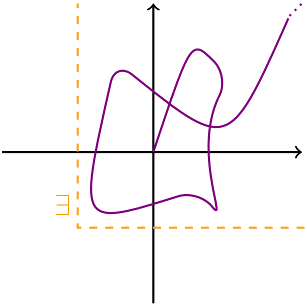
bounding



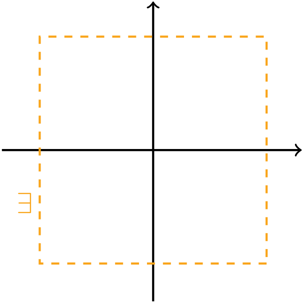
Energy and Bounding Games

Player 1's Objective

existential energy

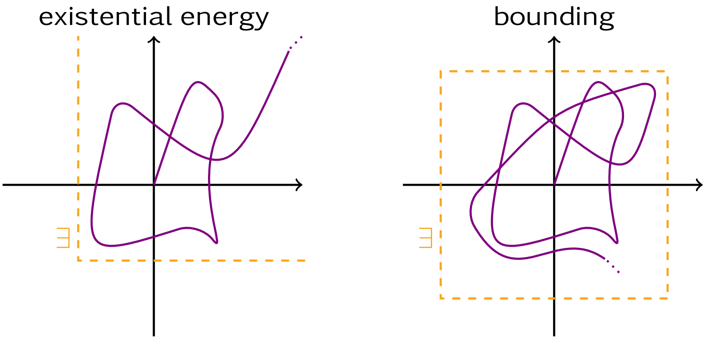


bounding



Energy and Bounding Games

Player 1's Objective



Energy and Bounding Games

Theorem

Bounding games of fixed dimension can be solved in pseudo polynomial time.

Corollary

1. *The given initial credit problem for multi-dimensional energy parity games is in 2EXP.*
2. *With fixed dimension and number of priorities, it is in pseudo polynomial time.*

Energy and Bounding Games

Theorem

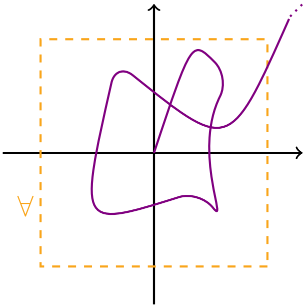
Bounding games of fixed dimension can be solved in pseudo polynomial time.

Corollary

1. *The given initial credit problem for multi-dimensional energy parity games is in 2EXP.*
2. *With fixed dimension and number of priorities, it is in pseudo polynomial time.*

Perfect Half Space Games

Player 2's Objective in a Bounding Game

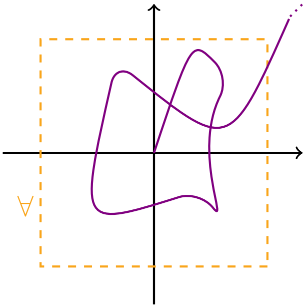


Key Intuition

Player 2 can escape in a perfect half space

Perfect Half Space Games

Player 2's Objective in a Bounding Game

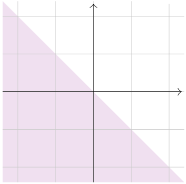


Key Intuition

Player 2 can escape in a perfect half space

Perfect Half Space Games

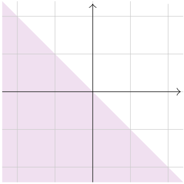
Perfect Half Space



$$\{(x,y) : x + y < 0\}$$

Perfect Half Space Games

Perfect Half Space

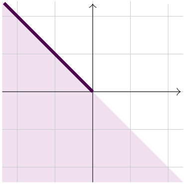


$$\{(x,y) : x + y < 0\}$$

boundary: $\{(x,y) : x + y = 0\}$

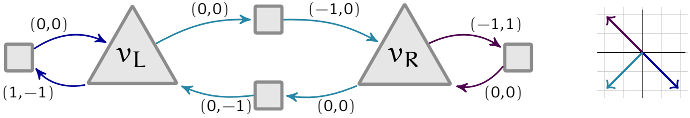
Perfect Half Space Games

Perfect Half Space



$$\{(x,y) : x + y < 0\}$$
$$\cup \{(x,y) : x + y = 0 \wedge x < 0\}$$

Perfect Half Space Games



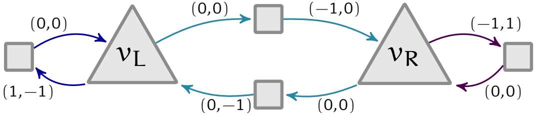
Plays

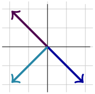
- pairs of vertices and perfect half spaces:

$$(v_0, \mathbf{H}_0) \xrightarrow{\mathbf{w}_1} (v_1, \mathbf{H}_1) \xrightarrow{\mathbf{w}_2} (v_2, \mathbf{H}_2) \cdots$$

- in his vertices, Player 2 chooses the current perfect half space

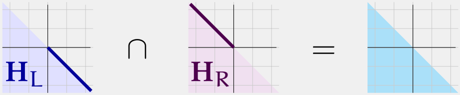
Perfect Half Space Games





- Player 2 wins if $\exists i$ s.t. $\sum_{j \geqslant 0} \mathbf{w}_j$ diverges into $\bigcap_{j>i} \mathbf{H}_j$

Example



Solving Perfect Half Space Games

Theorem

Perfect half space games on multi-weighted game graphs (V, E, d) *are solvable in* $(|V| \cdot \|E\|)^{O(d^3)}$.

Proof Idea

- reduce to a lexicographic energy game (Colcombet and Niwiński)
- $\approx$ perfect half space game with a single fixed $\mathbf{H}$
- itself reduced to a mean-payoff game

Solving Perfect Half Space Games

Theorem

Perfect half space games on multi-weighted game graphs (V, E, d) *are solvable in* $(|V| \cdot \|E\|)^{O(d^3)}$.

Proof Idea

- reduce to a lexicographic energy game (Colcombet and Niwiński)
- $\approx$ perfect half space game with a single fixed $\mathbf{H}$
- itself reduced to a mean-payoff game

Player 2 Strategies

Oblivious Strategy

Player 2 chooses the same $\mathbf{H}_v$ every time it visits vertex v

Theorem

If Player 2 has a winning strategy in a perfect half space game, then it has an oblivious one.

"Counterless" Strategy

Corollary (Brázdil et al., ICALP '10)

If Player 2 has a winning strategy in a multi-dimensional energy parity game, then it has a positional one.

Player 2 Strategies

Oblivious Strategy

Player 2 chooses the same $\mathbf{H}_v$ every time it visits vertex v

Theorem

If Player 2 has a winning strategy in a perfect half space game, then it has an oblivious one.

"Counterless" Strategy

Corollary (Brázdil et al., ICALP '10)

If Player 2 has a winning strategy in a multi-dimensional energy parity game, then it has a positional one.

Concluding Remarks

- tight 2-EXP bounds for multi-energy parity games
- impacts numerous problems
  - contractive $(\oplus,!)$-Horn linear logic (Kanovich, APAL '95)
  - (weak) simulation of finite-state systems by Petri nets (Abdulla et al., Concur '13)
  - model-checking Petri nets with a fragment of μ-calculus (Abdulla et al., Concur '13)
  - resource-bounded agent temporal logic RB$\pm$ATL* (Alechina et al., RP '16 & AI '17)
- fine understanding of Player 2's strategies:

  Player 2 can win by announcing in which perfect half space he will escape

Concluding Remarks

- tight 2-EXP bounds for multi-energy parity games
- impacts numerous problems
  - contractive $(\oplus,!)$-Horn linear logic
    (Kanovich, APAL '95)
  - (weak) simulation of finite-state systems by Petri nets
    (Abdulla et al., Concur '13)
  - model-checking Petri nets with a fragment of μ-calculus
    (Abdulla et al., Concur '13)
  - resource-bounded agent temporal logic RB±ATL*
    (Alechina et al., RP '16 & AI '17)
- fine understanding of Player 2's strategies:
  Player 2 can win by announcing in which perfect half space he will escape

Concluding Remarks

- tight 2-EXP bounds for multi-energy parity games
- impacts numerous problems
  - contractive $(\oplus, !)$-Horn linear logic
    (Kanovich, APAL '95)
  - (weak) simulation of finite-state systems by Petri nets
    (Abdulla et al., Concur '13)
  - model-checking Petri nets with a fragment of μ-calculus
    (Abdulla et al., Concur '13)
  - resource-bounded agent temporal logic RB±ATL*
    (Alechina et al., RP '16 & AI '17)
- fine understanding of Player 2's strategies:
  Player 2 can win by announcing in which perfect half space he will escape

References

Abdulla, P.A., Mayr, R., Sangnier, A., and Sproston, J., 2013. Solving parity games on integer vectors. In *Concur 2013*, volume 8052 of *LNCS*, pages 106–120. Springer. doi:10.1007/978-3-642-40184-8_9.

Alechina, N., Bulling, N., Demri, S., and Logan, B., 2016. On the complexity of resource-bounded logics. In *RP 2016*, volume 9899 of *LNCS*, pages 36–50. Springer. doi:10.1007/978-3-319-45994-3_3.

Alechina, N., Bulling, N., Logan, B., and Nguyen, H.N., 2017. The virtues of idleness: A decidable fragment of resource agent logic. *Artif. Intell.* doi:10.1016/j.artint.2016.12.005. to appear.

Brázdil, T., Jančar, P., and Kučera, A., 2010. Reachability games on extended vector addition systems with states. In *ICALP 2010*, volume 6199 of *LNCS*, pages 478–489. Springer. doi:10.1007/978-3-642-14162-1_40. arXiv version available from http://arxiv.org/abs/1002.2557.

Chatterjee, K., Randour, M., and Raskin, J.F., 2014. Strategy synthesis for multi-dimensional quantitative objectives. *Acta Inf.*, 51(3–4):129–163. doi:10.1007/s00236-013-0182-6.

Colcombet, T. and Niwiński, D., 2017. Lexicographic energy games. Manuscript.

Comin, C. and Rizzi, R., 2016. Improved pseudo-polynomial bound for the value problem and optimal strategy synthesis in mean payoff games. *Algorithmica*. doi:10.1007/s00453-016-0123-1. To appear.

Jurdziński, M., Lazić, R., and Schmitz, S., 2015. Fixed-dimensional energy games are in pseudo-polynomial time. In *ICALP 2015*, volume 9135 of *LNCS*, pages 260–272. Springer. doi:10.1007/978-3-662-47666-6_21. arXiv version available from https://arxiv.org/abs/1502.06875.

Kanovich, M.I., 1995. Petri nets, Horn programs, linear logic and vector games. *Ann. Pure App. Logic*, 75(1–2): 107–135. doi:10.1016/0168-0072(94)00060-G.

Lasota, S., 2009. EXPSPACE lower bounds for the simulation preorder between a communication-free Petri net and a finite-state system. *Information Processing Letters*, 109(15):850–855. doi:10.1016/j.ipl.2009.04.003.

Velner, Y., Chatterjee, K., Doyen, L., Henzinger, T.A., Rabinovich, A., and Raskin, J.F., 2015. The complexity of multi-mean-payoff and multi-energy games. *Inform. and Comput.*, 241:177–196. doi:10.1016/j.ic.2015.03.001.

Zwick, U. and Paterson, M., 1996. The complexity of mean payoff games on graphs. *Theor. Comput. Sci.*, 158(1): 343–359. doi:10.1016/0304-3975(95)00188-3.