Learning to Prove Safety over Parameterised Concurrent Systems

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Overview

Parameterised concurrent systems

Infinite families $\mathcal{F}$ of finite-state concurrent systems parameterised by the number of processes.

$$\mathcal{F} = \{\text{systems with } n \text{ processes} : n \in \mathbb{N}\}$$

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Checking safety for parameterised systems is undecidable in general.

In this talk, we will introduce a simple but effective heuristic to verify safety of parameterised systems based on automata learning.

Symbolic Framework

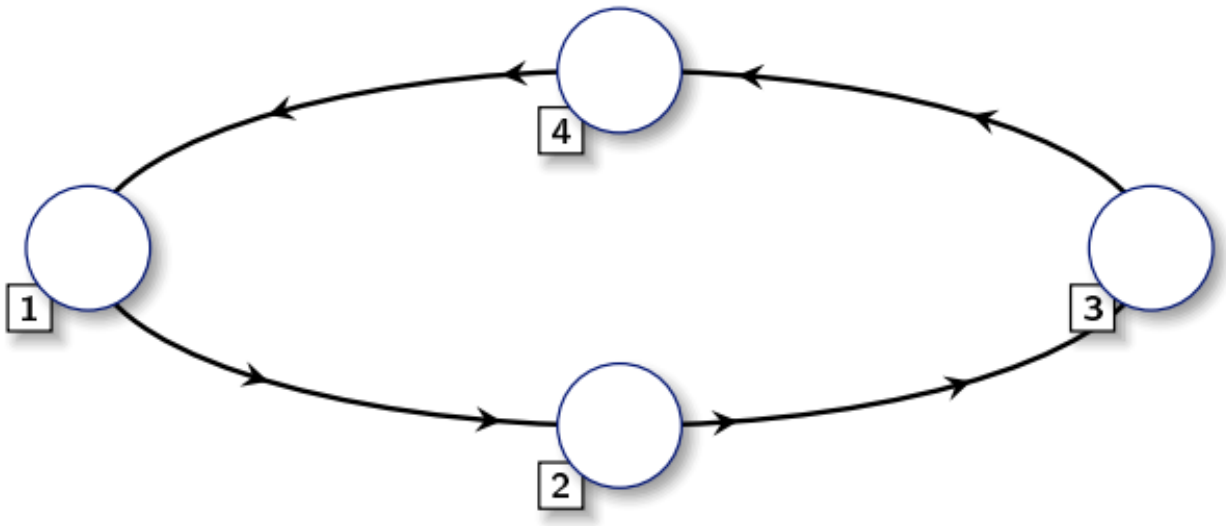
Modelling parameterised systems

Configurations: represented as finite words

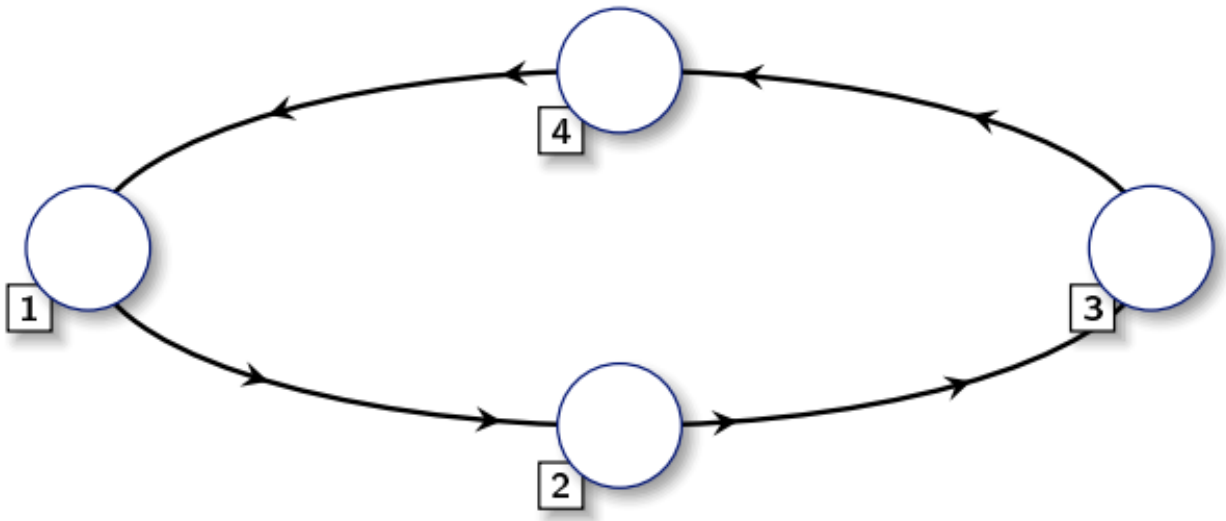
Sets of configurations: represented as finite automata

Transition relation: represented as a transducer

Token Ring Example



Token Ring Example



Configurations: 1000, 0100, 0010, 0001

Transitions: $\begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}\begin{bmatrix}0\\0\end{bmatrix}\begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}0\\0\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}\begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}0\\0\end{bmatrix}\begin{bmatrix}0\\0\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}\begin{bmatrix}0\\0\end{bmatrix}\begin{bmatrix}0\\0\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}.$

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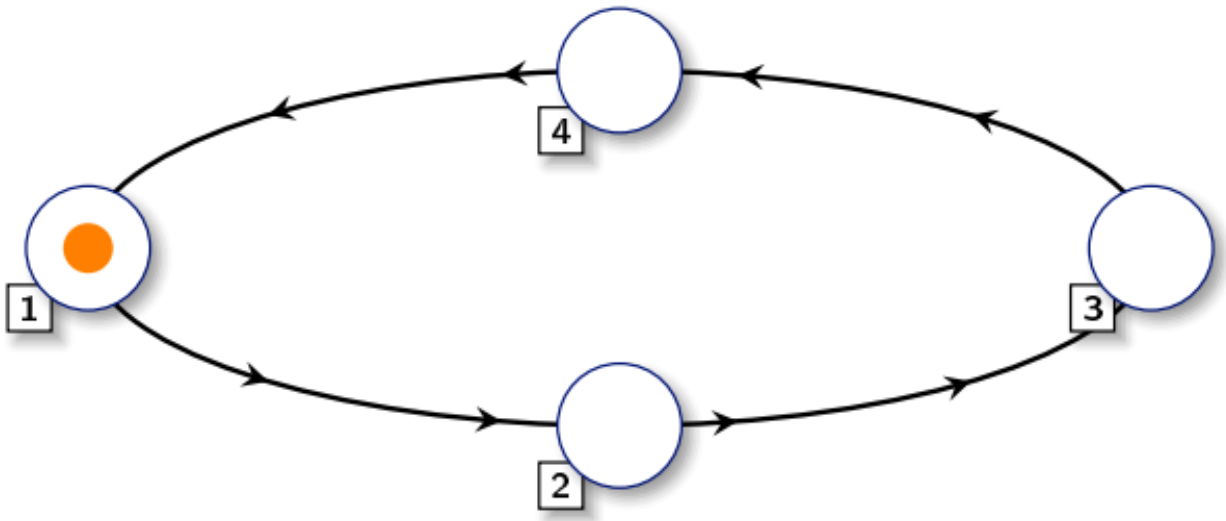

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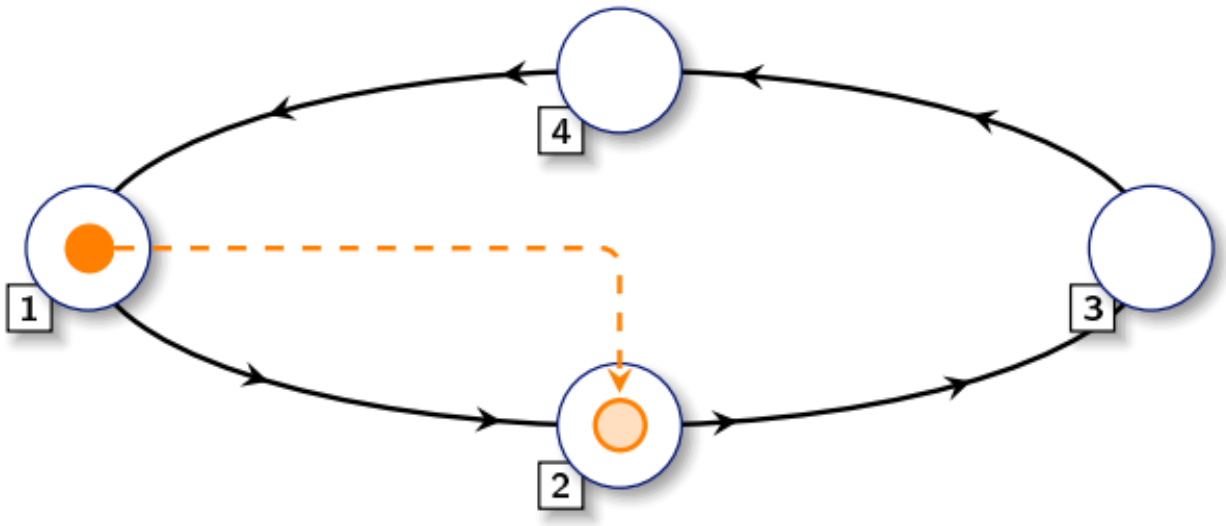

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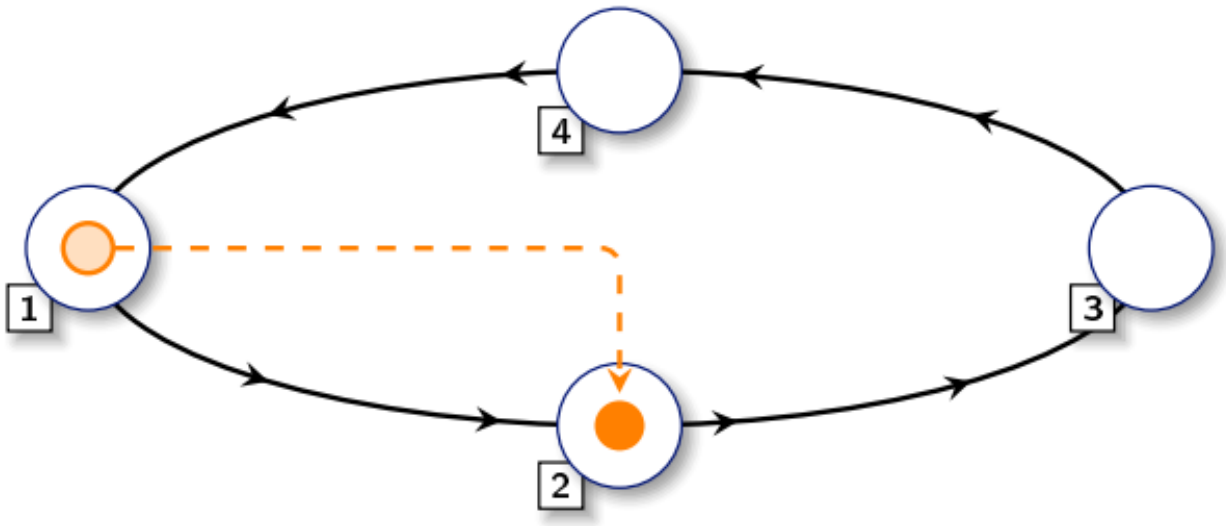

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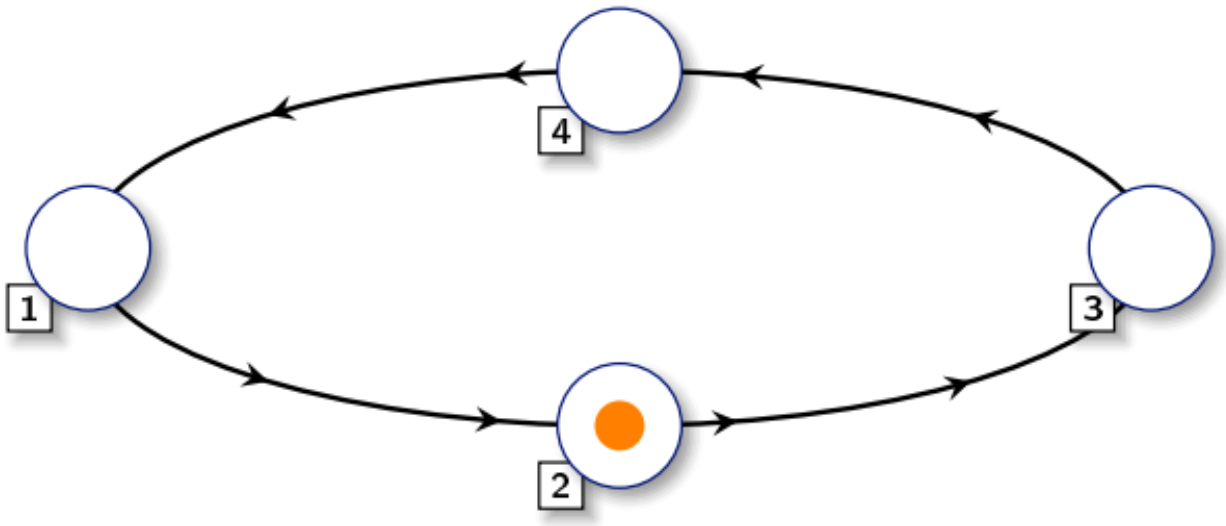

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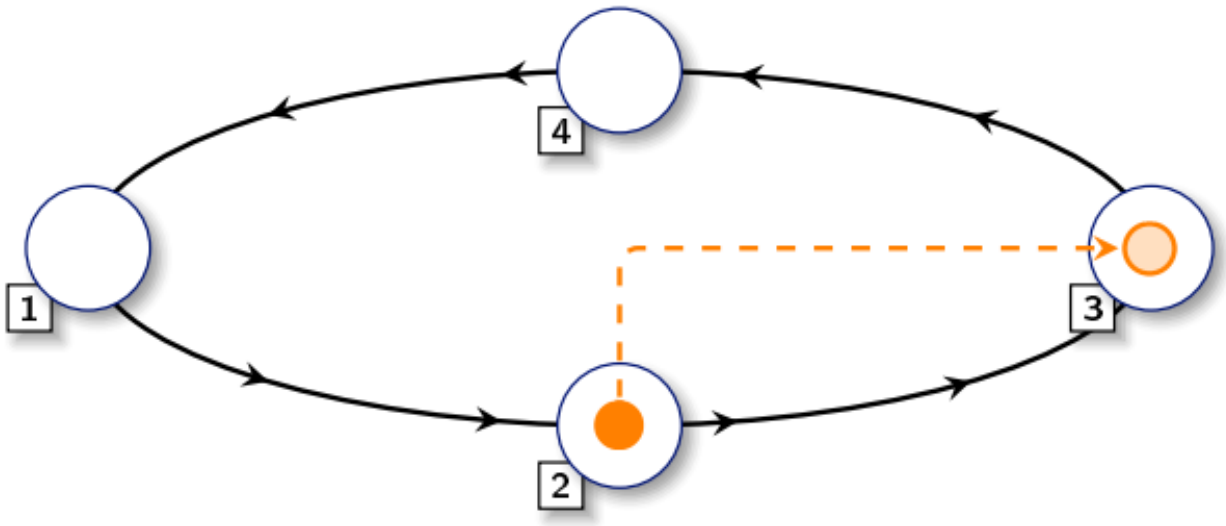

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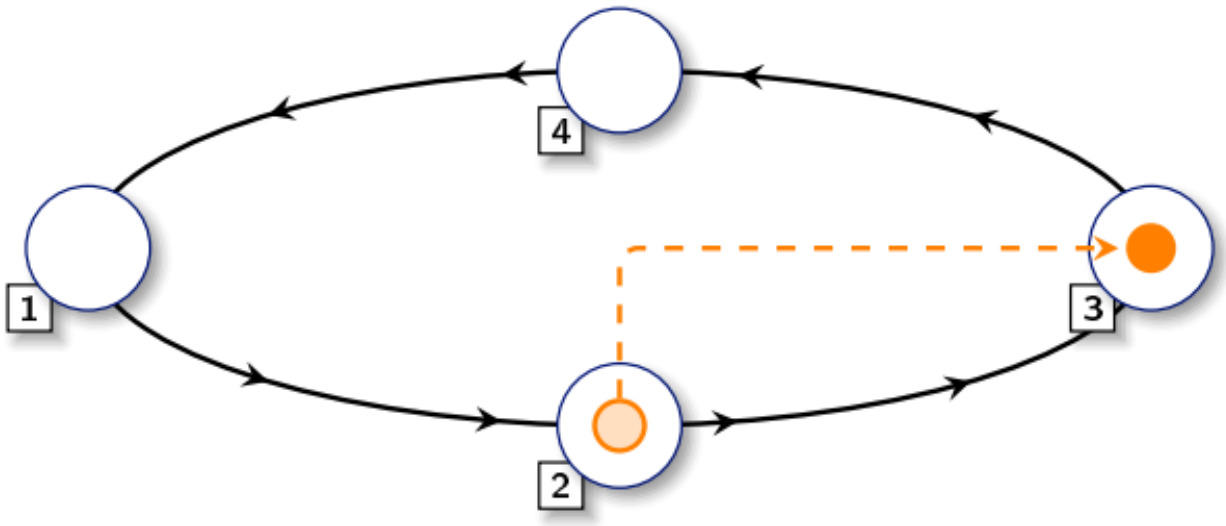

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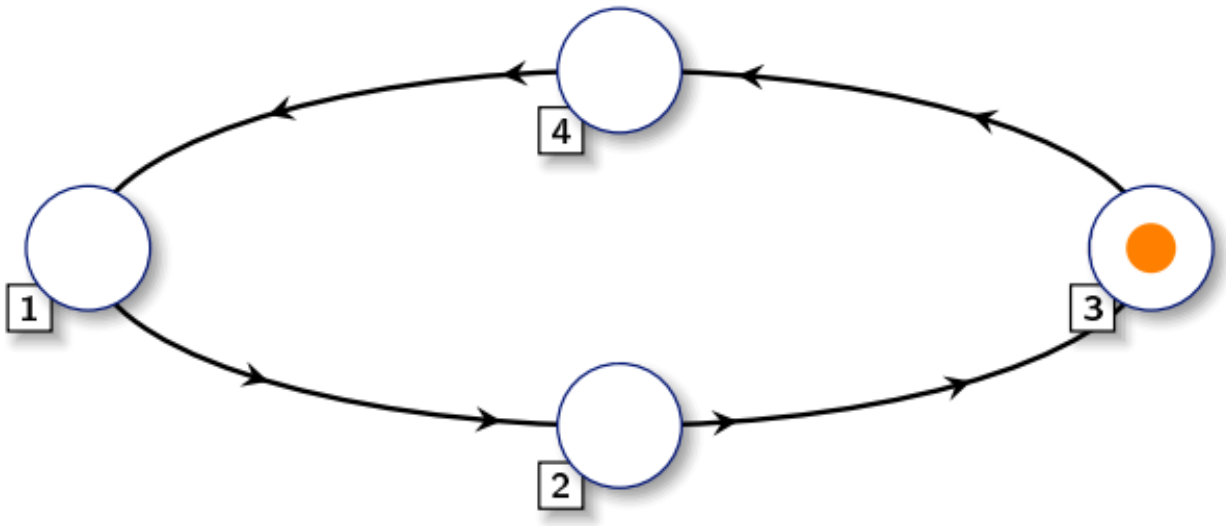

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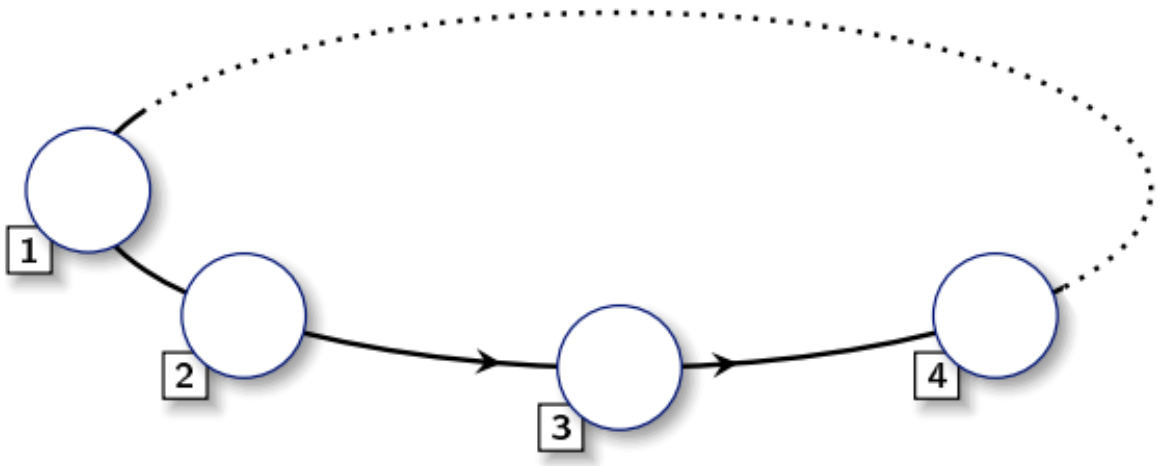


Configurations: $(0+1)^*$

Transitions: $\begin{bmatrix}0\\0\end{bmatrix}^*\begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}\begin{bmatrix}0\\0\end{bmatrix}^* + \begin{bmatrix}0\\1\end{bmatrix}\begin{bmatrix}0\\0\end{bmatrix}^*\begin{bmatrix}1\\0\end{bmatrix}$

Regular Model Checking

System model

A parameterised system is modelled by a tuple (I, T), where

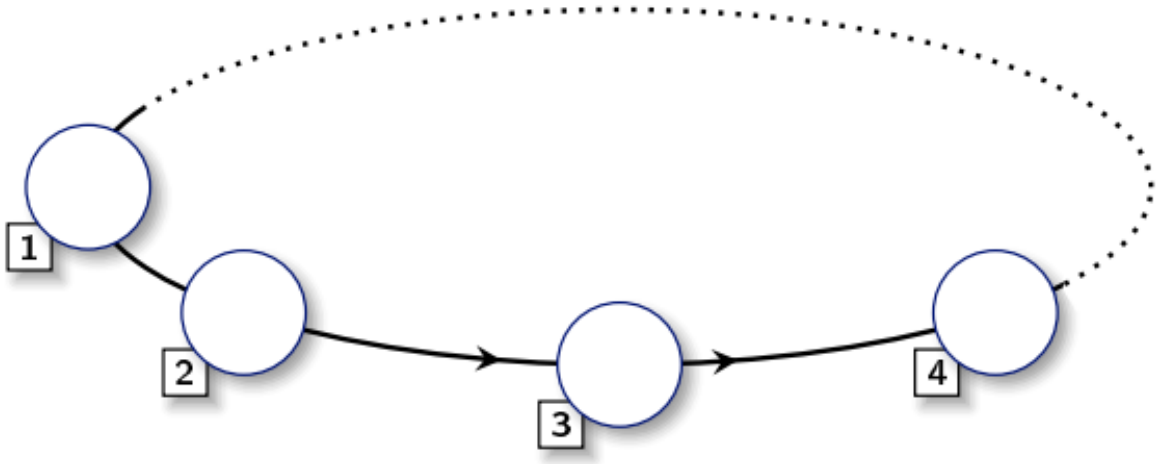
configurations are represented by words in Σ^*

$I \subseteq \Sigma^*$ is represented by a finite automaton

$T \subseteq \Sigma^* \times \Sigma^*$ is represented by a length-preserving transducer

$T^*(I) \equiv \bigcup_{n \geq 0} T^n(I)$ is called the reachability set of (I, T)

Regular Model Checking



I: 0^*10^*, T: $\begin{bmatrix}0\\0\end{bmatrix}^*\begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}\begin{bmatrix}0\\0\end{bmatrix}^* + \begin{bmatrix}0\\1\end{bmatrix}\begin{bmatrix}0\\0\end{bmatrix}^*\begin{bmatrix}1\\0\end{bmatrix}$, B: $0^*10^*10^*$

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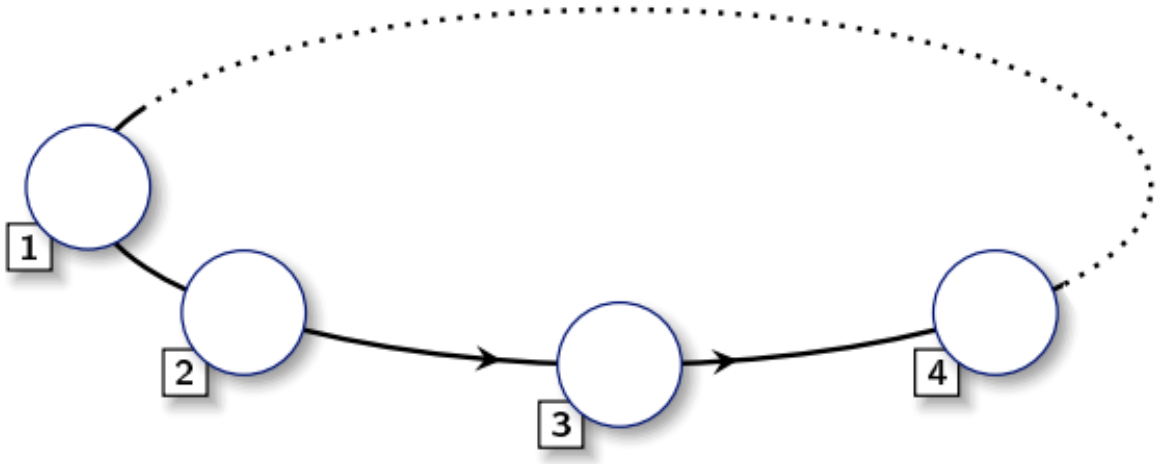

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Safety property: $T^*(I) \cap B = \emptyset$

Regular Model Checking

Safety verification

Given I, T, and B, does $T^*(I) \cap B = \emptyset$ hold?

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Proof rules

A regular set A is called a (regular) proof for safety iff

- $I \subseteq A$
- $A \cap B = \emptyset$
- $T(A) \subseteq A$

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We exploit these proof rules and the L* learning algorithm to synthesise a regular proof.

Learning Automata via Queries

L* learning algorithm

Proposed by Dana Angluin in 1987 to infer regular sets via querying.

To infer a regular set $\mathcal{R}$, L* makes two types of queries to an oracle:

- Membership query for a word w: Is w in $\mathcal{R}$?
- Equivalence query for a DFA $\mathcal{A}$: Is $L(\mathcal{A}) = \mathcal{R}$?

  If the answer is NO, L* will ask for a word $w \in L(\mathcal{A}) \ominus \mathcal{R}$.

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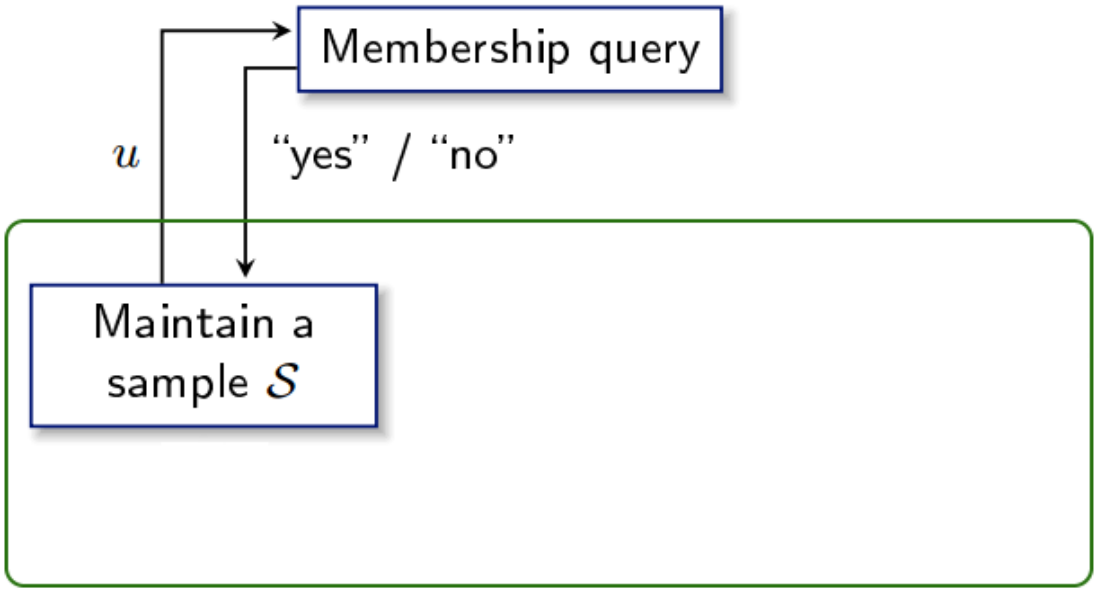
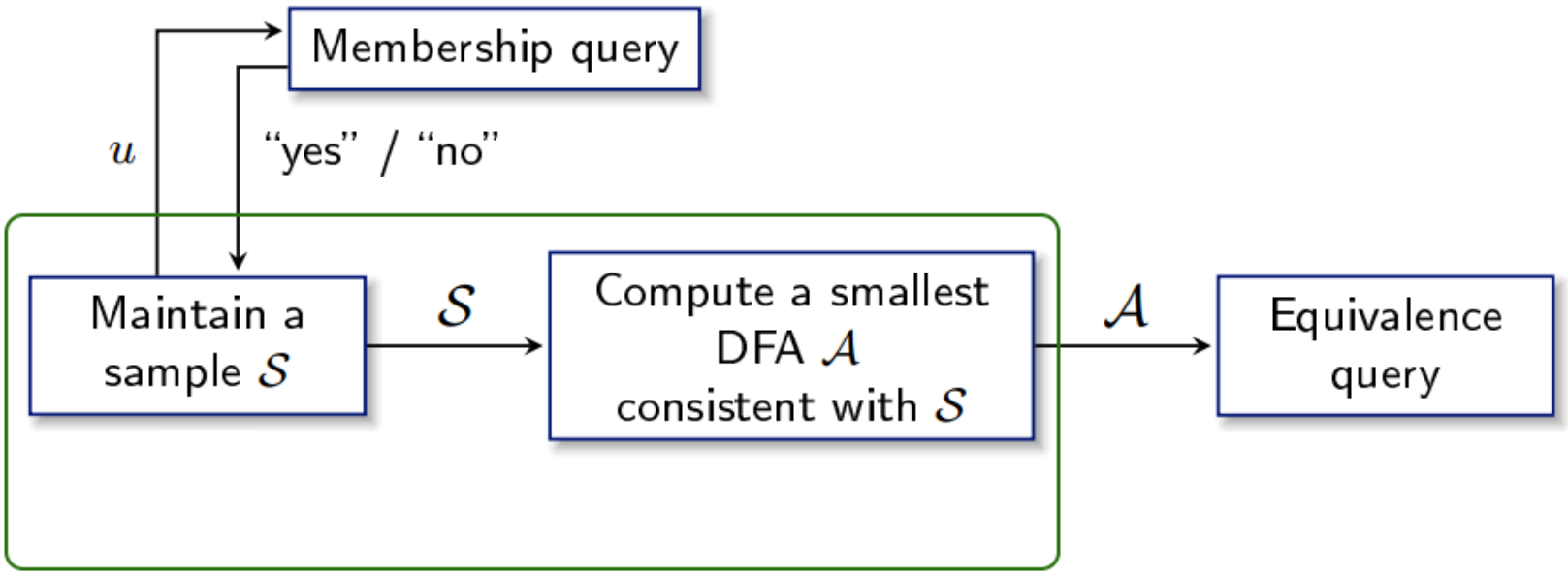
Guaranteed to learn a minimal DFA $\mathcal{A}$ for $\mathcal{R}$ with a polynomial number of queries (in the size of $\mathcal{A}$ and the returned words).

Learning Automata via Queries



Angluin's Automata Learning Algorithm

Learning Automata via Queries



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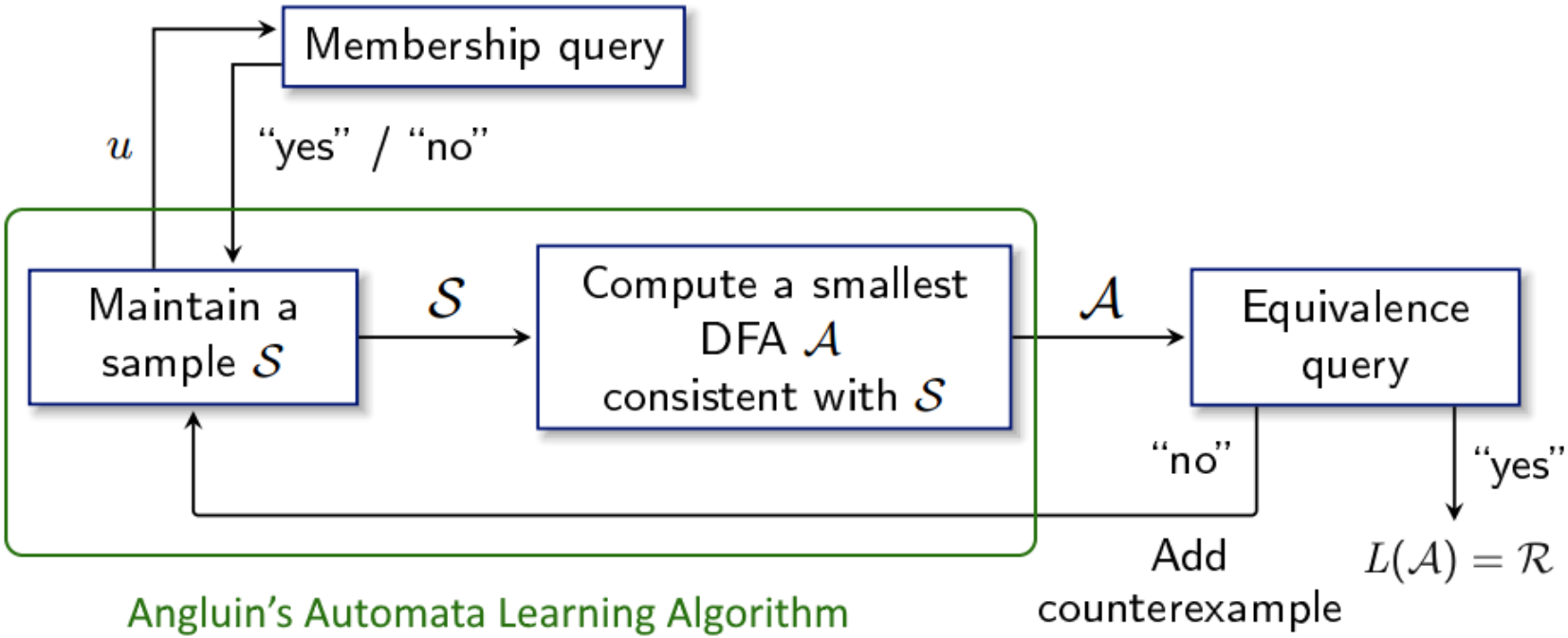

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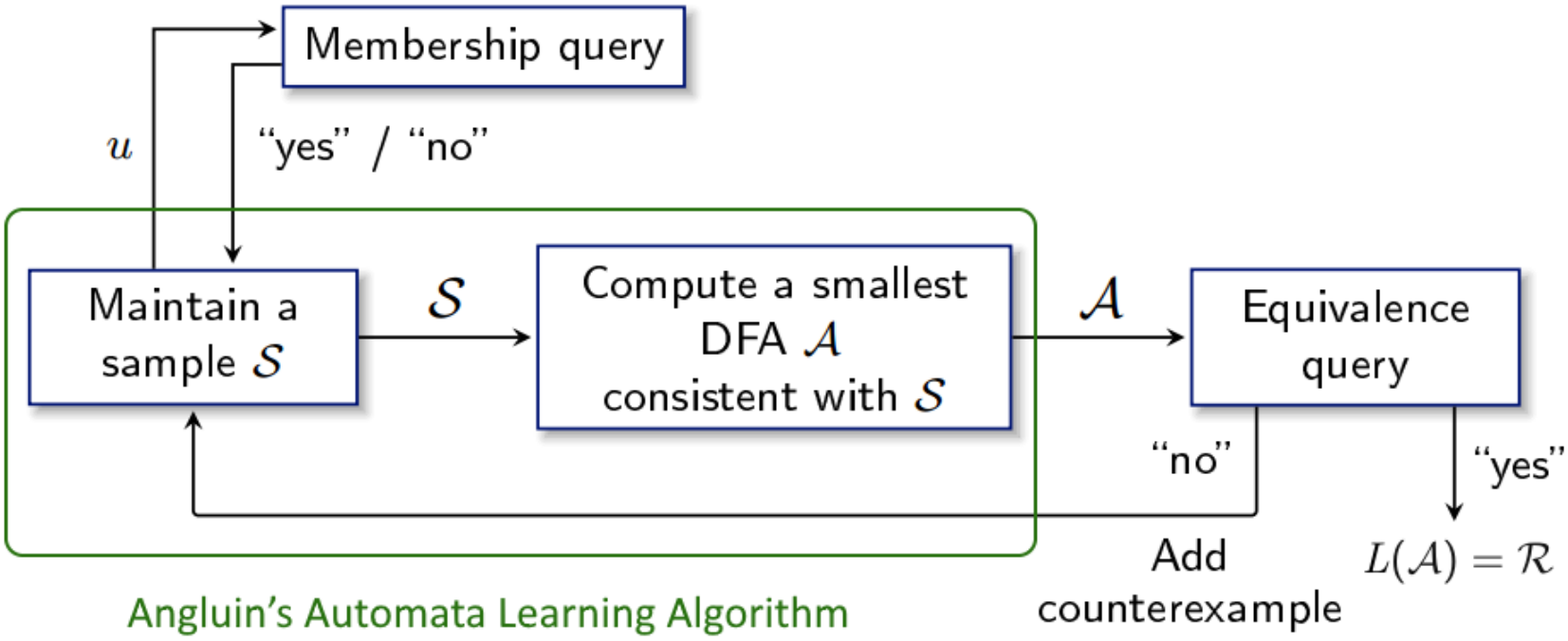
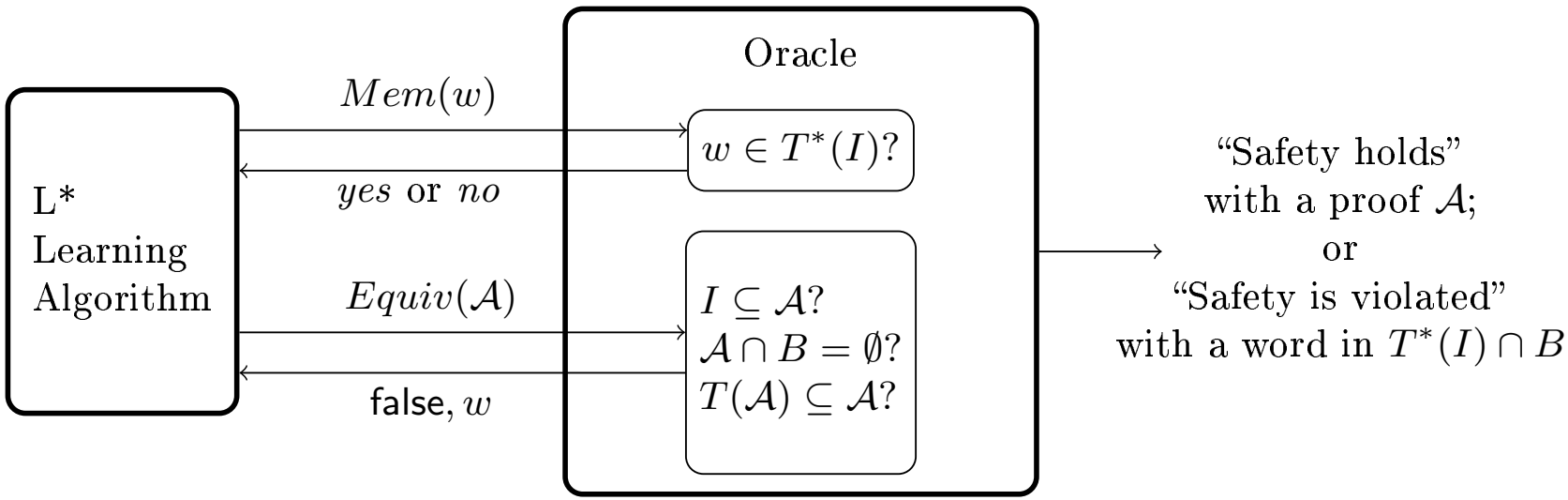


We propose an oracle for L* to learn a regular proof for safety.

An overview of Oracle



Comparisons with Other Methods

Methodology	Complete Subclass	Transition Relation
Learning-based synthesis[1]	$T^*(I)$ is regular	l.-p. / rational*
SAT-based refinement[2]	regular proof exists	rational
Widening / Accelerating[3]	unknown	length-preserving
Predicate abs. refinement[4]	unknown	rational

1. Chen et al.'17, Vardhan'04, Habermehl and Vojnar'05
2. Neider and Jansen'13, Lin and Rümmer'16
3. Nilsson'00, Legay'08
4. Bouajjani et al.'06

$\star$ Without termination guarantee [Vardhan, Habermehl and Vojnar]

Comparisons with Other Methods

RMC problems	Learning-based			SAT refinement			Widening			PAR
Name	Time	S_{inv}	T_{inv}	Time	S_{inv}	T_{inv}	Time	S_{inv}	T_{inv}	Time
Bakery	0.0s	6	18	0.5s	2	5	0.0s	6	11	0.0s
Burns	0.2s	8	96	1.1s	2	10	0.1s	7	38	0.0s
Szymanski	0.3s	43	473	1.6s	2	21	2.0s	51	102	0.1s
German	4.8s	14	8134	TO	-	-	TO	-	-	10s
Dijkstra	0.1s	9	378	1.7s	2	24	6.1s	8	83	0.3s
Dijkstra, ring	1.4s	22	264	0.9s	2	14	TO	-	-	0.1s
Dining Crypto.	0.1s	32	448	TO	-	-	TO	-	-	7.2s
Coffee Can	0.0s	3	18	0.2s	2	7	0.1s	6	13	0.0s
Herman, linear	0.0s	2	4	0.2s	2	4	0.0s	2	4	0.0s
Herman, ring	0.0s	2	4	0.4s	2	4	0.0s	2	4	0.0s
Israeli-Jalfon	0.0s	4	8	0.1s	2	4	0.0s	4	8	0.0s
Lehmann-Rabin	0.1s	8	48	0.5s	2	11	0.8s	19	105	0.0s
LR Dining Philo.	0.0s	4	16	0.2s	2	6	0.1s	7	18	0.0s
Mux Array	0.0s	5	30	0.4s	2	7	0.2s	4	14	0.0s
Res. Allocator	0.0s	5	15	0.0s	1	3	0.0s	4	9	0.0s
Kanban	TO	-	-	TO	-	-	TO	-	-	3.5s
Water Jugs	0.1s	24	264	TO	-	-	TO	-	-	0.0s

Timeout: 60 seconds

Summary

Automatic method for proving safety for parameterised systems

Regular model checking as symbolic framework

Automata learning to synthesise "regular" proofs

Simple but effective (50-line Java code based on existing learning and automata libraries)

Full paper can be found at FMCAD'17