Parity Objectives in Countable MDPs

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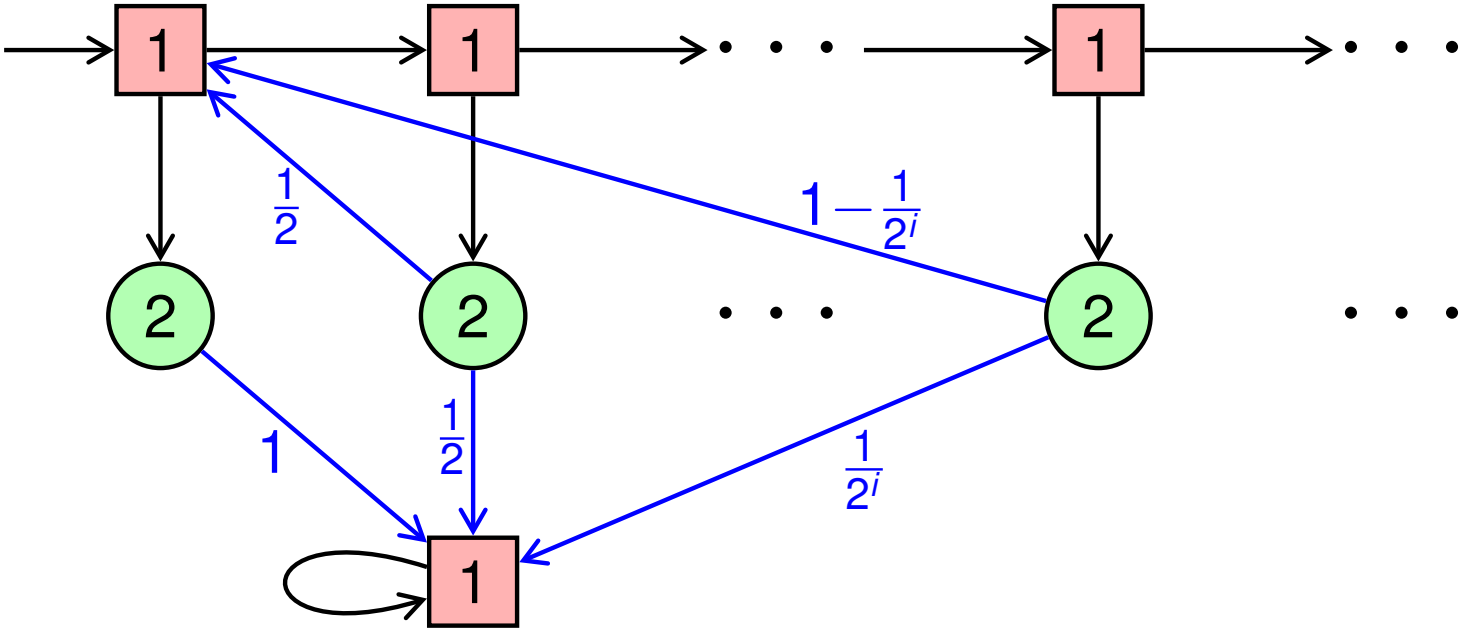
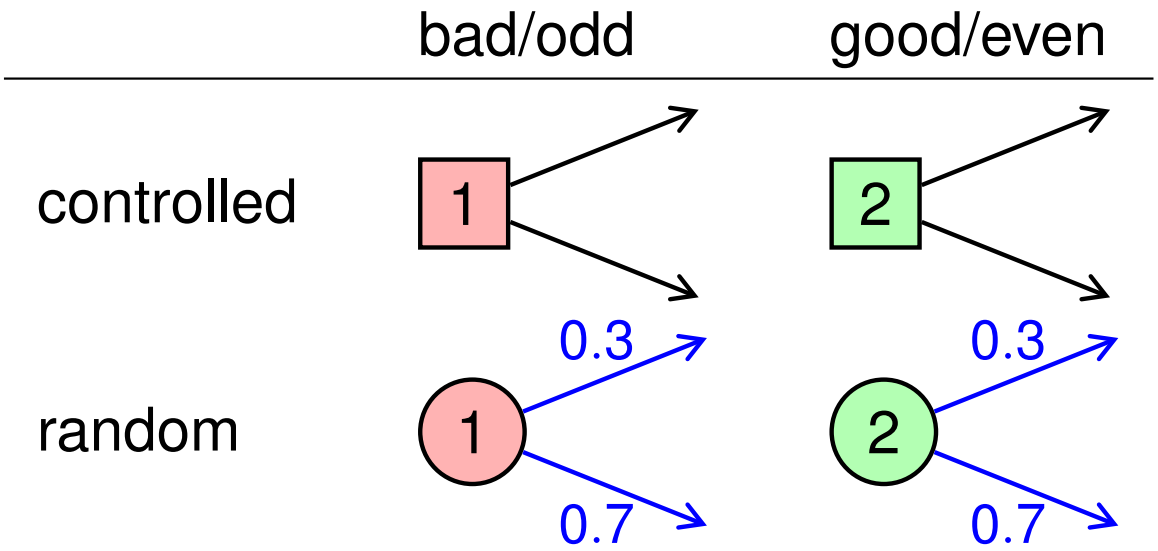
(joint work that appeared at LICS 2017)

Countable MDPs (with Finite Branching)





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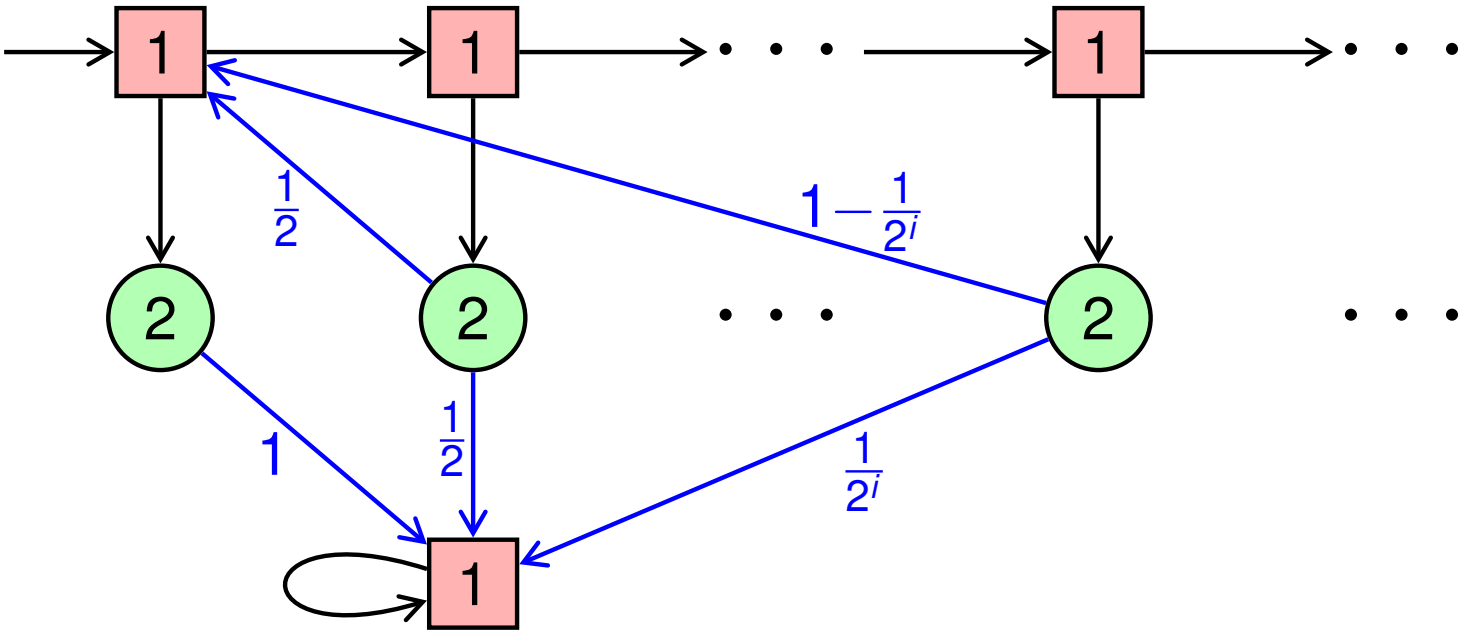
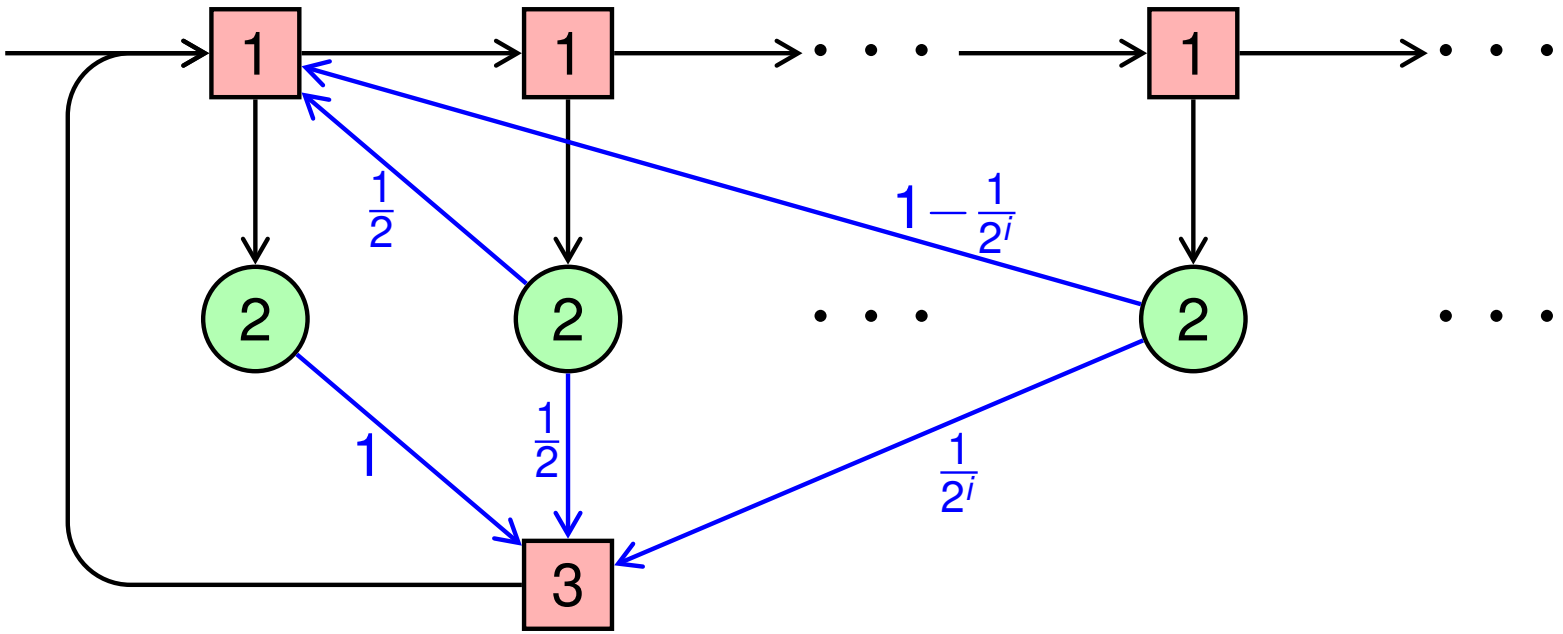


There is no almost-surely winning strategy.

$\sup_{\sigma} \Pr_{\sigma}(\text{Parity}) = 1$

All finite-memory strategies lose almost surely.

$\{1, 2, 3\}$-Parity



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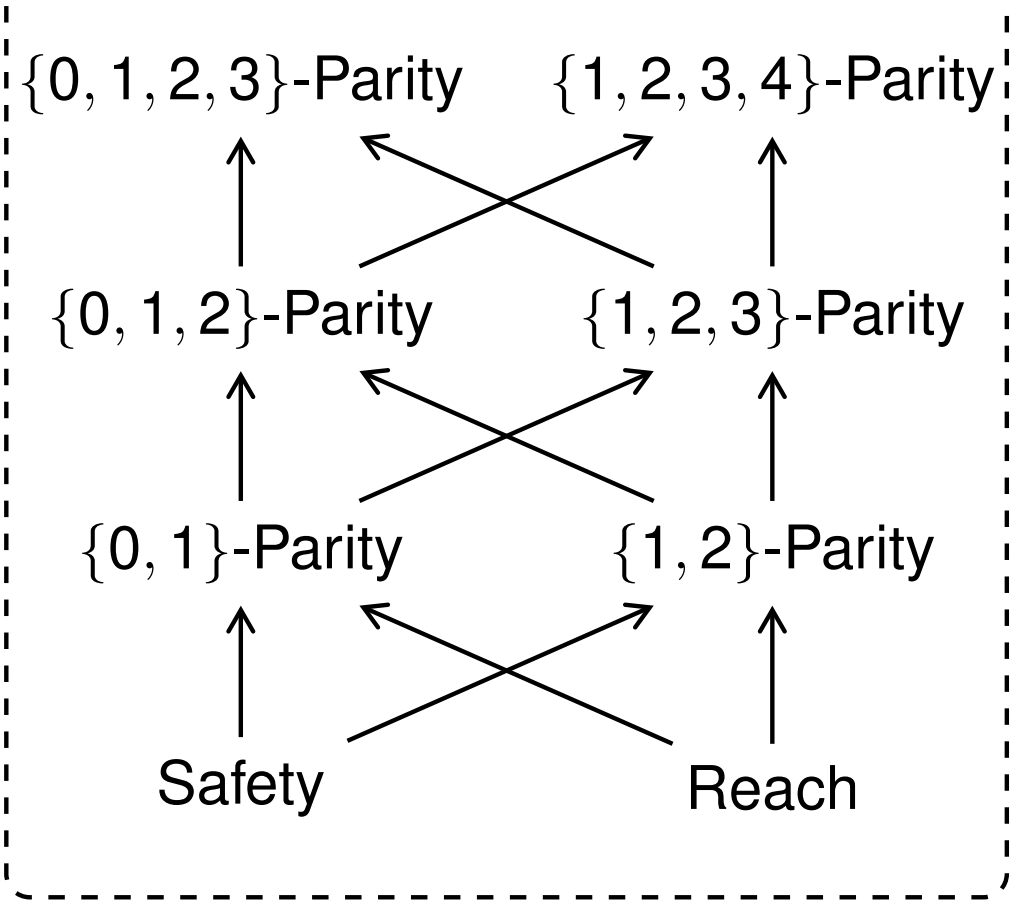
Our Results in the Mostowski Hierarchy



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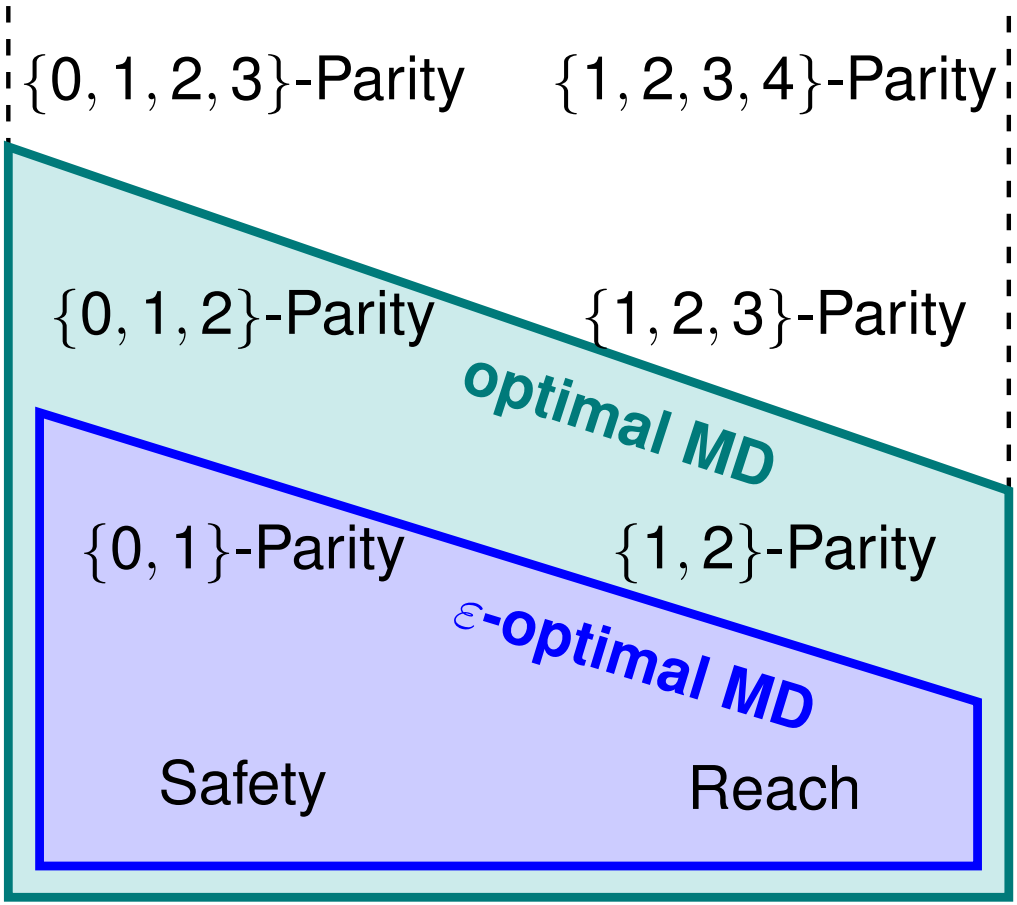

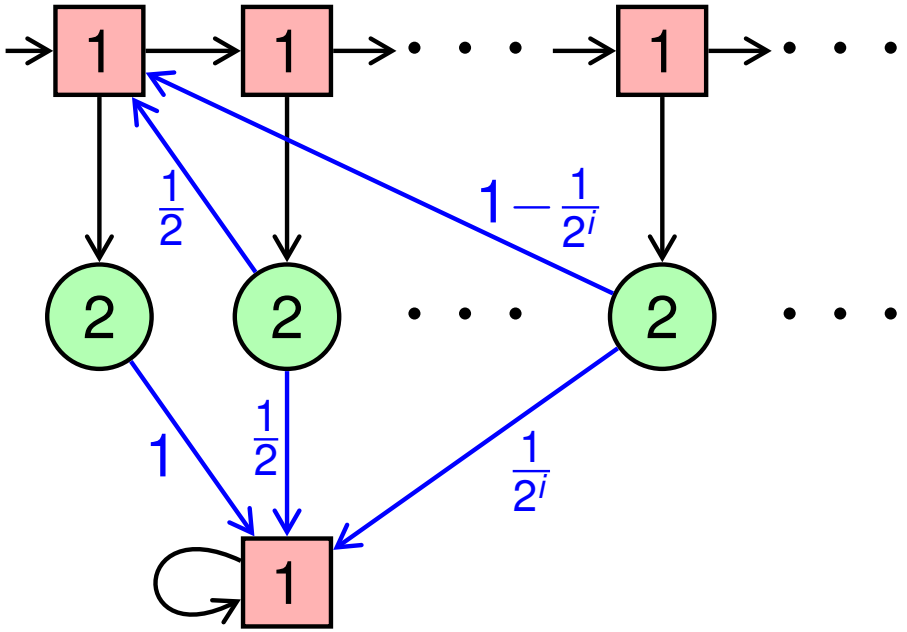

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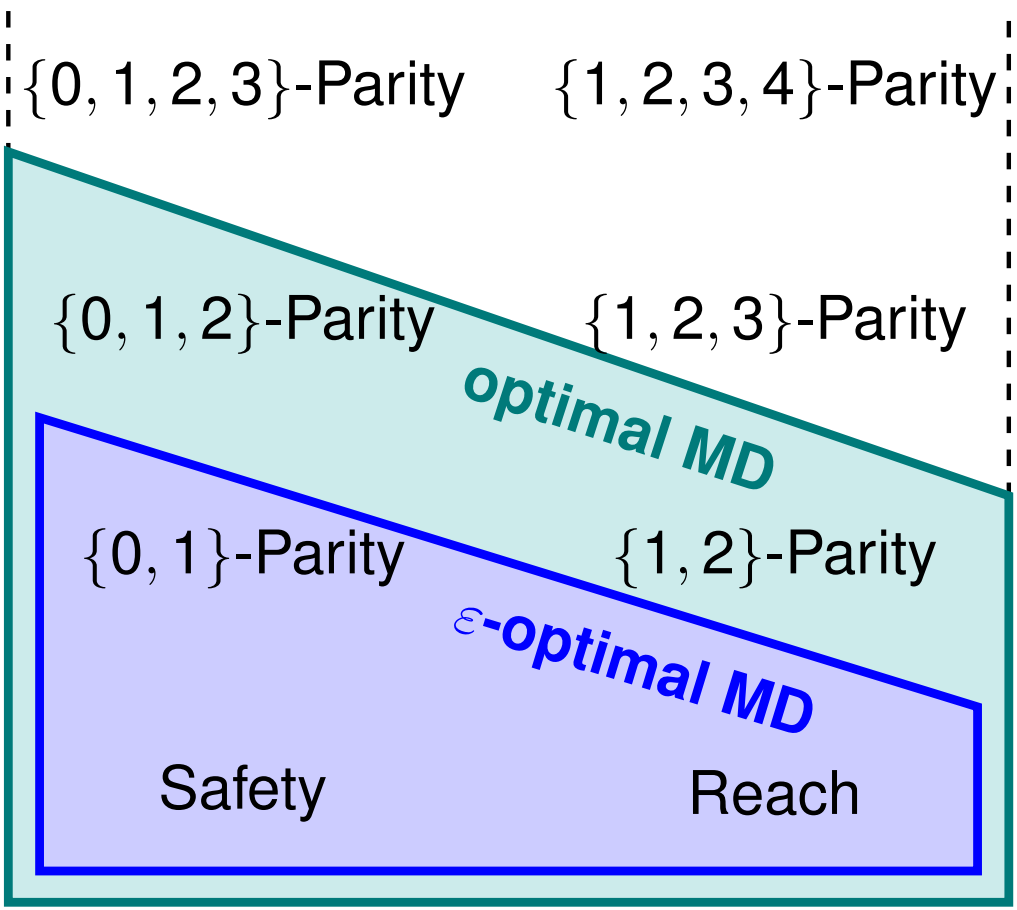

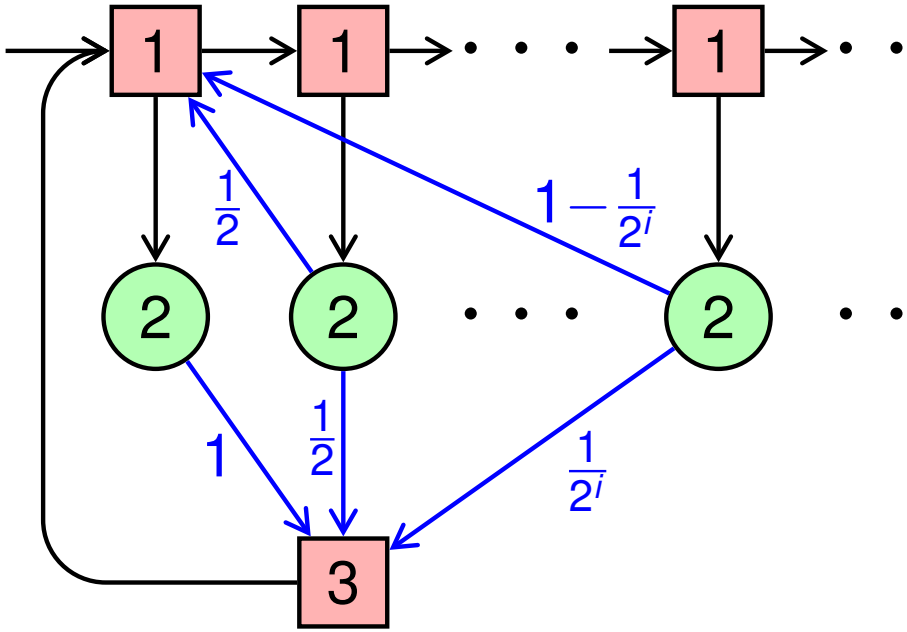

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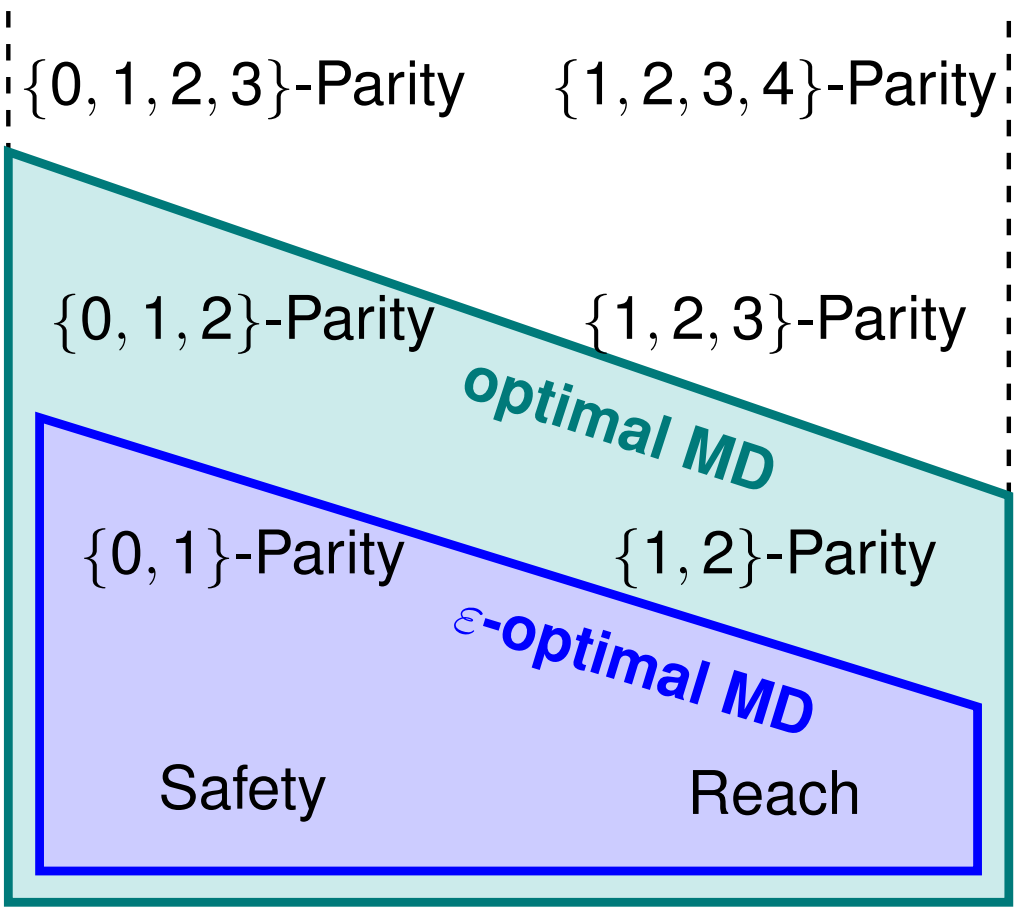

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Dichotomy between MD and infinite memory; contrast to finite MDPs

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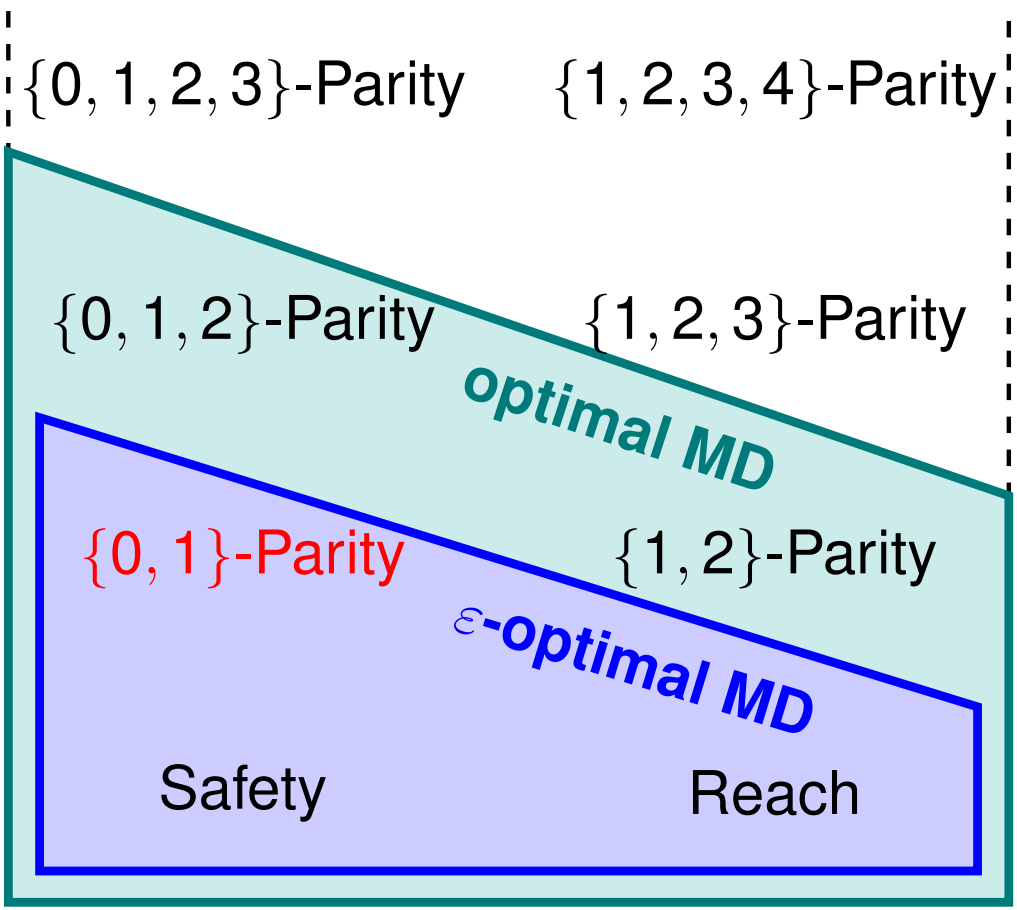

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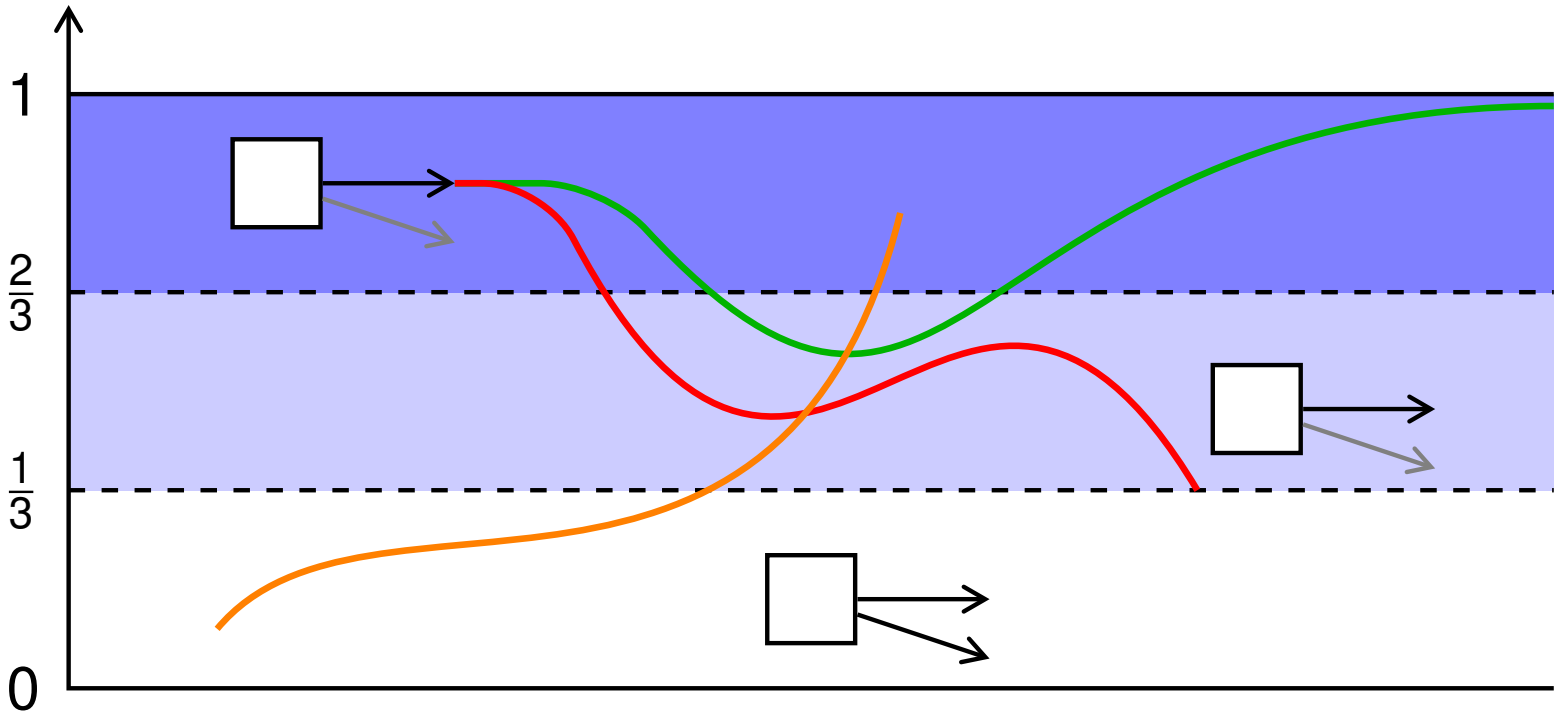
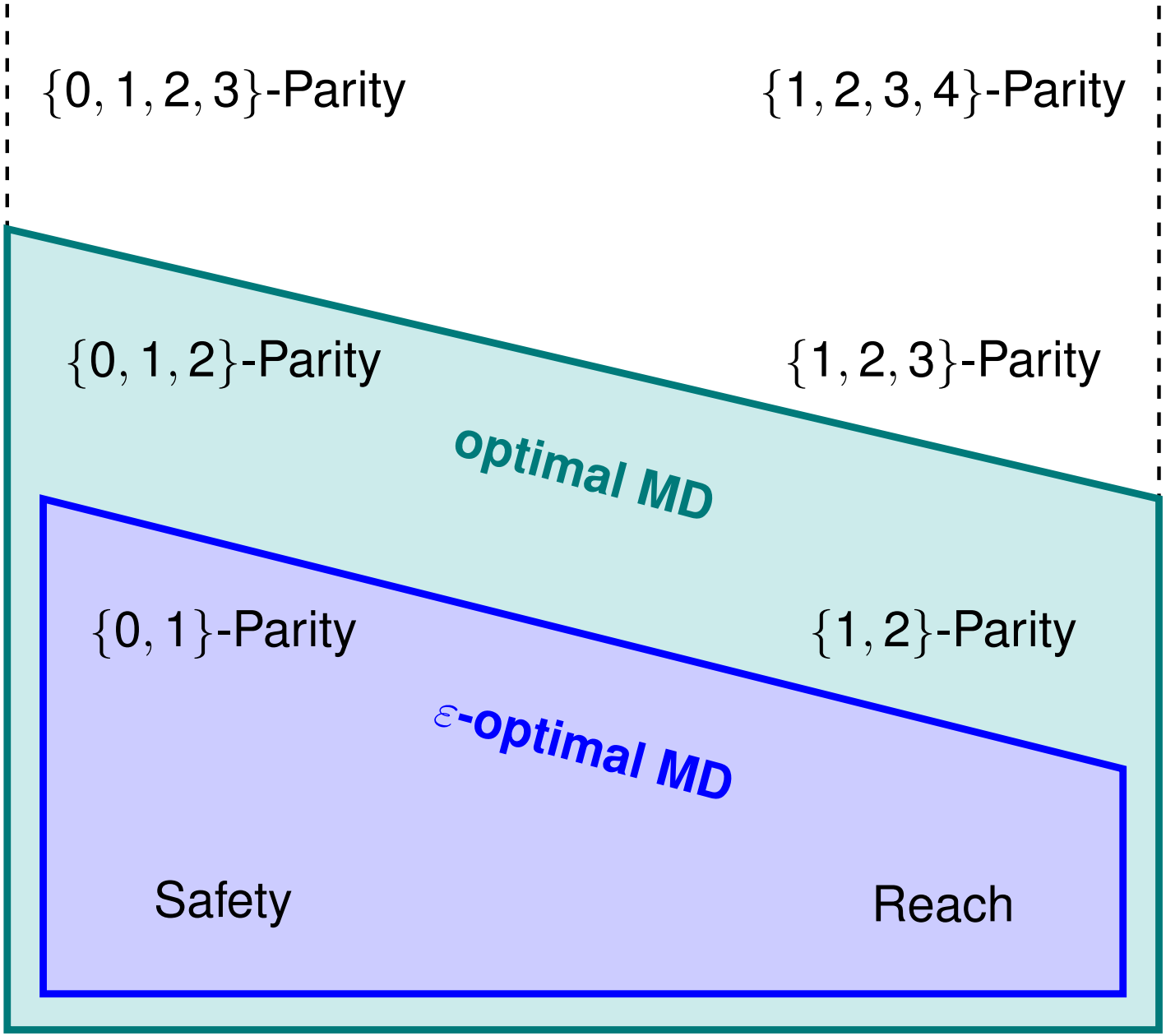
An Optimal MD-Strategy for Co-Büchi

$\max_\sigma \Pr_\sigma(\text{never see } \boxed{1} \text{ or } (1) \text{ again})$



0. Playing the safest action everywhere is not ok.
1. Fixing the safest action in the blue region is ok.
2. Once we are in dark blue : with prob $\geq \frac{1}{2}$ we stay in blue .
3. The a.s. winning strategy for 1. gets us in dark blue a.s.

When MD Suffices For Finitely Branching MDPs



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