

Learning to Prove Safety over Parameterised Concurrent Systems

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Parameterised concurrent systems

Infinite families \mathcal{F} of finite-state concurrent systems parameterised by the number of processes.

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In this talk, we will introduce a simple but effective heuristic to verify safety of parameterised systems based on [automata learning](#).

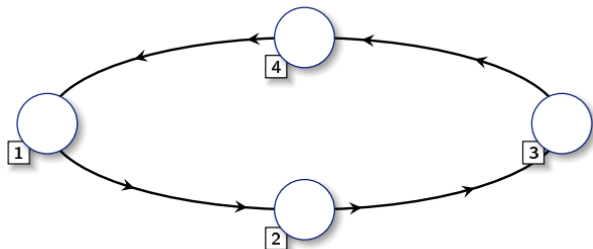
Modelling parameterised systems

Configurations: represented as finite words

Sets of configurations: represented as finite automata

Transition relation: represented as a transducer

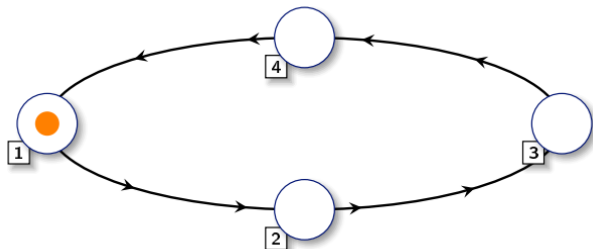
Token Ring Example



Configurations: 1000, 0100, 0010, 0001

Transitions: $\begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

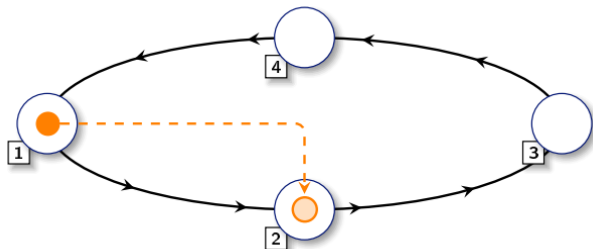
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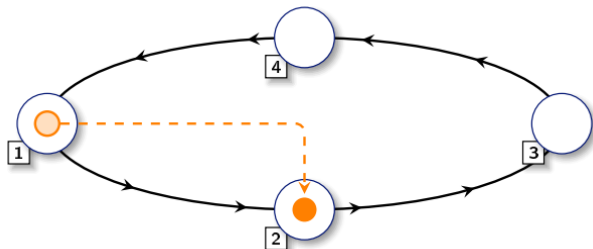
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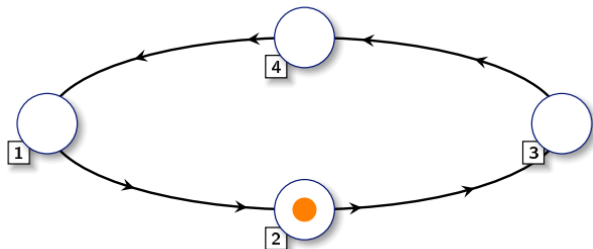
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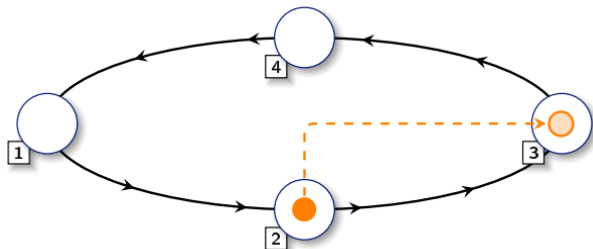
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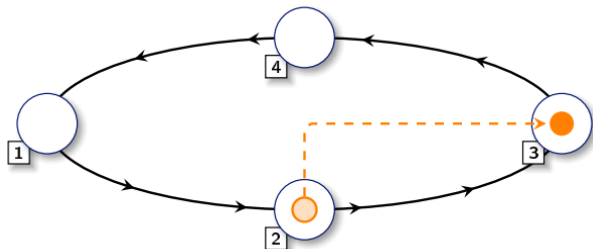
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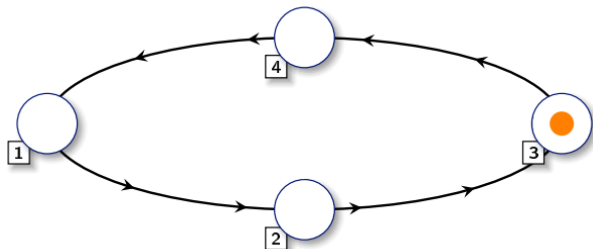
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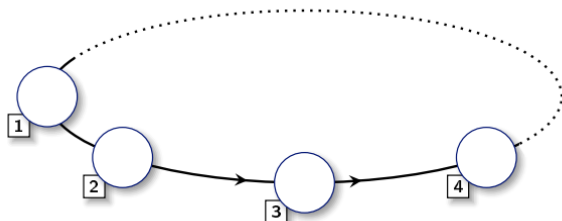
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Token Ring Example



Initial Configurations: $0^* 1 0^*$

Bad Configurations: $(0 + 1)^* 1 0^* 1 (0 + 1)^*$

Transitions: $\begin{bmatrix} 0 \\ 0 \end{bmatrix}^* \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix}^* + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix}^* \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Safety verification

Given I , T , and B , does $T^*(I) \cap B = \emptyset$ hold?

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Proof rules

A regular set A is called a (regular) **proof** for safety iff

- $I \subseteq A$
- $A \cap B = \emptyset$
- $T(A) \subseteq A$

Regular Model Checking

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Proof rules

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We exploit these proof rules and the **L* learning algorithm** to synthesise a regular proof.

Learning Automata via Queries

L* learning algorithm

Proposed by Dana Angluin in 1987 to infer regular sets via querying.

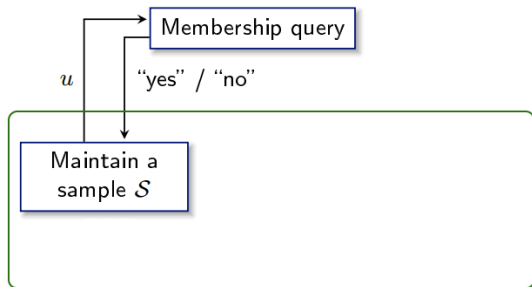
To infer a regular set \mathcal{R} , L* makes two types of queries to an oracle:

- **Membership query** for a word w : Is w in \mathcal{R} ?
- **Equivalence query** for a DFA \mathcal{A} : Is $L(\mathcal{A}) = \mathcal{R}$?

If the answer is NO, L* will ask for a word $w \in L(\mathcal{A}) \ominus \mathcal{R}$.

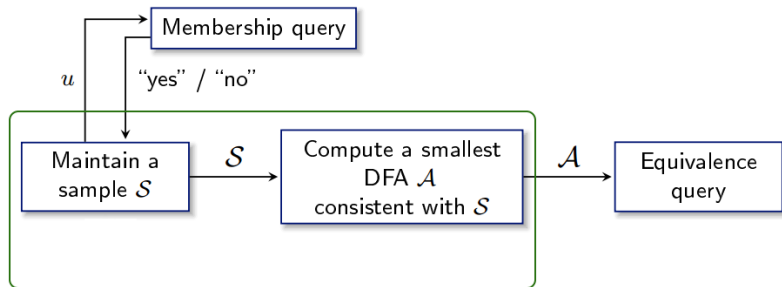
Guaranteed to learn a minimal DFA \mathcal{A} for \mathcal{R} with a polynomial number of queries (in the size of \mathcal{A} and the returned words).

Learning Automata via Queries



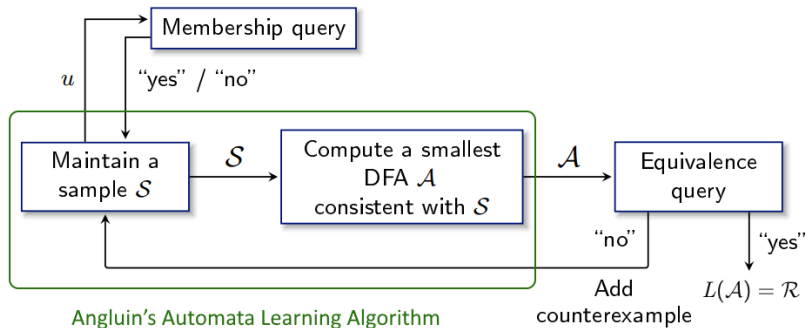
Angluin's Automata Learning Algorithm

Learning Automata via Queries

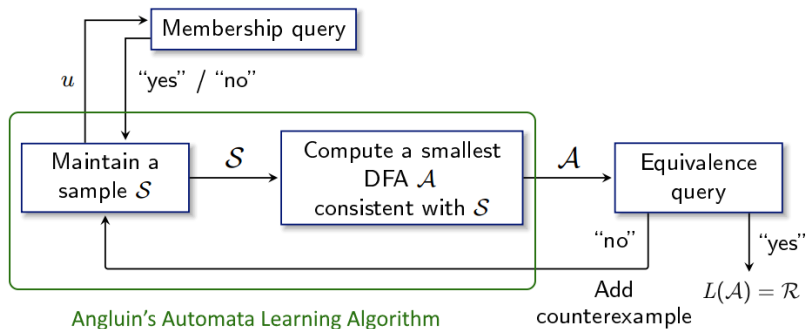


Angluin's Automata Learning Algorithm

Learning Automata via Queries

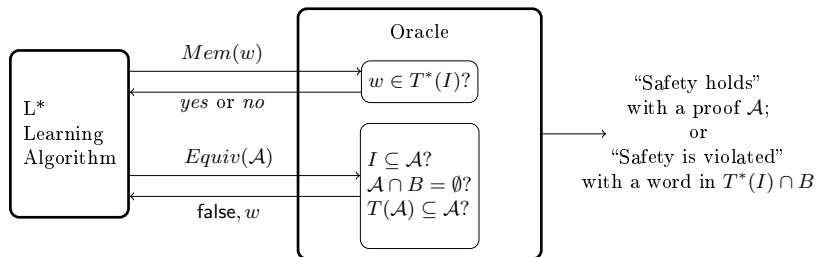


Learning Automata via Queries



We propose an oracle for L^* to learn a regular proof for safety.

An overview of Oracle



Comparisons with Other Methods

Methodology	Complete Subclass	Transition Relation
Learning-based synthesis ¹	$T^*(I)$ is regular	l.-p. / rational [*]
SAT-based refinement ²	regular proof exists	rational
Widening / Accelerating ³	unknown	length-preserving
Predicate abs. refinement ⁴	unknown	rational

1. Chen et al.'17, Vardhan'04, Habermehl and Vojnar'05
2. Neider and Jansen'13, Lin and Rümmer'16
3. Nilsson'00, Legay'08
4. Bouajjani et al.'06

★ Without termination guarantee [Vardhan, Habermehl and Vojnar]

Comparisons with Other Methods

RMC problems	Learning-based			SAT refinement			Widening			PAR
	Time	S _{inv}	T _{inv}	Time	S _{inv}	T _{inv}	Time	S _{inv}	T _{inv}	Time
Bakery	0.0s	6	18	0.5s	2	5	0.0s	6	11	0.0s
Burns	0.2s	8	96	1.1s	2	10	0.1s	7	38	0.0s
Szymanski	0.3s	43	473	1.6s	2	21	2.0s	51	102	0.1s
German	4.8s	14	8134	TO	-	-	TO	-	-	10s
Dijkstra	0.1s	9	378	1.7s	2	24	6.1s	8	83	0.3s
Dijkstra, ring	1.4s	22	264	0.9s	2	14	TO	-	-	0.1s
Dining Crypto.	0.1s	32	448	TO	-	-	TO	-	-	7.2s
Coffee Can	0.0s	3	18	0.2s	2	7	0.1s	6	13	0.0s
Herman, linear	0.0s	2	4	0.2s	2	4	0.0s	2	4	0.0s
Herman, ring	0.0s	2	4	0.4s	2	4	0.0s	2	4	0.0s
Israeli-Jalfon	0.0s	4	8	0.1s	2	4	0.0s	4	8	0.0s
Lehmann-Rabin	0.1s	8	48	0.5s	2	11	0.8s	19	105	0.0s
LR Dining Philo.	0.0s	4	16	0.2s	2	6	0.1s	7	18	0.0s
Mux Array	0.0s	5	30	0.4s	2	7	0.2s	4	14	0.0s
Res. Allocator	0.0s	5	15	0.0s	1	3	0.0s	4	9	0.0s
Kanban	TO	-	-	TO	-	-	TO	-	-	3.5s
Water Jugs	0.1s	24	264	TO	-	-	TO	-	-	0.0s

Timeout: 60 seconds

Regular model checking as symbolic framework

Automata learning to synthesise “regular” proofs

Simple but effective (50-line Java code based on existing learning and automata libraries)

Full paper can be found at FMCAD'17