

Universality of Partially Ordered NFAs

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Universality

Universality

Input: an automaton \mathcal{A} over Σ

Output: **Yes** if and only if $\mathcal{L}(\mathcal{A}) = \Sigma^*$

Why universality?

↪ Lower bound for **inclusion** and **equivalence**

Input: Two automata \mathcal{A} and \mathcal{B}

Inclusion: Is $\mathcal{L}(\mathcal{A}) \subseteq \mathcal{L}(\mathcal{B})$?

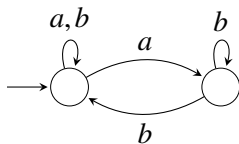
Equivalence: Is $\mathcal{L}(\mathcal{A}) = \mathcal{L}(\mathcal{B})$?

Partially Ordered NFAs

Motivation

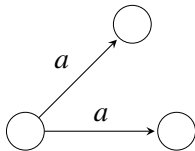
Motivation

NFAs as models for Discrete-Event Systems (DES)



Motivation

The simplest DES \rightsquigarrow acyclic NFA



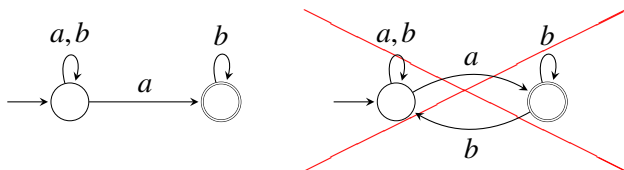
Motivation

The simplest **deadlock-free** DES \rightsquigarrow partially ordered NFA?

Motivation

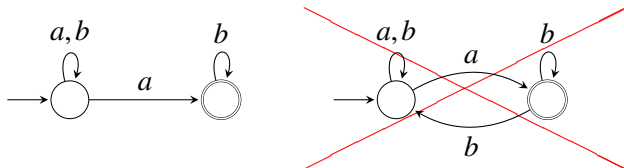
The simplest **deadlock-free** DES \rightsquigarrow partially ordered NFA?

Partially ordered – acyclic, but with self-loops



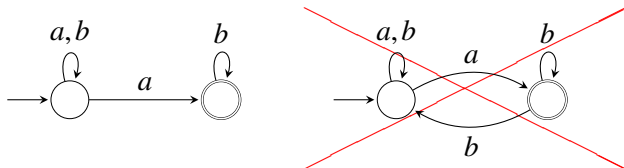
Partially Ordered NFAs (poNFAs)

Partially ordered – acyclic, but with self-loops



Partially Ordered NFAs (poNFAs)

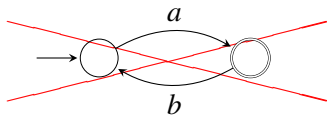
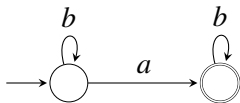
Partially ordered – acyclic, but with self-loops



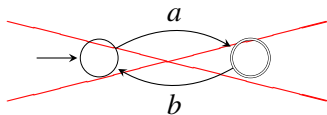
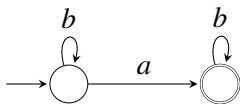
Expressive power:

- ▶ Level $\frac{3}{2}$ of the Straubing-Thérien hierarchy
- ▶ Σ_2 of the quantifier alternation hierarchy of FO[<]
- ▶ Alphabetic Pattern Constrains

Partially Ordered DFAs (poDFAs)



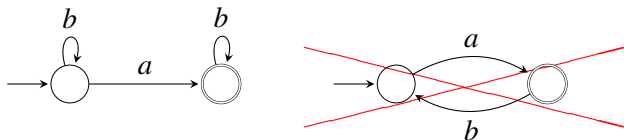
Partially Ordered DFAs (poDFAs)



Expressive power:

- ▶ \mathcal{R} -trivial languages

Partially Ordered DFAs (poDFAs)



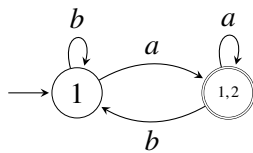
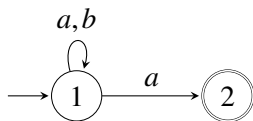
Expressive power:

- ▶ \mathcal{R} -trivial languages

poNFAs vs poDFAs:

- ▶ Determinization of poNFA may contain a cycle – not a poDFA
- ▶ poNFAs can encode cycles

Example

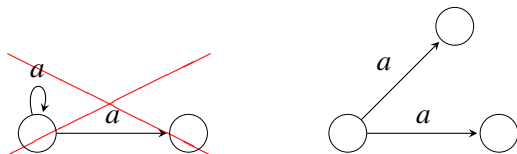


Self-loop deterministic poNFAs

Self-loop deterministic poNFAs (rpoNFAs)

Definition

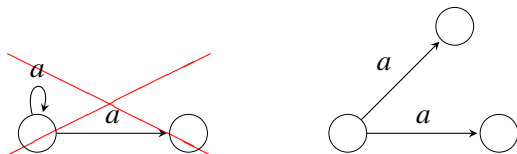
An **rpoNFA** is a poNFA such that, for every state q and symbol a , if $q \in \delta(q, a)$ then $\delta(q, a) = \{q\}$.



Self-loop deterministic poNFAs (rpoNFAs)

Definition

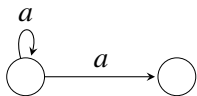
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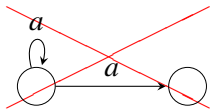
Expressive power:

- ▶ \mathcal{R} -trivial languages
- ▶ \rightsquigarrow equivalent to poDFAs

poNFAs vs. rpoNFAs



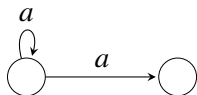
\rightsquigarrow unbounded nondeterminism



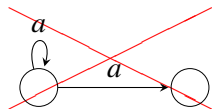
\rightsquigarrow bounded nondeterminism

Deciding universality

Deciding universality of poNFAs (rpoNFAs)



\rightsquigarrow PSPACE-complete ($|\Sigma| \geq 2$)



\rightsquigarrow $\begin{cases} \text{coNP-complete} & |\Sigma| = k \geq 2 \\ \text{PSPACE-complete} & \text{arbitrary} \end{cases}$

Results – Thank you!

- ▶ Complexity of universality

	Unary alph.	Fixed alph.	Arbitrary alph.
DFA	L-c	NL-c	NL-c
rpoNFA	NL-c	coNP-c	PSPACE-c
poNFA	NL-c	PSPACE-c	PSPACE-c
NFA	coNP-c	PSPACE-c	PSPACE-c

- ▶ rpoNFA \rightsquigarrow DRE-definable (expressions in schema langs for XML data)