# Markov Decision Processes with Energy-Parity Objectives

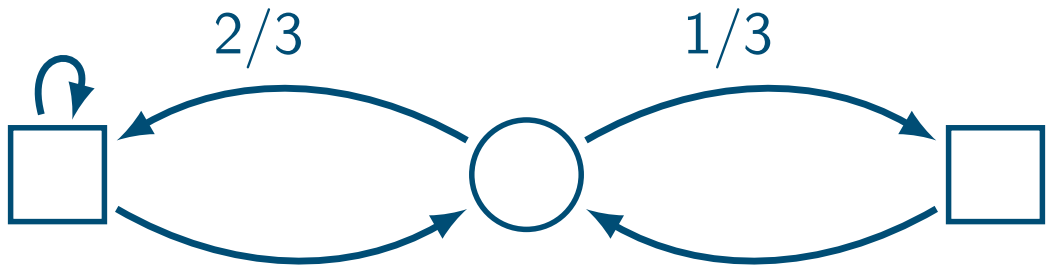
Richard Mayr, Patrick Totzke,
Sven Schewe, Dominik Wojtczak

Edinburgh/Liverpool, UK

LICS'17

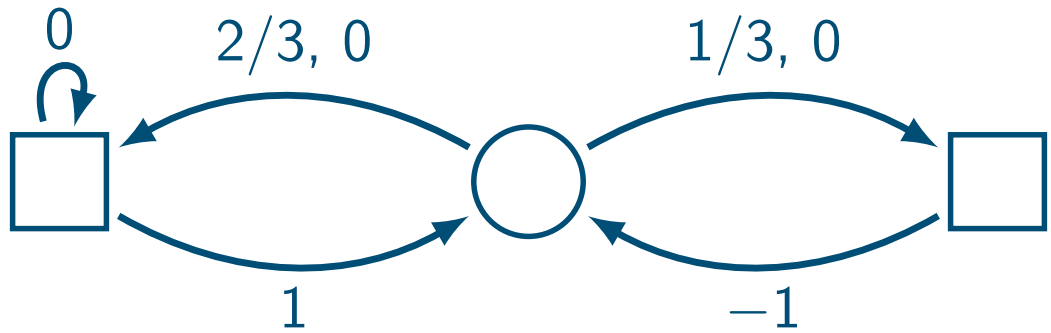
# MDPs with Energy-Parity Objectives

Finite graphs, partitioned into *controlled* □ and *random* ○ states;
A prob. dist. over successors for every random state.



# MDPs with Energy-Parity Objectives

Finite graphs, partitioned into *controlled* □ and *random* ○ states;
A prob. dist. over successors for every random state.

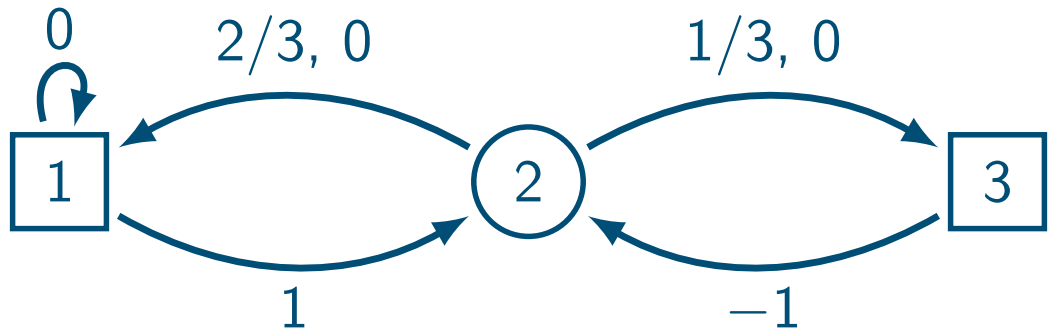



ENERGY

$\forall n \in \mathbb{N}.\ \sum_{i=0}^{n} \mathsf{cost}(e_i) \geq 0$

# MDPs with Energy-Parity Objectives

Finite graphs, partitioned into *controlled* □ and *random* ○ states;
A prob. dist. over successors for every random state.




ENERGY

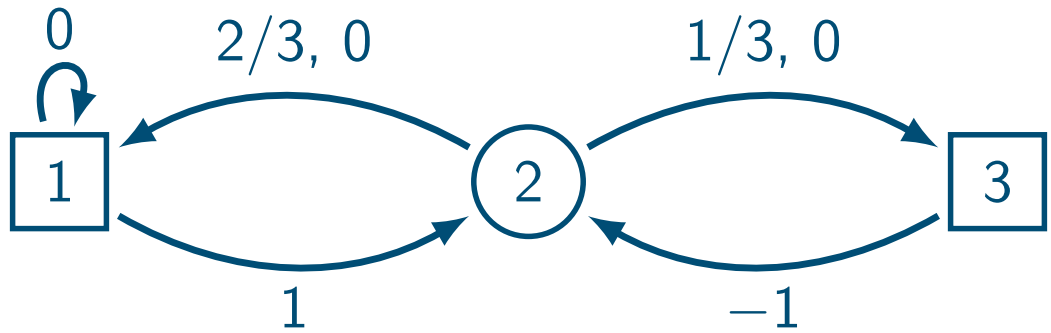
$\forall n \in \mathbb{N}.\ \sum_{i=0}^{n} \mathsf{cost}(e_i) \geq 0$

PARITY

maximal colour visited infinitely often is even.

# MDPs with Energy-Parity Objectives

Finite graphs, partitioned into *controlled* □ and *random* ○ states;
A prob. dist. over successors for every random state.




## ENERGY

$\forall n \in \mathbb{N}.\ \sum_{i=0}^{n} \mathrm{cost}(e_i) \geq 0$

## PARITY

maximal colour visited infinitely often is even.

## Positive Mean Payoff

$\lim_{n\to\infty} \sum_{i=0}^{n} \mathrm{cost}(e_i)/n > 0$

# Results on almost-sure problems for finite MDPs

ENERGY

# Results on almost-sure problems for finite MDPs

**ENERGY**

- FD determined
- NP $\cap$ coNP
- pseudo P

# Results on almost-sure problems for finite MDPs

**ENERGY**

- FD determined
- NP $\cap$ coNP
- pseudo P

**Positive Mean-Payoff**

# Results on almost-sure problems for finite MDPs

**ENERGY**

- FD determined
- NP $\cap$ coNP
- pseudo P

**Positive Mean-Payoff**

- MD determined
- P-time

# Results on almost-sure problems for finite MDPs

**ENERGY**

- FD determined
- NP ∩ coNP
- pseudo P

**Positive Mean-Payoff**

- MD determined
- P-time

**PARITY**

# Results on almost-sure problems for finite MDPs

**ENERGY**

- FD determined
- NP $\cap$ coNP
- pseudo P

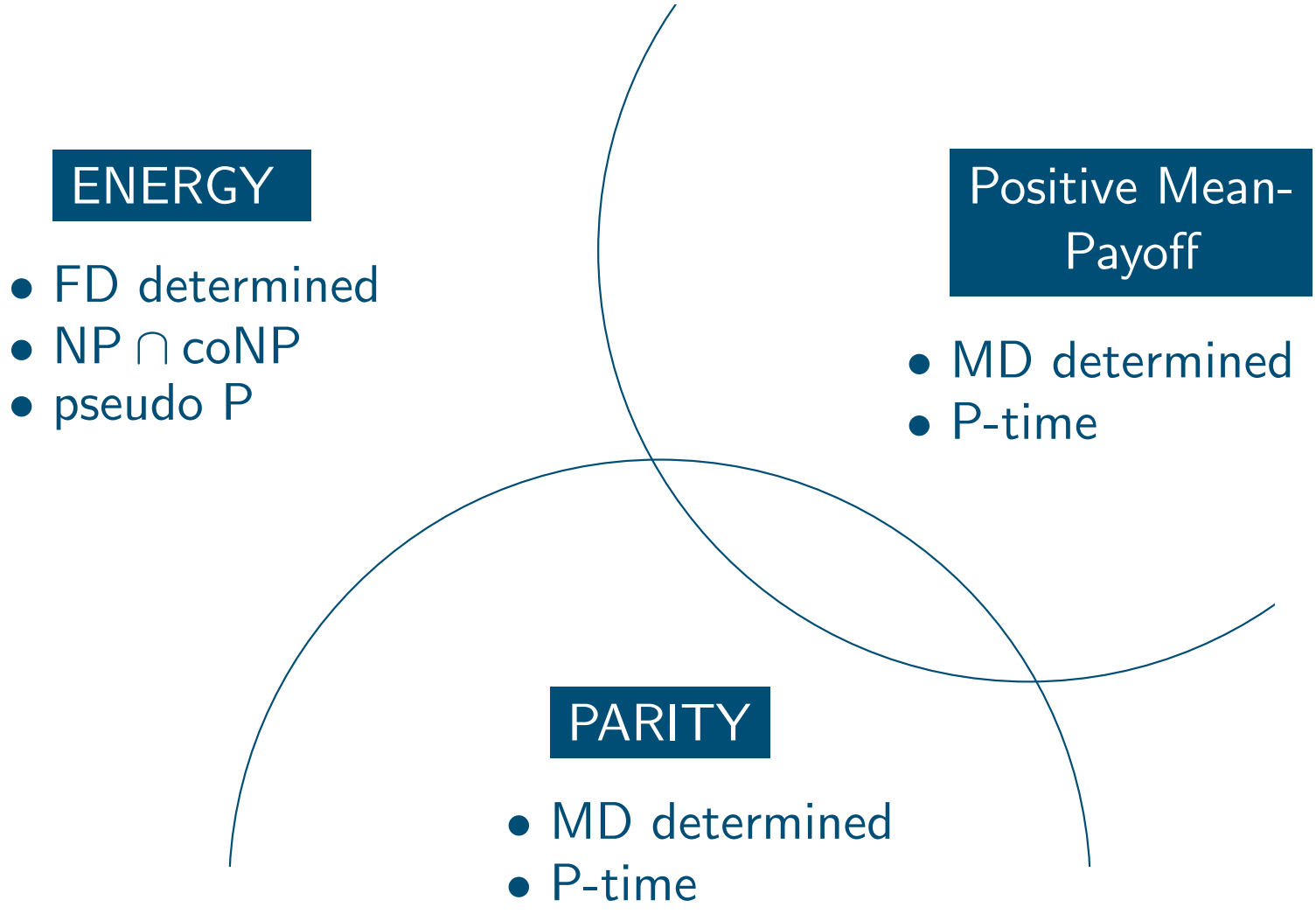
**Positive Mean-Payoff**

- MD determined
- P-time

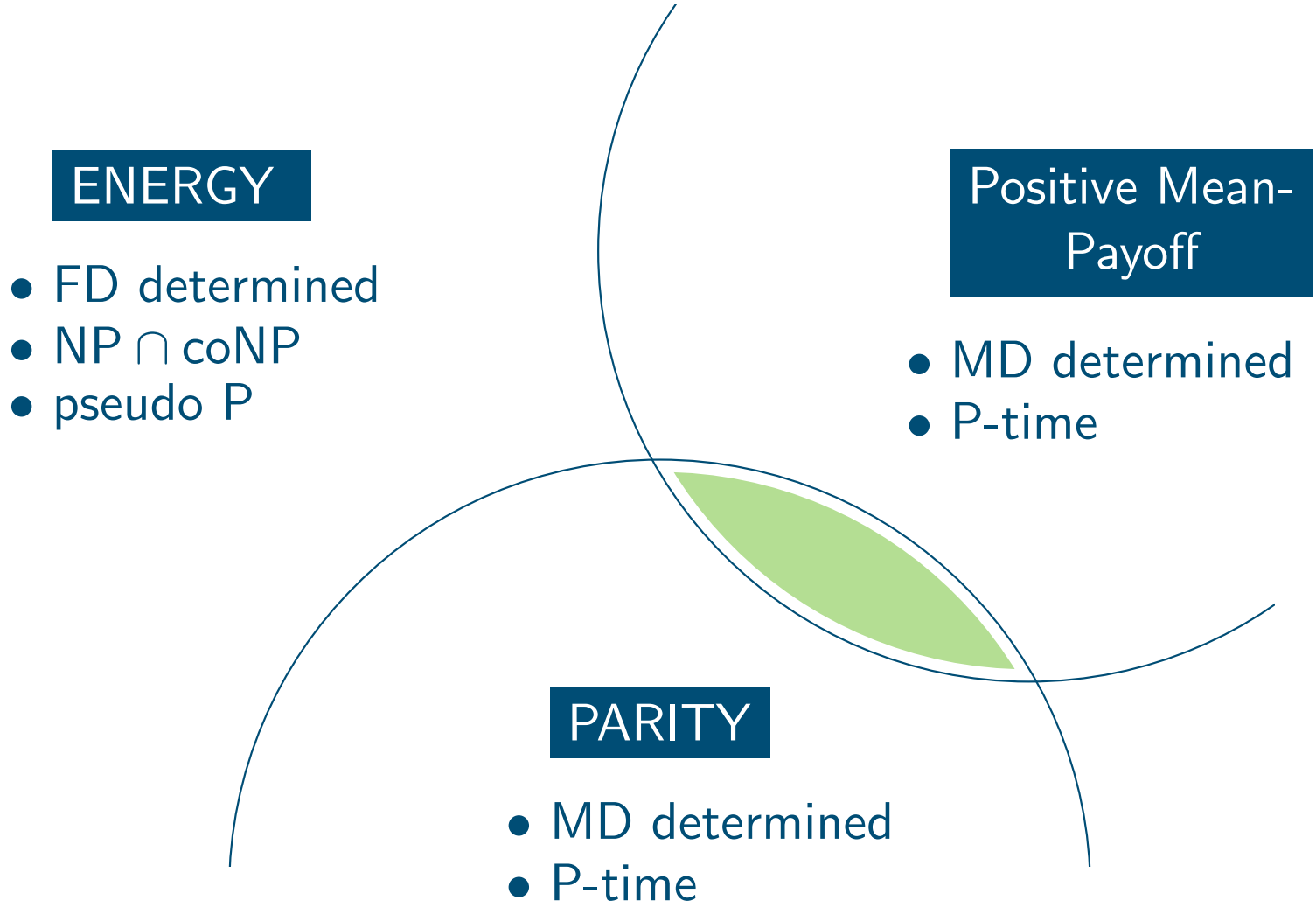
**PARITY**

- MD determined
- P-time

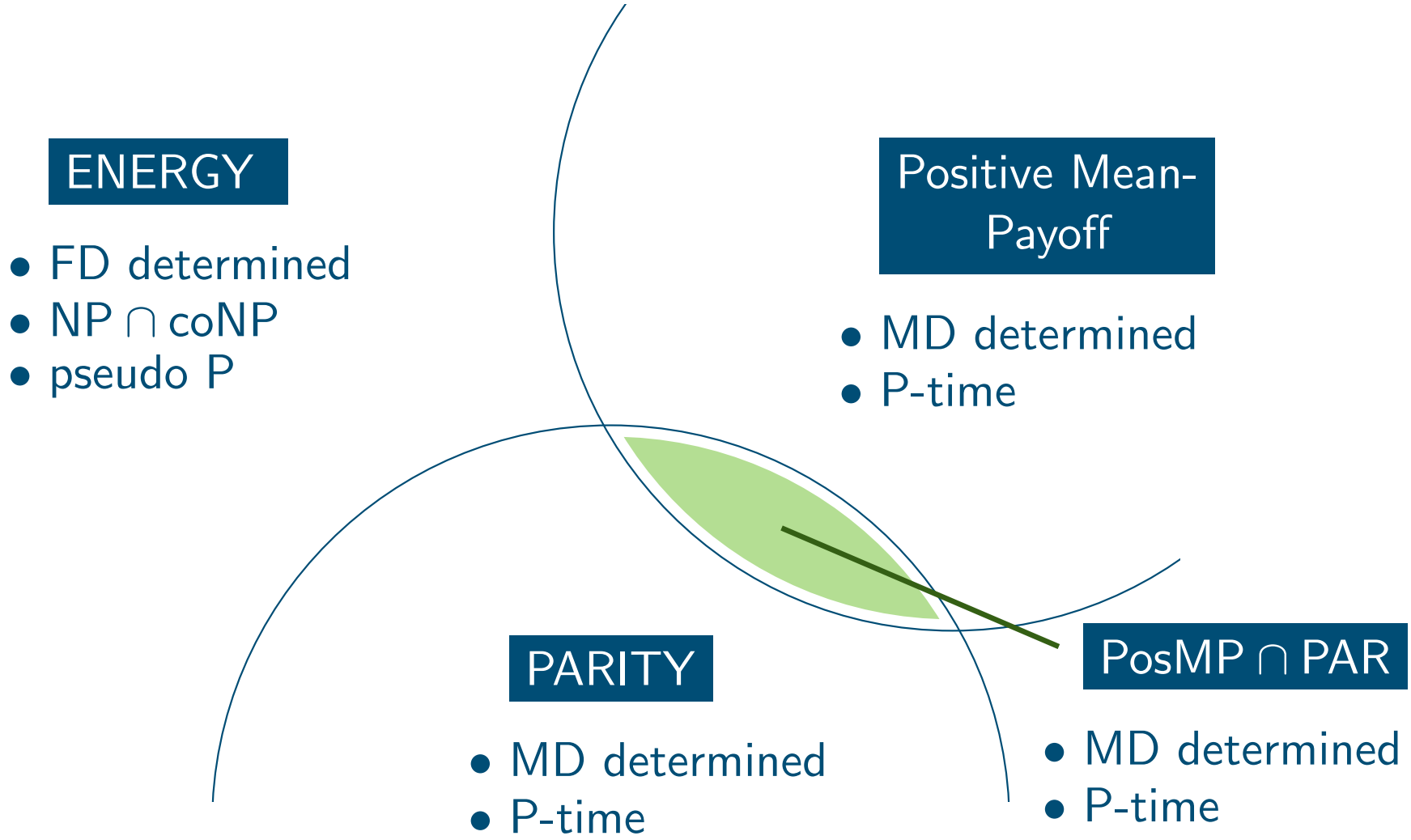
# Results on almost-sure problems for finite MDPs



# Results on almost-sure problems for finite MDPs

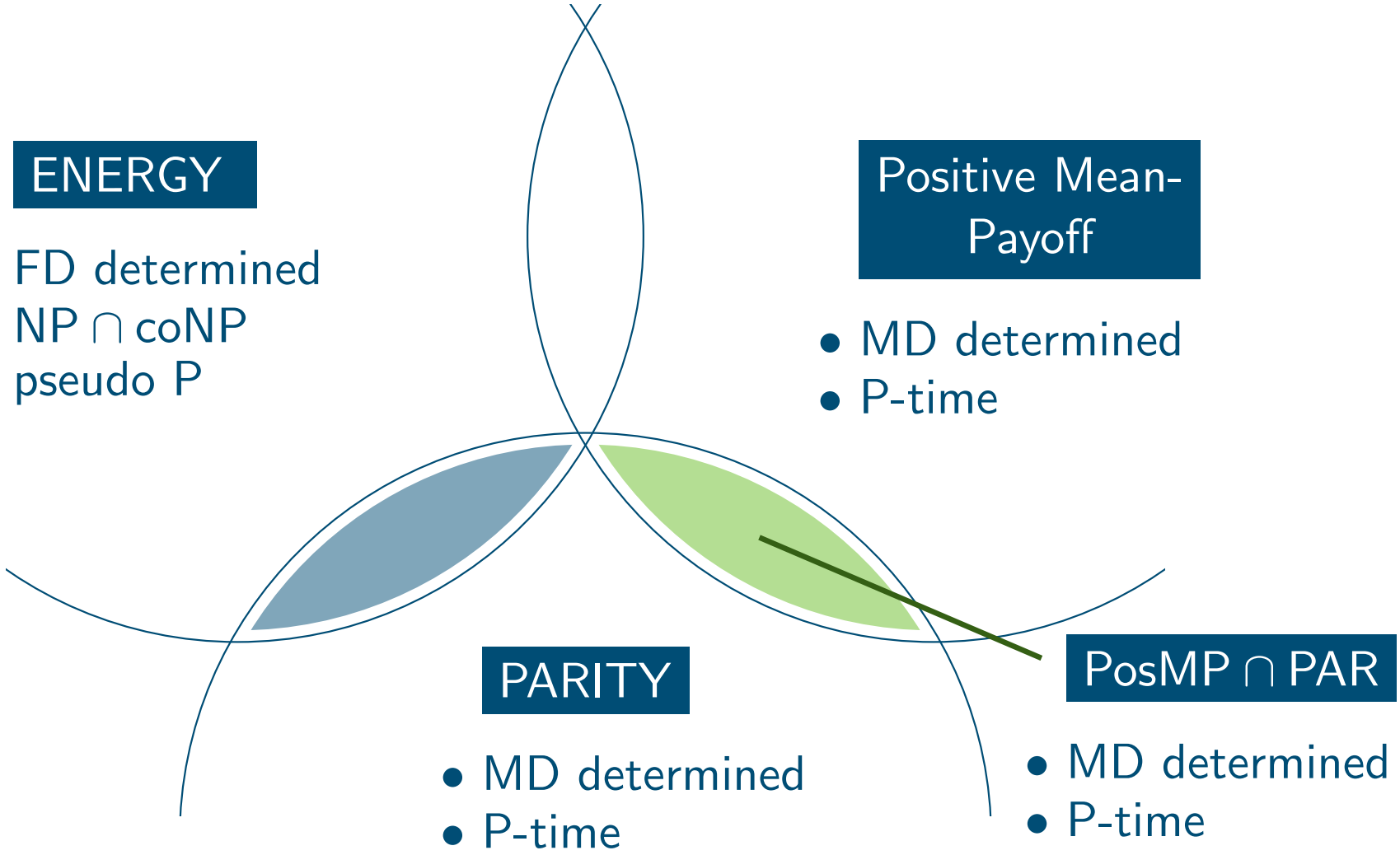


# Results on almost-sure problems for finite MDPs



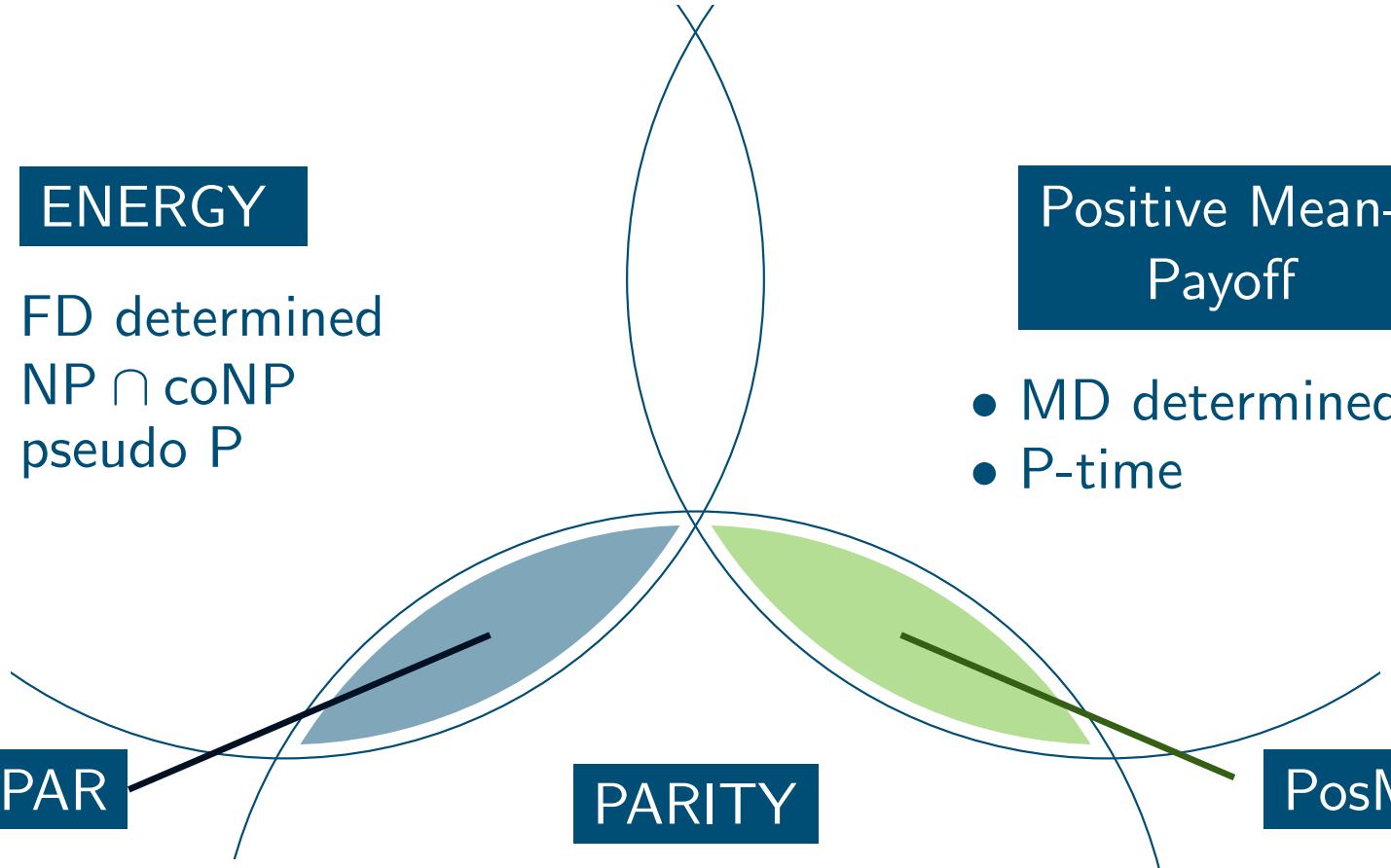

Chatterjee/Doyen, MFCS'11

# Results on almost-sure problems for finite MDPs




Chatterjee/Doyen, MFCS'11

# Results on almost-sure problems for finite MDPs



**ENERGY**

- FD determined
- NP ∩ coNP
- pseudo P

**Positive Mean-Payoff**

- MD determined
- P-time

**EN ∩ PAR**

- reduces to EN ∩ Büchi
- reduces to EN games
- FD determined

**PARITY**

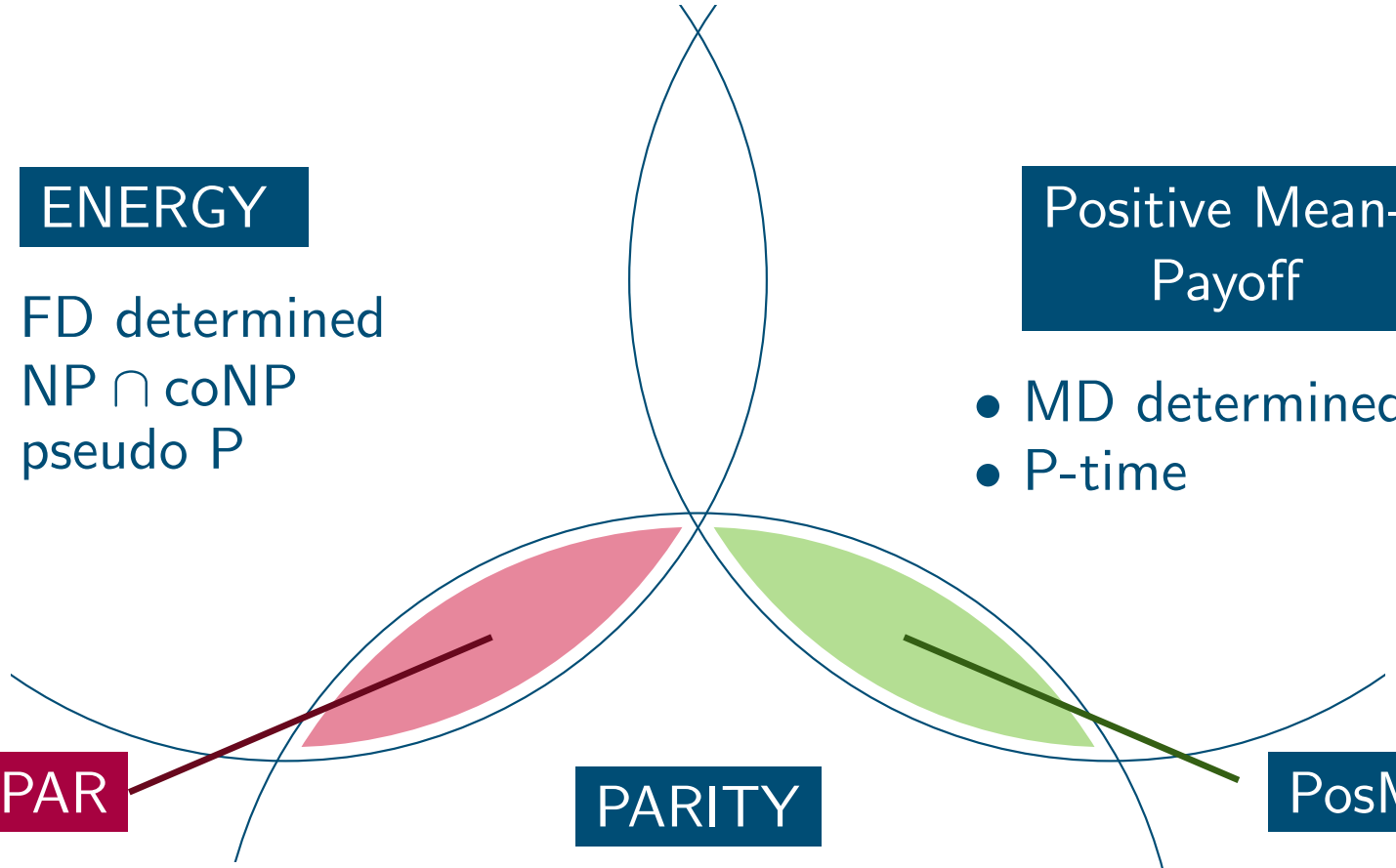
- MD determined
- P-time

**PosMP ∩ PAR**

- MD determined
- P-time

Chatterjee/Doyen, MFCS'11

# Results on almost-sure problems for finite MDPs



**ENERGY**

- FD determined
- NP ∩ coNP
- pseudo P

**Positive Mean-Payoff**

- MD determined
- P-time

**EN ∩ PAR**

- ✗ reduces to EN ∩ Büchi
- ✗ reduces to EN games
- ✗ FD determined

**PARITY**

- MD determined
- P-time

**PosMP ∩ PAR**

- MD determined
- P-time

Chatterjee/Doyen, MFCS'11

# Our Contribution

ENERGY $\cap$ PARITY objectives for finite MDPs:

1. Almost-sure optimal strategies may need infinite memory.

# Our Contribution

ENERGY $\cap$ PARITY objectives for finite MDPs:

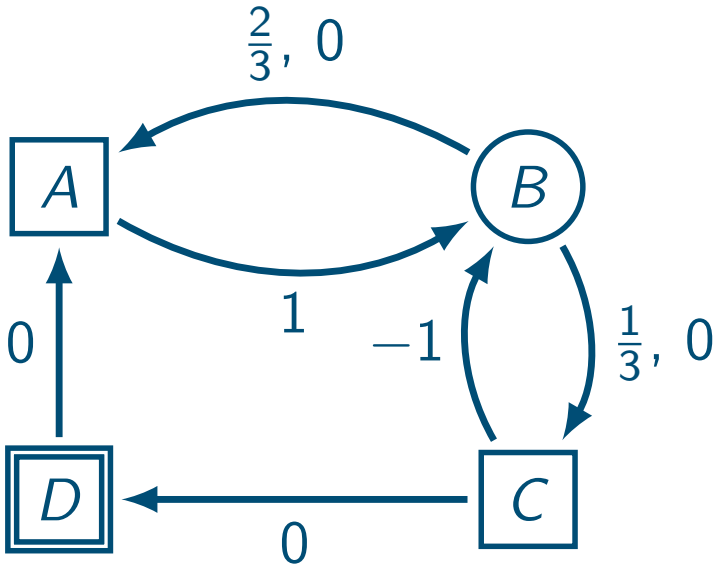
1. Almost-sure optimal strategies may need infinite memory.
2. A.s. winning sets are computable in NP $\cap$ coNP and (pseudo) P-time, by (a new!) reduction to Mean-Payoff games.

# Our Contribution

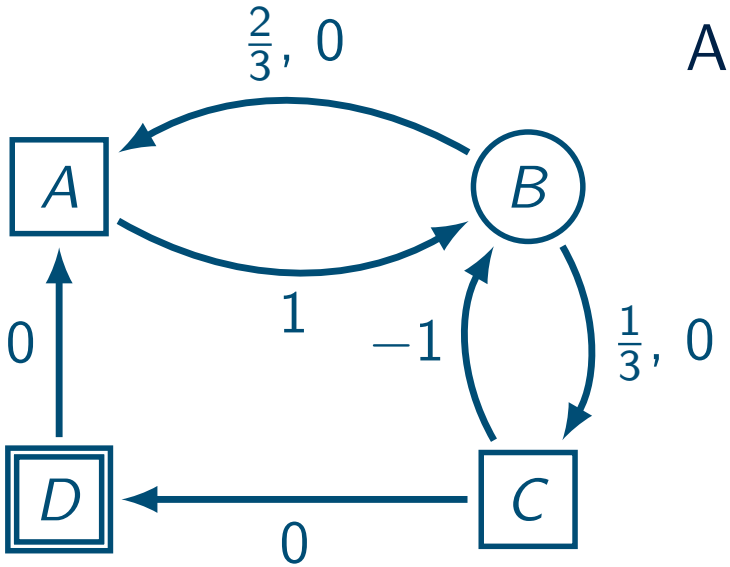
ENERGY $\cap$ PARITY objectives for finite MDPs:

1. Almost-sure optimal strategies may need infinite memory.
2. A.s. winning sets are computable in NP $\cap$ coNP and (pseudo) P-time, by (a new!) reduction to Mean-Payoff games.
3. Same bounds hold for the limit-sure problem ( $\sup_\sigma \mathbb{P}^\sigma(\text{ENERGY} \cap \text{PARITY}) = 1$ ?)

# 1. What's the Problem?

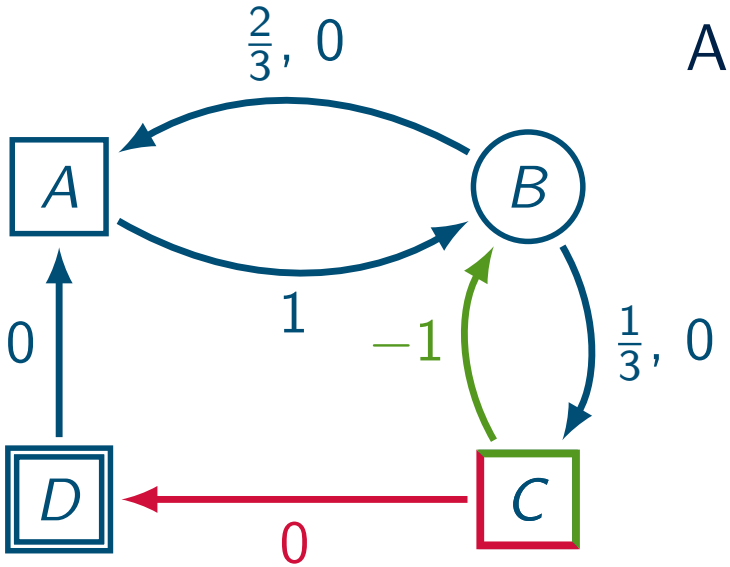


# 1. What's the Problem?



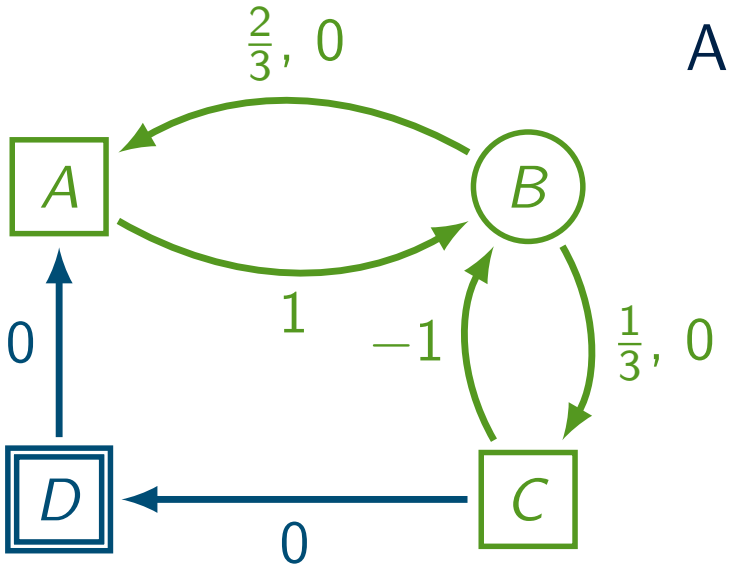

Aim: Satisfy ENERGY and avoid $D$

# 1. What's the Problem?




Aim: Satisfy ENERGY and avoid $D$

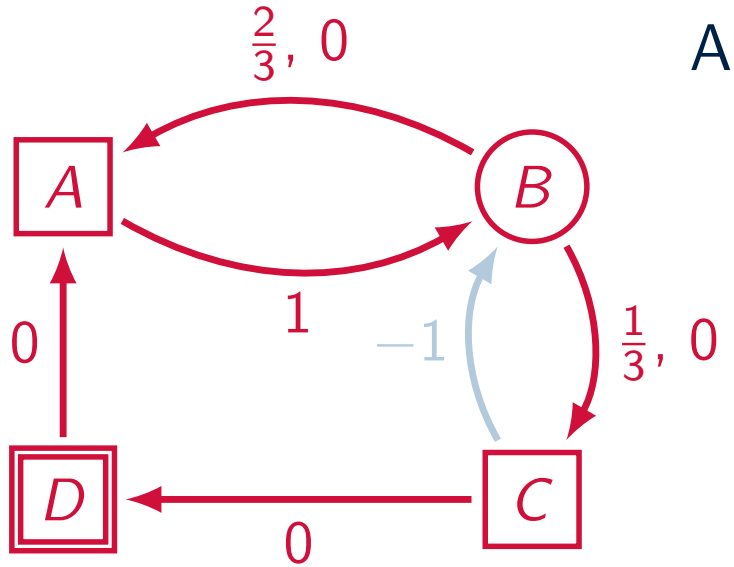
# 1. What's the Problem?




Aim: Satisfy ENERGY and avoid $D$

- stay in $\{A, B, C\}$ and lose ENERGY

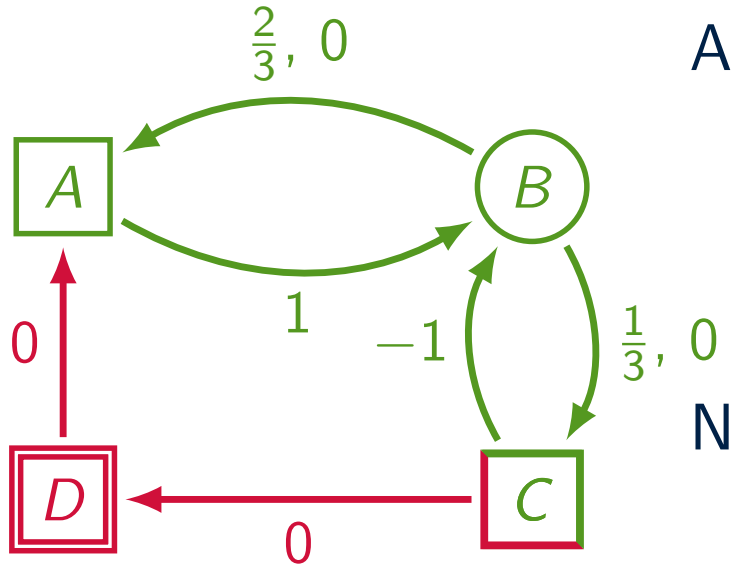
# 1. What's the Problem?




Aim: Satisfy ENERGY and avoid $D$

- stay in $\{A, B, C\}$ and lose ENERGY
- prefer $D$ over $B$ and lose PARITY

# 1. What's the Problem?



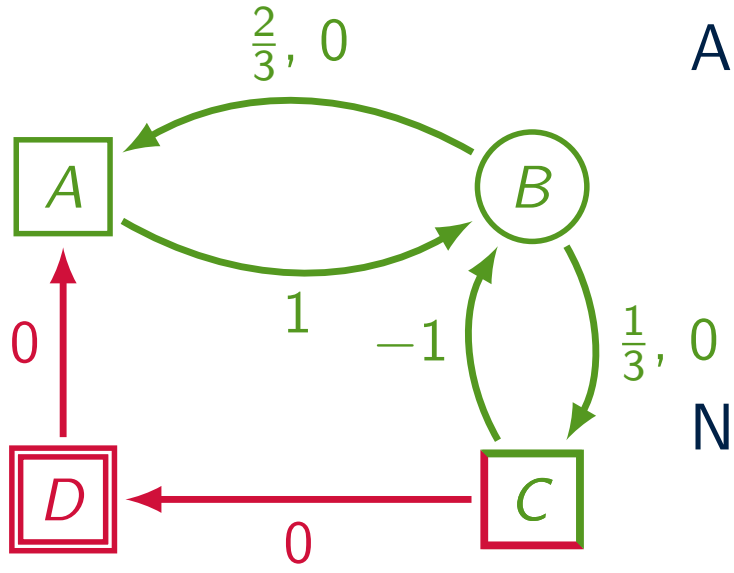

Aim: Satisfy ENERGY and avoid $D$

- stay in $\{A, B, C\}$ and lose ENERGY
- prefer $D$ over $B$ and lose PARITY

No FM-strategy wins (a.s.)

- eventually commits to red or green
- maintain bounded distance to $D$

# 1. What's the Problem?




Aim: Satisfy ENERGY and avoid $D$

- stay in $\{A, B, C\}$ and lose ENERGY
- prefer $D$ over $B$ and lose PARITY

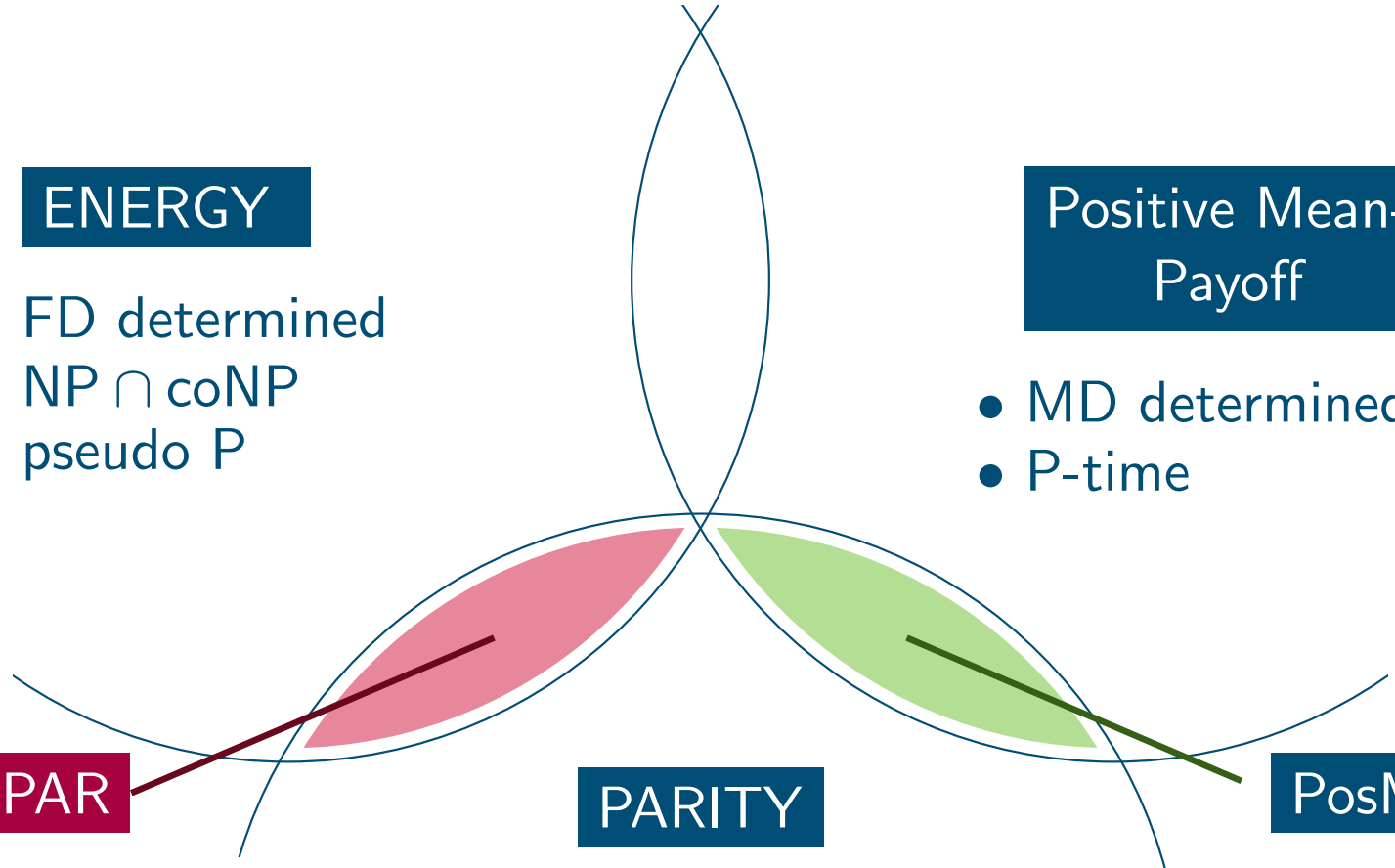
No FM-strategy wins (a.s.)

- eventually commits to red or green
- maintain bounded distance to $D$

An (a.s.) winning strategy

- "move to $D$ only if energy level is 0"
- works because $\mathbb{P}^{green}(\text{always} > 0) > 1/2$

# Results on almost-sure problems for finite MDPs



**ENERGY**

- FD determined
- NP ∩ coNP
- pseudo P

**Positive Mean-Payoff**

- MD determined
- P-time

**EN ∩ PAR**

- needs ∞ memory
- NP ∩ coNP
- pseudo P

**PARITY**

- MD determined
- P-time

**PosMP ∩ PAR**

- MD determined
- P-time

thank you.