Quantitative Reductions and Vertex-Ranked Games

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Reachability Games

Winning condition: Play reaches either \( \star \) or \( \star\star \) or \( \star\star\star \)
Reachability Games

Winning condition: Play reaches either \( \bigstar \) or \( \bigstar \) or \( \bigstar \)
Winning condition: Play reaches either 🌸 or 🌻 or 🌼
Winning condition: Play reaches either \( \star \) or \( \diamond \) or \( \triangle \)
Reachability Games

Winning condition: Play reaches either \( \square \) or \( \pentagon \) or \( \triangle \)
Reachability

✓
Generalized Reachability: The Problem

Winning condition:

![Diagram of a reachability game with nodes and edges representing the game's structure.](image-url)
Generalized Reachability: The Problem

Winning condition:
Reach one from \( \{\text{pentagon}, \text{square}\} \) and one from \( \{\text{triangle}, \text{circle}\} \).
Winning condition: Reach some memory state $S$ with $S \cap \{0, 0\} \neq \emptyset$ and with $S \cap \{0, 0\} \neq \emptyset$. 

**Reachability Condition**
Winning condition: Reach some memory state $S$ with $S \cap \{0, 0\} \neq \emptyset$ and with $S \cap \{0, 0\} \neq \emptyset$.
Generalized Reachability: One Solution

Winning condition: Reach some memory state $S$ with $\mathcal{S} \cap \{0, 0\} \neq \emptyset$ and with $\mathcal{S} \cap \{0, 0\} \neq \emptyset$.
Winning condition: Reach some memory state $S$ with $S \cap \{0\} \neq \emptyset$ and with $S \cap \{0\} \neq \emptyset$. 

Reachability Condition

Generalized Reachability: One Solution
Winning condition: Reach some memory state \( S \) with \( S \cap \{0\} \neq \emptyset \) and with \( S \cap \{1\} \neq \emptyset \).
Winning condition: Reach some memory state $S$ with $S \cap \{0, 0\} \neq \emptyset$ and $S \cap \{0, 0\} \neq \emptyset$.
Winning condition:
Generalized Reachability: One Solution

Winning condition: Reach some memory state $S$ with

$$S \cap \{\text{pentagon, hexagon}\} \neq \emptyset \text{ and with } S \cap \{\text{hexagon, pentagon}\} \neq \emptyset$$
Winning condition: Reach some memory state \( S \) with \( S \cap \{,\} \neq \emptyset \) and with \( S \cap \{,\} \neq \emptyset \)

Reachability Condition
Reachability

✔
The Big Picture

Reachability

Generalized Reachability
The Big Picture

Reachability $\checkmark$ Generalized Reachability
The Big Picture

Quantitative

Qualitative

Reachability

Generalized Reachability

✓
Assign cost to each play.
Assign cost to each play.

\[ C_{\rho} = \begin{cases} 
0 & \text{if } \{0,0\} \text{ and } \{0,0\} \text{ are visited} \\
1 & \text{if one of them is visited} \\
2 & \text{if neither is visited} 
\end{cases} \]
Assign cost to each play.

\[
Cst(\rho) = \begin{cases} 
0 & \text{if } \{\text{pink}, \text{yellow}\} \text{ and } \{\text{yellow}, \text{blue}\} \text{ are visited} \\
1 & \text{if one of them is visited} \\
2 & \text{if neither is visited}
\end{cases}
\]
The Big Picture

Quantitative
- Generalized
- Reachability

Qualitative

Quantitative Reductions and Vertex-Ranked Games
The Big Picture

Quantitative Reachability

Qualitative

Quantitative

Generalized Reachability

Cst = 0

Cst = 1

Cst = 2

Reachability

Generalized Reachability

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The Big Picture

Quantitative Reductions and Vertex-Ranked Games
The Big Picture

- **Vertex-Ranked Reachability**
- **Quantitative Generalized Reachability**

- **Quantitative**
  - \( Cst = 0 \)
  - \( Cst = 1 \)
  - \( Cst = 2 \)

- **Qualitative**

- **Reachability**
- **Generalized Reachability**
The Big Picture

Vertex-Ranked Reachability → Quantitative Generalized Reachability

Cst = 0

Cst = 1

Cst = 2

Qualitative

Reachability

Generalized Reachability

Quantitative
The Big Picture

Vertex-Ranked Reachability

Quantitative

Generalized Reachability

Cst = 0

Cst = 1

Cst = 2

Reachability

Generalized Reachability

Qualitative

Quantitative Reductions and Vertex-Ranked Games
Conclusion

Contribution

- Lifted reductions to quantitative games
Contribution

- Lifted reductions to quantitative games
- Solved wide range of general-purpose quantitative games
Conclusion

Contribution

- Lifted reductions to quantitative games
- Solved wide range of general-purpose quantitative games

Next Steps
Conclusion

Contribution

- Lifted reductions to quantitative games
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Conclusion

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Next Steps