

# Mean-Payoff Optimization in Continuous-Time Markov Chains with Parametric Alarms

Christel Baier, Clemens Dubslaff,  
Ľuboš Korenčiak, Antonín Kučera, Vojtěch Řehák

TU Dresden, Germany  
Faculty of Informatics, Masaryk University, Brno, Czech Republic

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# The Aim

Parametric model,  $\mathbf{p} \in D$

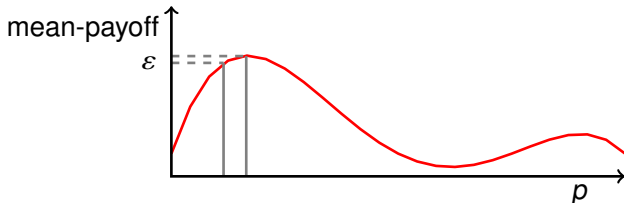
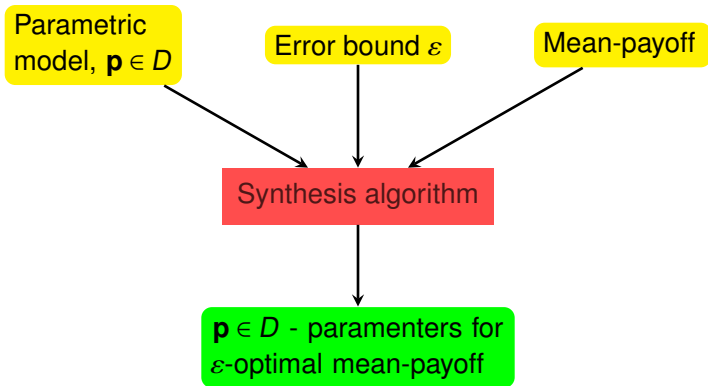
Mean-payoff

Synthesis algorithm

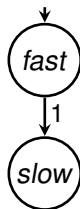
$\mathbf{p} \in D$  - parameters for optimal mean-payoff



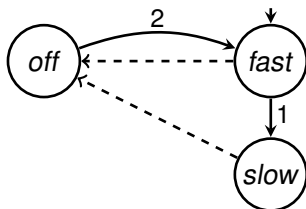
# The Aim



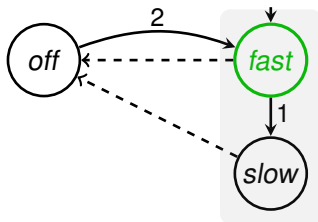
# Continuous-time Markov Chain with Alarms (ACTMC)



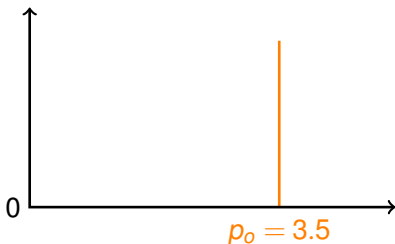
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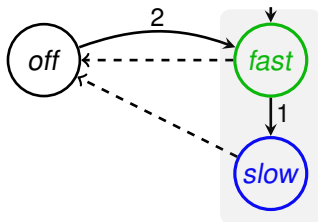
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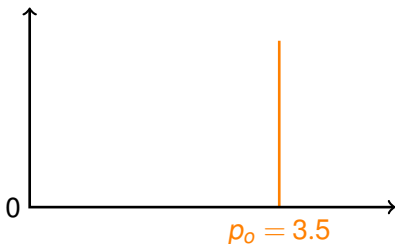
$$\omega = (o=3.5, s=2),$$



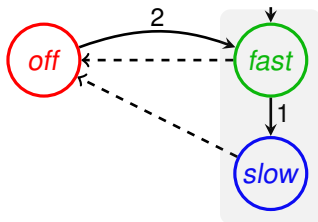
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$$\omega = (o=3.5, s=2), (o=1.5),$$



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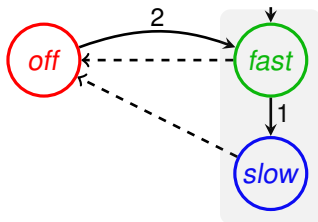


$$\omega = (o=3.5, s=2), (o=1.5), (f=4), (\dots), \dots$$

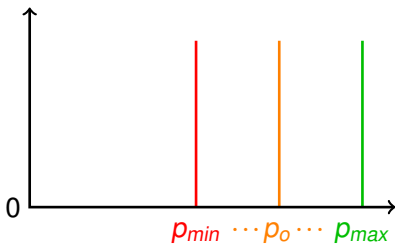




# Parametric ACTMC

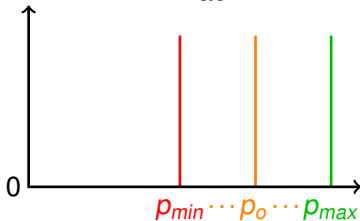


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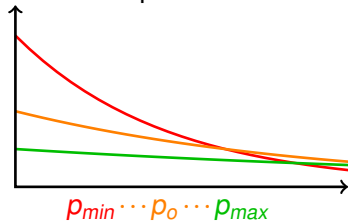


# Parametric distributions

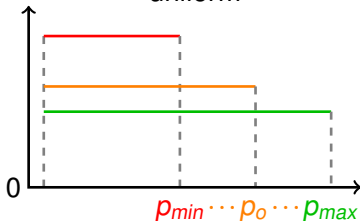
Dirac



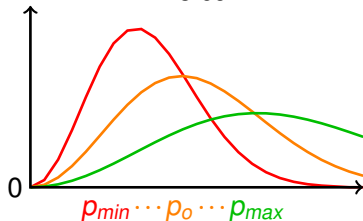
exponential



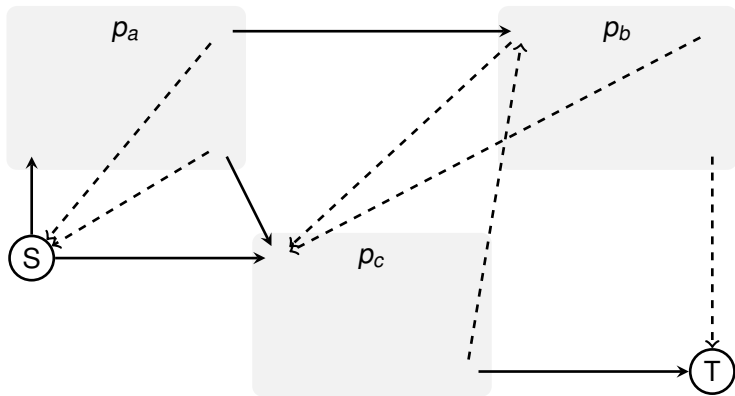
uniform



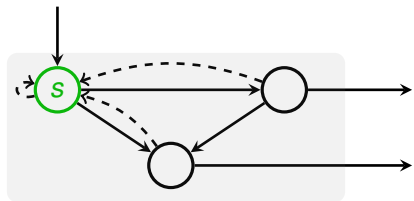
Weibull



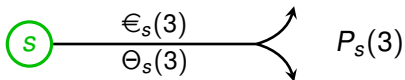
# Restrictions - disjoint regions



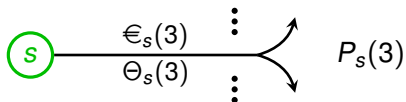
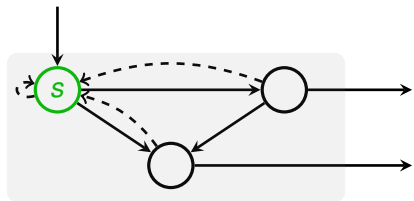
# Explicit Algorithm



ACTMC and  $\varepsilon$ ;  $\mathbf{p} = ?$



# Explicit Algorithm

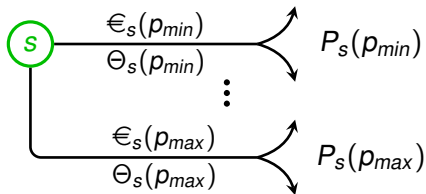
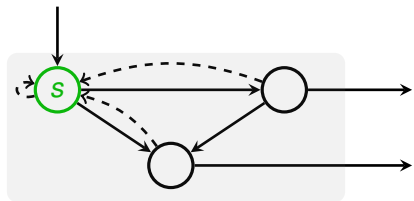


ACTMC and  $\epsilon$ ;  $\mathbf{p} = ?$



uncountable action  
space MDP

# Explicit Algorithm



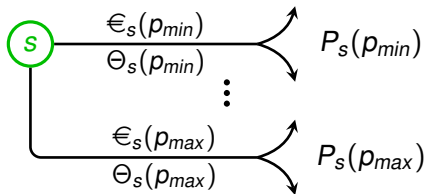
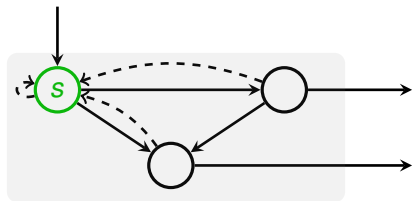
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$\epsilon$ -optimal  $\mathbf{p}$

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For  $\epsilon = 0.1$  we get at least  $10^{20}$  actions.

Our contribution is based on two ideas:

- policy iteration
- symbolic representation of actions

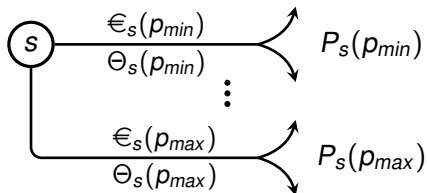


# Symbolic Algorithm

## policy iteration

$$\arg \max_{p \in D} F_s(p)$$

$$F_s(p) = \dots \epsilon_s(p) \dots \Theta_s(p) \dots P_s(p) \dots$$



ACTMC and  $\epsilon$ ;  $\mathbf{p} = ?$

↓  
uncountable action  
space MDP

↓  
finite MDP

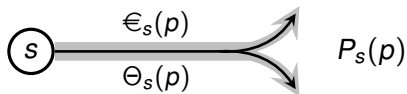
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↓  
MDP with  
symbolic actions

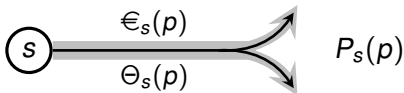
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### Message:

Sometimes it is better to use **continuous** approach than finite discretization.

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uncountable action  
space MDP

↓  
MDP with  
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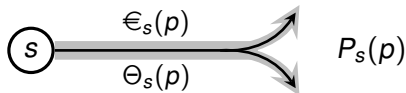
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Thank you for your attention!