ML and Extended Branching VASS

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EBVASS [Jacquemard, Segoufin, Dimino; LMCS 2016]

BVASS

$$\frac{(q, \vec{v_1} + \vec{v_2})}{(q_1, \vec{v_1}) \qquad (q_2, \vec{v_2})}$$

EBVASS

$$C = \{c_1, \cdots, c_m\} \qquad\qquad c_i = (l_i, r_i, p_i) \in \{1, \cdots, k\}^3$$

$$\frac{(q, \vec{v_1} + \vec{v_2} + \sum_i n_i \cdot \overline{e_{\pi_3(c_i)}})}{(q_1, \vec{v_1} + \sum_i n_i \cdot \overline{e_{\pi_1(c_i)}}) \qquad (q_2, \vec{v_2} + \sum_i n_i \cdot \overline{e_{\pi_2(c_i)}})}$$

Paradigm

- imperative programming with higher-order procedures
- functional (higher-order) programming with state

Interactions of program with context (game semantics)

$$M \mapsto [\![M]\!]$$

1. What is the automata-theoretic nature of $[\![M]\!]$?
2. When can we decide $M_1 \cong M_2$?

CBV: finitary Reduced ML

Types

$$\theta ::= \mathsf{int} \mid \mathsf{unit} \mid \mathsf{int\ ref} \mid \theta \to \theta$$

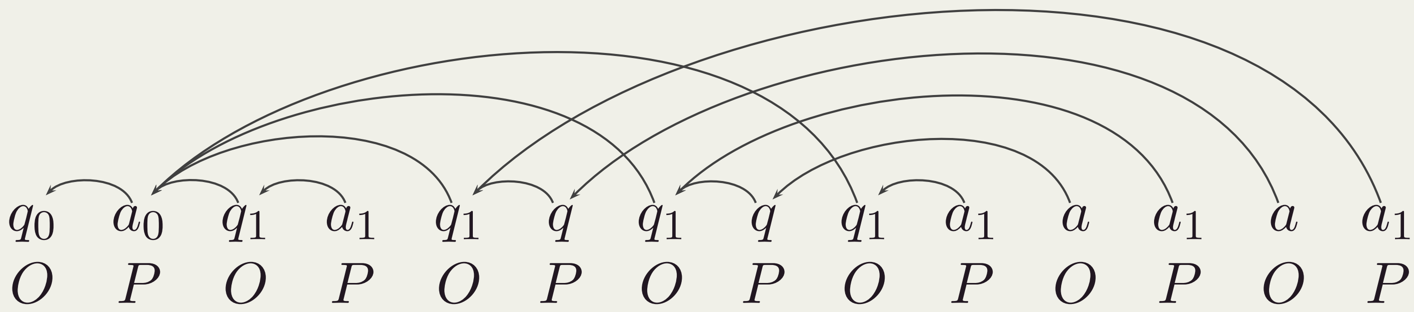
Terms (selection)

$$x \qquad \lambda x^{\theta}.M \qquad M\,N$$

$$\mathsf{ref}\ M \qquad !M \qquad M := N \qquad \mathbf{while}\ M\ \mathbf{do}\ N$$

Finitary restriction

$\mathsf{int} = \{0, \cdots, \mathrm{max}\}$ no recursion



Complete plays capture contextual equivalence [Abramsky, McCusker; CSL'97]

$$M_1 \cong M_2$$

if and only if

$$\text{comp}\,[\![M_1]\!] = \text{comp}\,[\![M_2]\!]$$

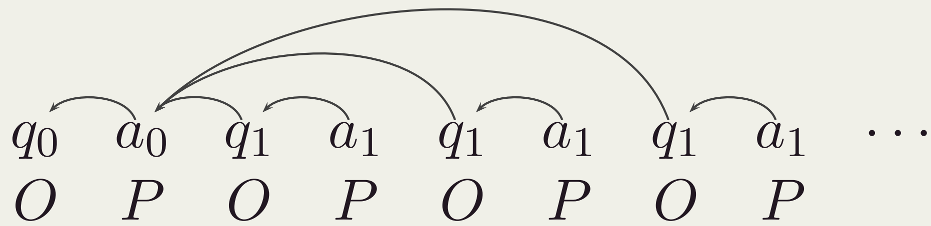
Classification

Reduced ML (CBV)

- no direct dependence on order
- need for automata over infinite alphabets
- connections to numerous classes of automata
- links to unsolved decision problems

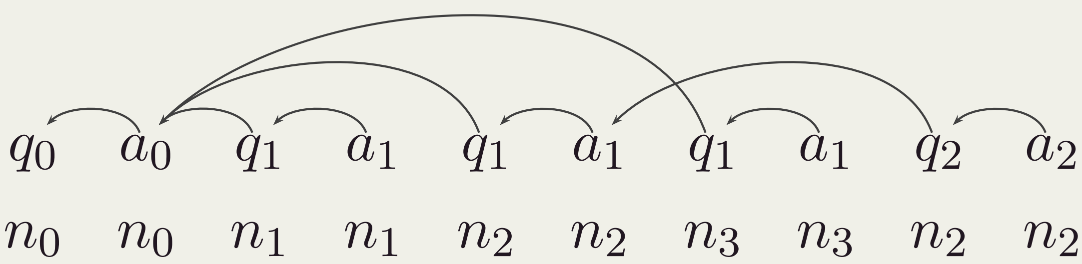
First-order types

- $\vdash M$: unit $\to$ unit



NB: pointers are determined uniquely

- $\vdash M$: unit $\to$ unit $\to$ unit



$$(q_0, n_0)(a_0, n_0)(q_1, n_1)(a_1, n_1)(q_1, n_2)(a_1, n_2)(q_1, n_3)(a_1, n_3)(q_2, n_2)(a_2, n_2)$$

Cotton-Barratt, Hopkins, M., Ong [FOSSACS'16]

- $\mathsf{unit} \to \mathsf{unit}$: *Finite Automata*

- $\mathsf{unit} \to \mathsf{unit} \to \mathsf{unit}$: *Weak Class Memory Automata* (Björklund, Schwentick)

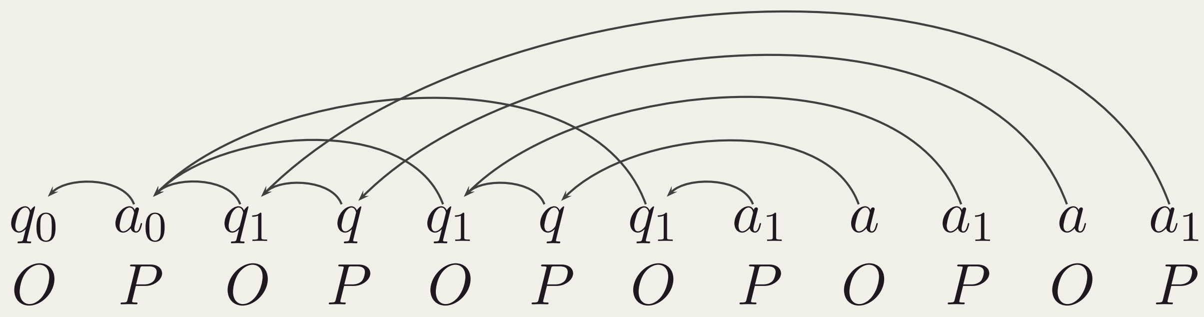
  Locally Prefix-Closed Data Automata (Decker, Habermehl, Leucker, Thoma), *Class Counting Automata* (Manuel, Ramanujam), *Non-reset History Register Automata* (Tzevelekos, Grigore)

- $\mathsf{unit} \to \mathsf{unit} \to \cdots \to \mathsf{unit}$: *Weak Nested Class Memory Automata*

  Locally Prefix-Closed Nested Data Automata (Decker, Habermehl, Leucker, Thoma)

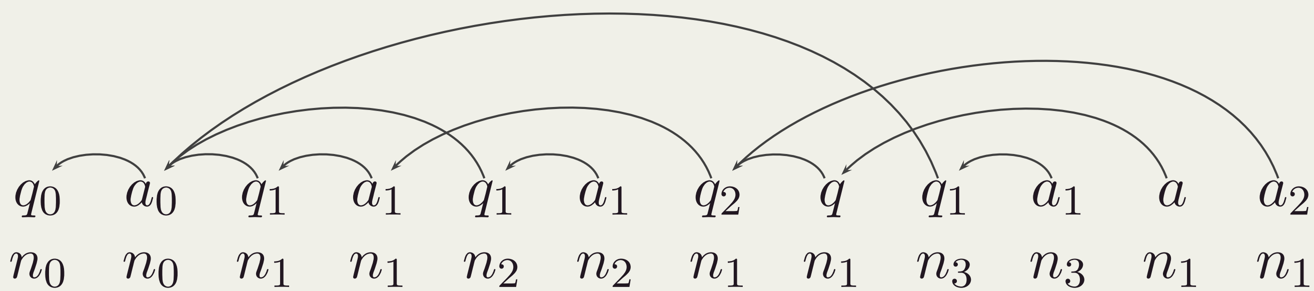
Hopkins, M., Ong [ICALP'11]

- $\vdash M : (\mathsf{unit} \to \mathsf{unit}) \to \mathsf{unit}$: *Visibly Pushdown Automata* (Alur, Madhusudan)



Cotton-Barratt, M., Ong [ESOP'17]

- $\vdash M$: unit $\to$ (unit $\to$ unit) $\to$ unit: *Visibly Pushdown Class Memory Automata*



$$(q_0,n_0)(a_0,n_0)(q_1,n_1)(a_1,n_1)(q_1,n_2)(a_1,n_2)(q_2,n_1)(q,n_1)(q_1,n_3)(a_1,n_3)(a,n_1)(a_2,n_1)$$

Theorem VPCMA emptiness is equivalent to EBVASS reachability.

Classification

- **decidable**

$$\mathsf{unit} \to \cdots \to \mathsf{unit}$$
$$(\mathsf{unit} \to \cdots \to \mathsf{unit}) \to \mathsf{unit}$$

- **EBVASS-equivalent**

$$\mathsf{unit} \to (\mathsf{unit} \to \cdots \to \mathsf{unit}) \to \mathsf{unit}$$

- **undecidable**

$$(\mathsf{unit} \to \mathsf{unit}) \to \mathsf{unit} \to \mathsf{unit}$$
$$\mathsf{unit} \to \mathsf{unit} \to (\mathsf{unit} \to \cdots \to \mathsf{unit}) \to \mathsf{unit}$$
$$((\mathsf{unit} \to \mathsf{unit}) \to \mathsf{unit}) \to \mathsf{unit}$$

Summary

class of ML programs $\sim$ EBVASS $\sim$ $FO^2(<, +1, \sim)$