

Untangling two-way transducers in elementary time



Gabriele Puppis

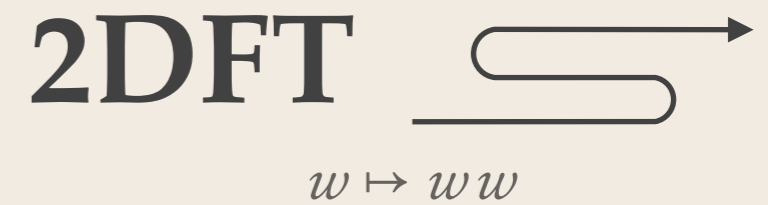
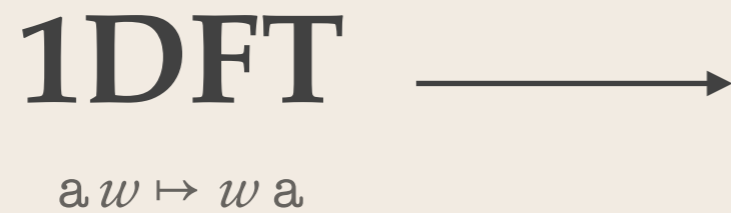
joint works with Félix Baschenis
Olivier Gauwin
Anca Muscholl

Finite-state Transducers = automata with outputs on transitions

1 way

2 way

Deterministic



Non
deterministic




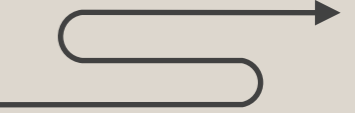
Finite-state Transducers = automata with outputs on transitions

1 way

2 way

Deterministic

1DFT 
 $aw \mapsto wa$

2DFT 
 $w \mapsto ww$

Non
deterministic

$wa \mapsto aw$
1NFT 

2NFT 

Finite-state Transducers = automata with outputs on transitions

1 way

2 way

Deterministic

1DFT

$aw \mapsto wa$

2DFT

$w \mapsto ww$

Non
deterministic

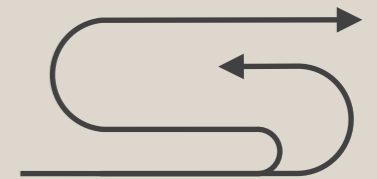
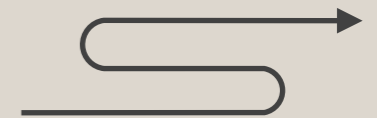
$wa \mapsto aw$

1NFT

2NFT

Choffrut '77
PTIME

Filiot & al.'13
non-elementary



Finite-state Transducers = automata with outputs on transitions

1 way

2 way

Deterministic

1DFT

$aw \mapsto wa$

2DFT

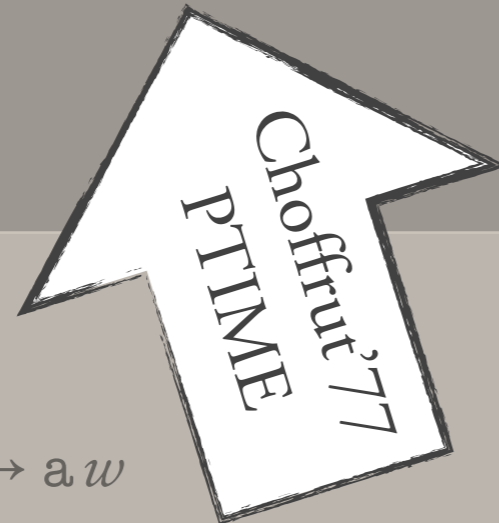
$w \mapsto ww$

Non
deterministic

1NFT

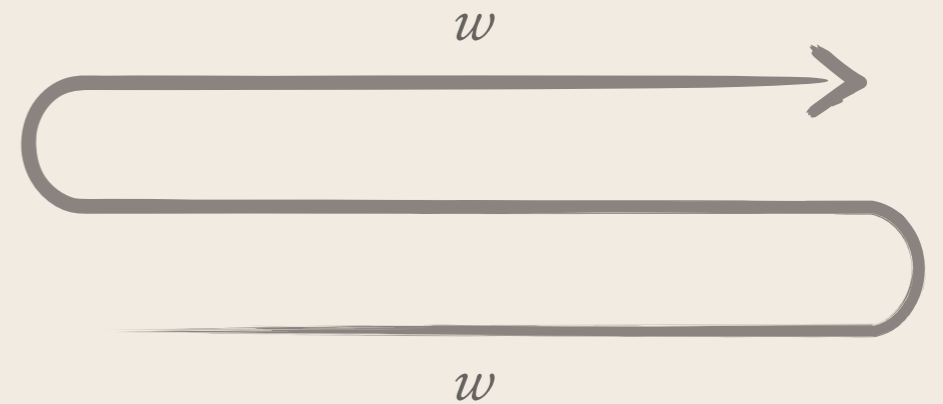
$wa \mapsto aw$

2NFT



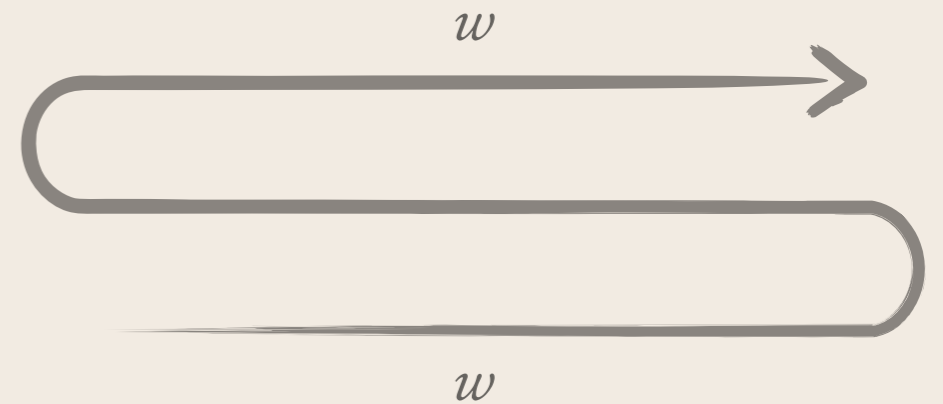
Fix a regular language R

$$T(w) = \begin{cases} w w & \text{if } w \in R \\ \perp & \text{otherwise} \end{cases}$$



Fix a regular language R

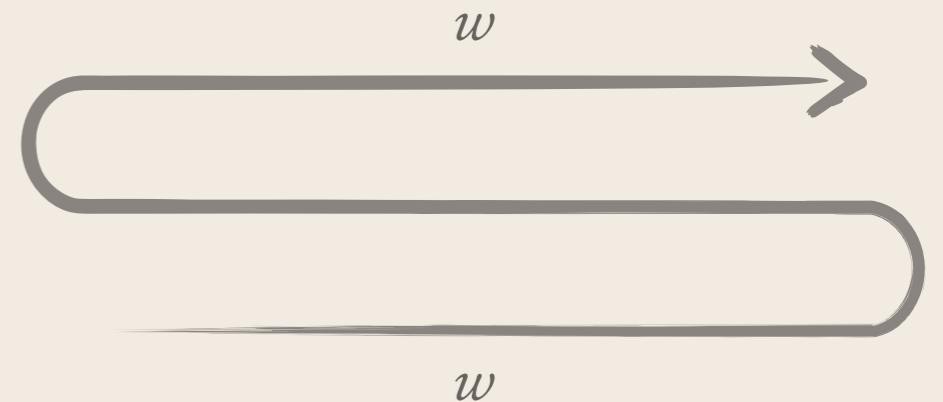
$$T(w) = \begin{cases} w w & \text{if } w \in R \\ \perp & \text{otherwise} \end{cases}$$



❖ $R = \Sigma^*$ \longrightarrow T not 1-way definable

Fix a regular language R

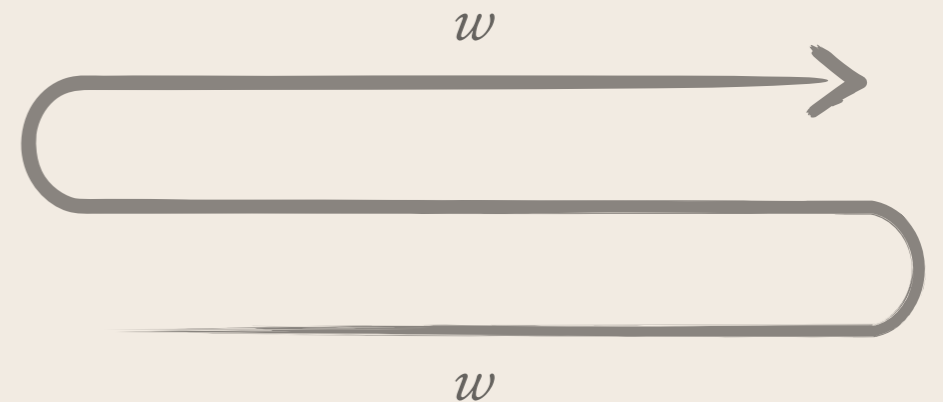
$$T(w) = \begin{cases} w w & \text{if } w \in R \\ \perp & \text{otherwise} \end{cases}$$



- ❖ $R = \Sigma^*$ \longrightarrow T not 1-way definable
- ❖ R finite \longrightarrow T 1-way definable

Fix a regular language R

$$T(w) = \begin{cases} w w & \text{if } w \in R \\ \perp & \text{otherwise} \end{cases}$$



❖ $R = \Sigma^*$ \longrightarrow T not 1-way definable

❖ R finite \longrightarrow T 1-way definable

❖ $R = \{ abc \}^*$ \longrightarrow T 1-way definable

(output “ab, ca, bc, ab, ...” while reading w)

Given a *functional* 2NFT T ,

- ❖ we can construct a 1NFT $T' \subseteq T$ (3EXPTIME)
- ❖ T is 1-way definable iff $T' = T$
- ❖ we can decide the latter (2EXPSPACE)

Note:

the 1-way definability problem for *relational* 2NFT is undecidable.

Questions?



Simulating 2NFT by 1NFT



Simulating 2NFT by 1NFT



Simulating 2NFT by 1NFT



Simulating 2NFT by 1NFT



Simulating 2NFT by 1NFT

