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Decidable Weighted Expressions with Presburger Combinators

Boolean vs Quantitative Languages

$$L : \Sigma^* \to \{0, 1\}$$

Classical decision problems

Emptiness	$\exists u. f(u) \geq 1$
Universality	$\forall u. f(u) \geq 1$
Inclusion	$\forall u. f(u) \geq g(u)$
Equivalence	$\forall u. f(u) = g(u)$

Boolean vs Quantitative Languages

$$L : \Sigma^* \to \cancel{\{0,1\}}\ \mathbb{Z} \cup \{-\infty\}$$

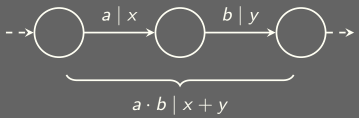
Classical quantitative decision problems

Emptiness	$\exists u. f(u) \geq \cancel{1}\ \nu$	for some threshold ν
Universality	$\forall u. f(u) \geq \cancel{1}\ \nu$	for some threshold ν
Inclusion	$\forall u. f(u) \geq g(u)$	
Equivalence	$\forall u. f(u) = g(u)$	

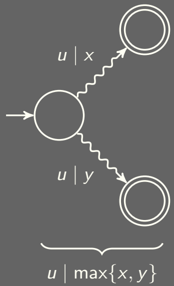
Classical Model: Weighted Automata

(max,+) WA

Transition sequence



Non-determinism



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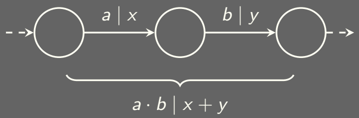

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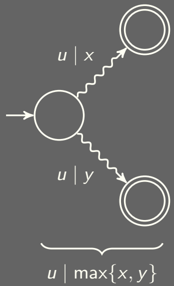

Undecidability [Krob 1994]

Quantitative language-inclusion is undecidable for $(\max,+)$ WA

- Even for linearly ambiguous automata [Colcombet 2010]

Decidable Formalisms: Restriction

Finitely ambiguous (max,+) WA [Filiot et al. 2012]

Define functions of the form,

$$u \mapsto \max\{\mathcal{A}_1(u), \ldots, \mathcal{A}_k(u)\}$$

$\mathcal{A}_i$: Unambiguous WA

- ☺ Quantitative decision problems are DECIDABLE
- ☺ Closed under *max* and *sum*
- ☹ Limited expressive power (*min*, *minus*, ...)

Decidable Formalisms: New model

Mean-payoff expressions [Chatterjee et al. 2010]

$$E ::= \mathcal{A} \mid \max(E, E) \mid \min(E, E) \mid E + E \mid -E$$

$\mathcal{A}$: Deterministic WA

- ☺ Quantitative decision problems are PSPACE-COMPLETE [Velner 2012]
- ☺ Closed under *max*, *min*, *sum* and *minus*

- ☹ Determinism (define Lipschitz continuous functions)
- ☹ Does **not** contain all finitely ambiguous (max,+) WA
- ☹ Monolithism (apply on the whole word)

Contributions

❶ Simple expressions

$$E ::= \mathcal{A} \mid \phi(E, E)$$

$\mathcal{A}$: Unambiguous WA

ϕ : $\exists$FO[$\leq, +, 0, 1$] formula defining function with arity two

- ☺ Quantitative decision problems are PSpace-Complete
- ☺ Closed under Presburger definable functions
- ☺ Contain all finitely ambiguous (max,$+$) WA

- ☹ Monolithism (apply on the whole word)

Contributions

➋ Iterable expressions

$$E ::= \mathcal{A} \mid \phi(E, E) \mid E^{\circledast}$$

- Sum arbitrarily many factors
- Unique decomposition required

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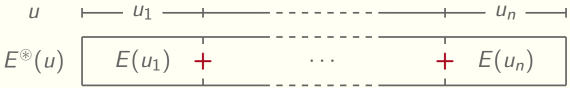


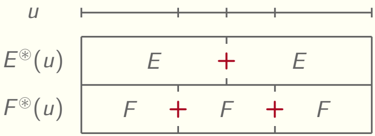
Remark

$E^{\circledast} \rightsquigarrow \heartsuit$	$u_1 \heartsuit\, u_2 \heartsuit \dots u_n \heartsuit \mapsto \sum_{i=1}^{n} \mathbf{E}(u_i \heartsuit)$
$F^{\circledast} \rightsquigarrow \clubsuit$	$u_1 \clubsuit\, u_2 \clubsuit \dots u_m \clubsuit \mapsto \sum_{i=1}^{m} \mathbf{F}(u_i \clubsuit)$

Results

Theorem (Iterable Expressions)

Quantitative decision problems are UNDECIDABLE



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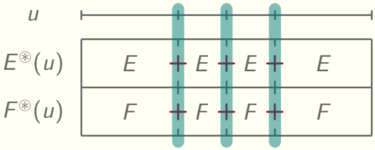


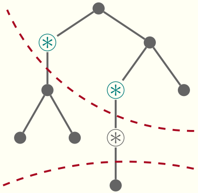
Results

Theorem (Iterable Expressions)

Quantitative decision problems are UNDECIDABLE

Theorem (Synchronised Iterable Expressions)

Synchronisation property is PTIME



Results

Theorem (Iterable Expressions)

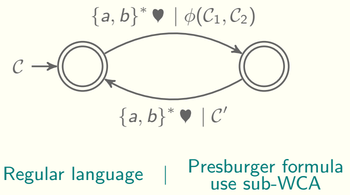
Quantitative decision problems are Undecidable

Theorem (Synchronised Iterable Expressions)

Synchronisation property is PTime

Quantitative decision problems are Decidable

- **Weighted Chop Automaton**



Results

Theorem (Iterable Expressions)

Quantitative decision problems are UNDECIDABLE

Theorem (Synchronised Iterable Expressions)

Synchronisation property is PTIME
Quantitative decision problems are DECIDABLE

- **Weighted Chop Automaton**

Theorem (Simple Expressions)

Quantitative decision problems are PSPACE-COMPLETE

- *Reversal bounded counter machines*

Thanks!