Games with lexicographically ordered ω-regular objectives

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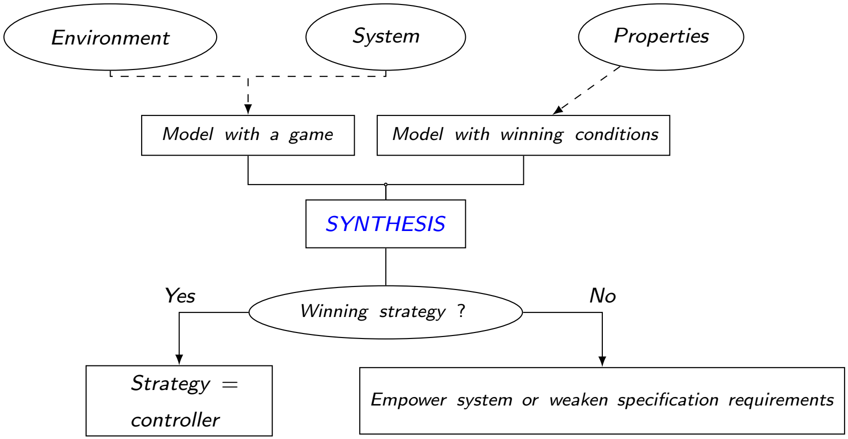
1. Introduction
2. Lexicographic games
3. Fixed parameter tractability
4. Conclusion

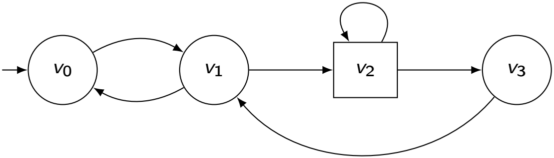
Synthesis via Game Theory



Games on graphs

- Two-player game:

system/player 1 (○) vs. the environment/player 2 (□)



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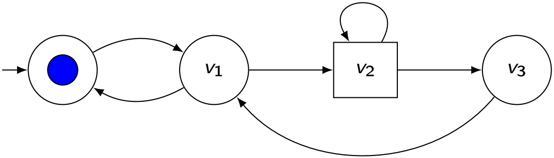

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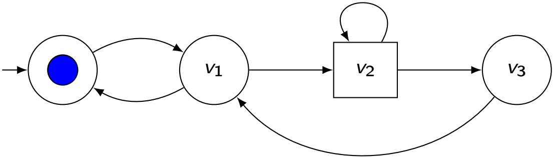

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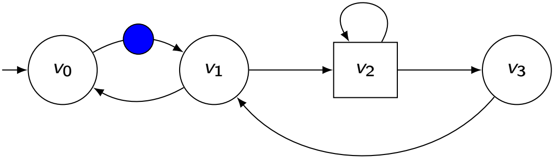

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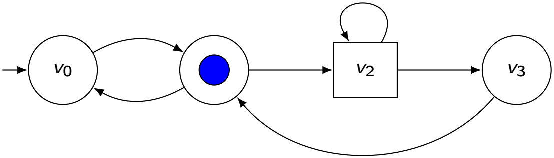

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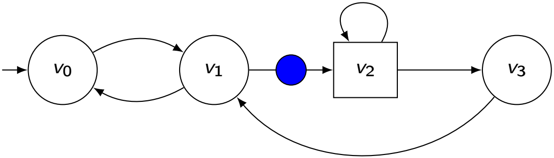

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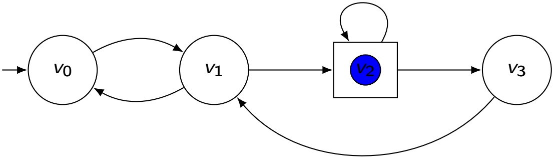

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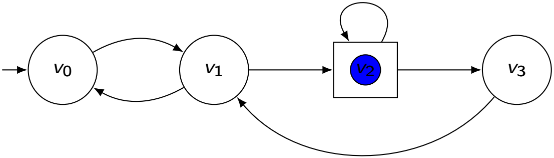

- Strategies: function that maps histories to vertex.
- Objective of player 1: set of plays.

→ Ex: v_2 has to be visited infinitely often.

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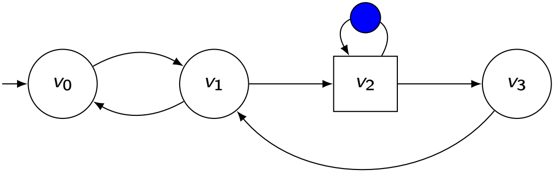

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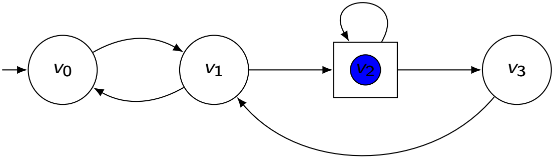

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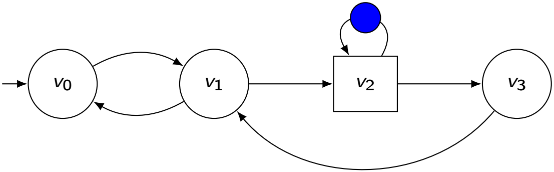

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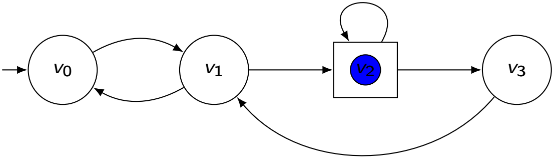

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Lexicographic game

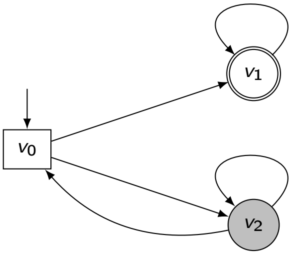
Game structure + n objectives ($n \geq 1$) $\rightsquigarrow$ lexicographic game

- every play is associated with a payoff $\in \{0, 1\}^n$,
- the i^{th} component equals 1 if the i^{th} objective is satisfied, and 0 otherwise.
- Player 1 wants to maximize the payoff (w.r.t. the lexicographic order).
- Player 2 wants to minimize the payoff (w.r.t. the lexicographic order).

$$x \prec y \iff (x_1 < y_1) \vee (x_1 = y_1 \wedge x_2 < y_2) \vee \ldots (\forall i < n, x_i = y_i \wedge x_n < y_n).$$

Example

$\Omega_1 = \text{Buchi}(\{v_1\})$, $\Omega_2 = \text{Buchi}(\{v_2\})$



Play $\pi_1 = v_0 v_1^\omega$ is s.t. $\text{Payoff}(\pi_1) = (1, 0)$.
Play $\pi_2 = v_0 v_2^\omega$ is s.t. $\text{Payoff}(\pi_2) = (0, 1)$.

Questions

Let $(G, \Omega_1, \ldots, \Omega_n)$ be a lexicographic game.

- Is the lexicographic game determined ?

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  $\Leftrightarrow$ does every vertex have a value ?
  Vertex has value $\Leftrightarrow$ highest payoff Player 1 can ensure $=$ lowest payoff Player 2 can concede

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- **Threshold problem:** Given a threshold, can Player 1 ensure a payoff $\succeq$ to this threshold ?
- Cost of computing the values.
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$\rightsquigarrow$ General approach + study of lexicographic ω-regular games

Approach

Let $(G, \Omega_1, \ldots, \Omega_n)$ be a lexicographic game.

- Reduce the threshold problem to solving a classical game with $\Omega = \cup \cap \Omega_i$.
- If $\Omega_1, \ldots, \Omega_n$ are Borel objectives, the lexicographic game is determined.
- An algorithm solving the threshold problem induces an algorithm to compute the values and the optimal strategies.

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Focus on well known ω-regular objectives: reachability, safety, Büchi, co-Büchi, Explicit Muller, parity, Streett, Rabin and Muller objectives.

ω-regular objectives

	Reachability, Safety	Büchi	Co-Büchi	Explicit Muller	Parity, Rabin, Streett, Muller
Threshold problem	PSPACE-complete	P-complete			PSPACE-complete
Values	exponential	polynomial			exponential
$\mathcal{P}_1$ memory	exponential	linear	memoryless	exponential	
$\mathcal{P}_2$ memory		memoryless	linear		

Threshold problem already studied in [BBMU12].
New here: study of Explicit Muller and lower bounds for parity, Streett and Rabin + values, optimal strategies and memory requirements and most importantly, deterministic algorithms.

FPT

Idea of an FPT algorithm:

- Input x, parameter y
- algorithm running in time $f(y) \cdot |x|^c$, f computable function, c constant.

Some results:

- Calude and al. [CJK+17] recently provided a quasipolynomial algorithm for parity games, as well as an FPT algorithm.
- There is an FPT algorithm for parity games iff there is one for Muller, Streett and Rabin games [BSV03].

  $\rightsquigarrow$ Lexicographic games with ω-regular objectives are in FPT.

- Future work: quantitative objectives

Paper available on ArXiV: `https://arxiv.org/abs/1707.05968`

Submitted to FSTTCS 2017.

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Thank you!

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