Regular separability of Parikh automata languages

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Regular separability of Parikh automata languages \( \mathbb{Z} \)-Petri nets

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1. Problem: Regular separability of $\mathbb{Z}$-Petri nets languages.

2. Result: The problem is decidable.

Main techniques

1. Deterministic = nondeterministic. (Presented before by Sławek Lasota.)

2. Regular partitioning.

3. Reduce to separability of bounded languages. (Bounded languages are of the form, e.g., $x^*y^*z^*$ for $x, y, z \in \Sigma^*$.)
Regular separability

A

B
Regular separability

“A” and “B” with “simple” (regular) arrow pointing to the right.
Regular separability

A

B

“simple” (regular)

recognised by Petri nets
Decidable separability

1. 1CN [Czerwiński, Lasota LICS’17] (one \(\mathbb{N}\)-counter, with zero test).

   Via the \textit{Regular Overapproximation} technique

\[\rightarrow\] next talk by Wojtek Czerwiński.
Decidable separability

1. 1CN [Czerwiński, Lasota LICS’17] (one $\mathbb{N}$-counter, with zero test).

   Via the Regular Overapproximation technique → next talk by Wojtek Czerwiński.

2. $\mathbb{Z}$-PN [C., Czerwiński, Lasota, Paperman ICALP’17] (many $\mathbb{Z}$-counters; no zero test).
Decidable separability

1. 1CN [Czerwiński, Lasota LICS’17] (one \(\mathbb{N}\)-counter, with zero test).

   Via the Regular Overapproximation technique
   \(\rightarrow\) next talk by Wojtek Czerwiński.

2. \(\mathbb{Z}\)-PN [C., Czerwiński, Lasota, Paperman ICALP’17]
   (many \(\mathbb{Z}\)-counters; no zero test).

3. [Conjecture] Separability of \(\mathbb{N}\)-PN languages is decidable.
Decidable separability

1. 1CN [Czerwiński, Lasota LICS’17] (one $\mathbb{N}$-counter, with zero test).

   Via the *Regular Overapproximation* technique → next talk by Wojtek Czerwiński.

2. $\mathbb{Z}$-PN [C., Czerwiński, Lasota, Paperman ICALP’17] (many $\mathbb{Z}$-counters; no zero test).

3. [Conjecture] Separability of $\mathbb{N}$-PN languages is decidable.
Regular separability of \( \mathbb{Z} \text{-PN} \)

What is a \( \mathbb{Z} \)-Petri net?

Nondeterministic finite automaton \( A \) + finitely many \( \mathbb{Z} \)-counters (no zero test).

Accepting runs: counters start and end with value zero.
Regular separability of $\mathbb{Z}$-PN

What is a $\mathbb{Z}$-Petri net?

Nondeterministic finite automaton $A$ + finitely many $\mathbb{Z}$-counters (no zero test).

Accepting runs: counters start and end with value zero.

Theorem. Regular separability is decidable for $\mathbb{Z}$-PN.
1. Deterministic = nondeterministic. (Presented before by Sławek Lasota.) → assume DFA
Solution

1. Deterministic = nondeterministic. (Presented before by Sławek Lasota.)

2. Regular partitioning.

assume DFA
Solution

1. Deterministic = nondeterministic. \[\Rightarrow\] assume DFA (Presented before by Sławek Lasota.)

2. Regular partitioning.

3. Reduce to separability of *bounded languages*. (Bounded languages are of the form, e.g., \(x^*y^*z^*\) for \(x, y, z \in \Sigma^*\).)
1. Deterministic = nondeterministic.  
(Presented before by Sławek Lasota.)

2. Regular partitioning.

3. Reduce to separability of bounded languages.  
(Bounded languages are of the form, e.g., $x^*y^*z^*$ for $x, y, z \in \Sigma^*$.)
Regular partitioning

A, B separable iff, for every i, $A \cap R_i$, $B \cap R_i$ separable
Regular partitioning: Application
Regular partitioning: Application
Regular partitioning: Application
Regular partitioning: Application

\[ R(A) \cap R(B) \]
Regular partitioning: Application

\[ \Sigma^* \setminus R(A) \cap R(B) \]

We can assume the \textit{same underlying DFA} for the two \( \mathbb{Z} \)-PNs.
1. Deterministic = nondeterministic. 
   (Presented before by Sławek Lasota.) 
   \[\rightarrow\] assume DFA

2. Regular partitioning. ✔

3. Reduce to separability of \textit{bounded languages}. 
   (Bounded languages are of the form, e.g., \(x^*y^*z^*\) for \(x, y, z \in \Sigma^*\).)
Consider *simple cycles* instead of single transitions:

- Cycles can be rearranged (once enough states have been visited).
- We can fix an order for cycles $\rightarrow$ bounded language.
Lemma. \( L(A), L(B) \) are regular separable iff \( L(A), L(B) \) are regular separable in \( x^*y^*z^* \)
Reduction to bounded languages

regular separability of \( \mathbb{Z} \text{-PN languages} \)
Reduction to bounded languages

Regular separability of $\mathbb{Z}$-PN languages $\implies$ bounded regular separability of bounded $\mathbb{Z}$-PN languages
Reduction to bounded languages

\[ \Pi(L(A)) \rightarrow \Pi(x^* y^* z^*) \rightarrow \Pi(L(B)) \]

Regular separability of \( \mathbb{Z} \)-PN languages \( \rightarrow \) bounded regular separability of bounded \( \mathbb{Z} \)-PN languages
Reduction to bounded languages

\[ \Pi(L(A)) \quad \text{semilinear} \quad \Pi(L(B)) \]

\[ \Pi(R) \quad \text{unary} \]

regular separability of \( \mathbb{Z} \)-PN languages \( \rightarrow \) bounded regular separability of bounded \( \mathbb{Z} \)-PN languages
Reduction to bounded languages

- Regular separability of $\mathbb{Z}$-PN languages

- Bounded regular separability of bounded $\mathbb{Z}$-PN languages

- Unary separability of semilinear sets

[Choffrut, Grigorieff ILP’06]
Solution

1. Deterministic = nondeterministic. ➞ assume DFA
   (Presented before by Sławek Lasota.)

2. Regular partitioning. ✓

3. Reduce to separability of bounded languages. ✓
   (Bounded languages are of the form, e.g., $x^*y^*z^*$ for $x, y, z \in \Sigma^*$.)
Conclusions

1. [Decidable] 1CN [Czerwiński, Lasota LICS’17] (one $\mathbb{N}$-counter, with zero test).

   Via the Regular Overapproximation technique
   → next talk by Wojtek Czerwiński.

2. [Decidable] $\mathbb{Z}$-PN [C., Czerwiński, Lasota, Paperman ICALP’17] (many $\mathbb{Z}$-counters; no zero test). ✔

3. [Conjecture] Separability of $\mathbb{N}$-PN languages is decidable.