Regular separability of Parikh automata languages

L. Clemente, W. Czerwiński, S. Lasota (University of Warsaw)
Ch. Paperman (University of Bordeaux)

London, September 2017

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At a glance

1. Problem: Regular separability of $\mathbb{Z}$-Petri nets languages.
2. Result: The problem is decidable.
3. Future: Decidable for $\mathbb{N}$-Petri nets? Conjecture: YES.

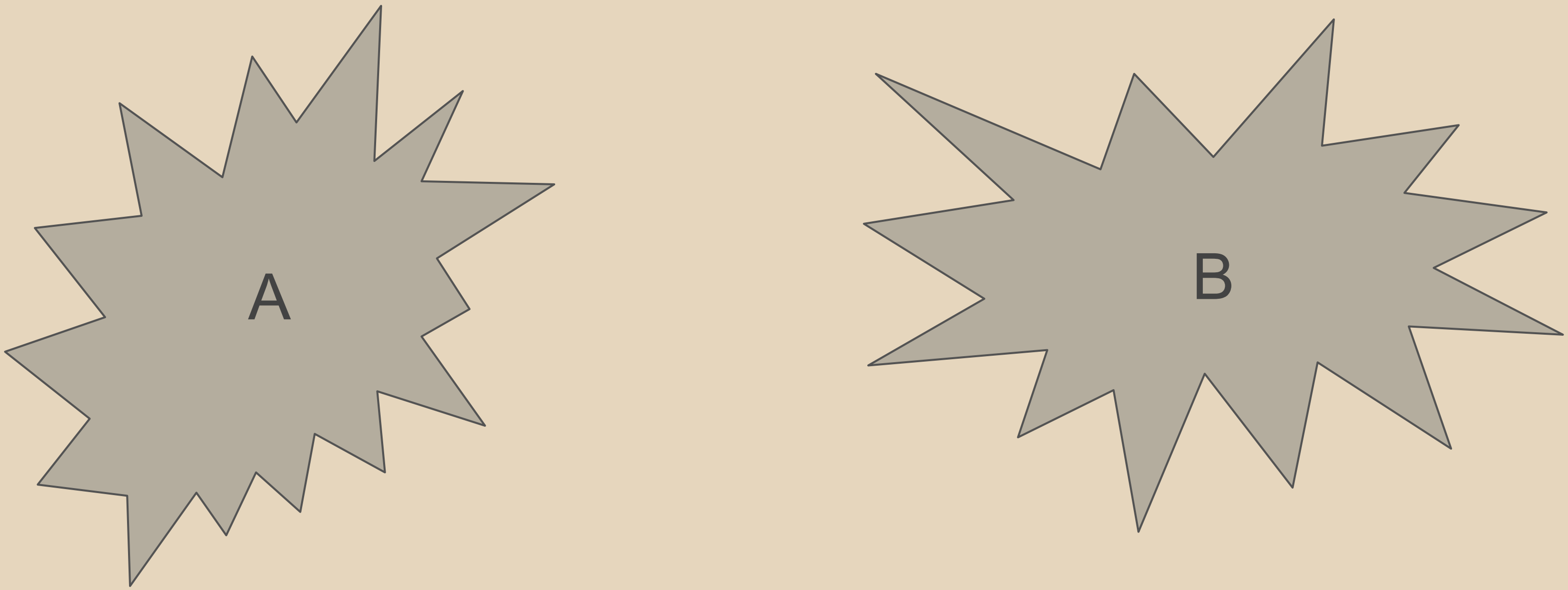
Main techniques

1. Deterministic = nondeterministic.
   (Presented before by Sławek Lasota.)

2. Regular partitioning.

3. Reduce to separability of *bounded languages*.
   (Bounded languages are of the form, e.g., $x^*y^*z^*$ for $x, y, z \in \Sigma^*$.)

Regular separability



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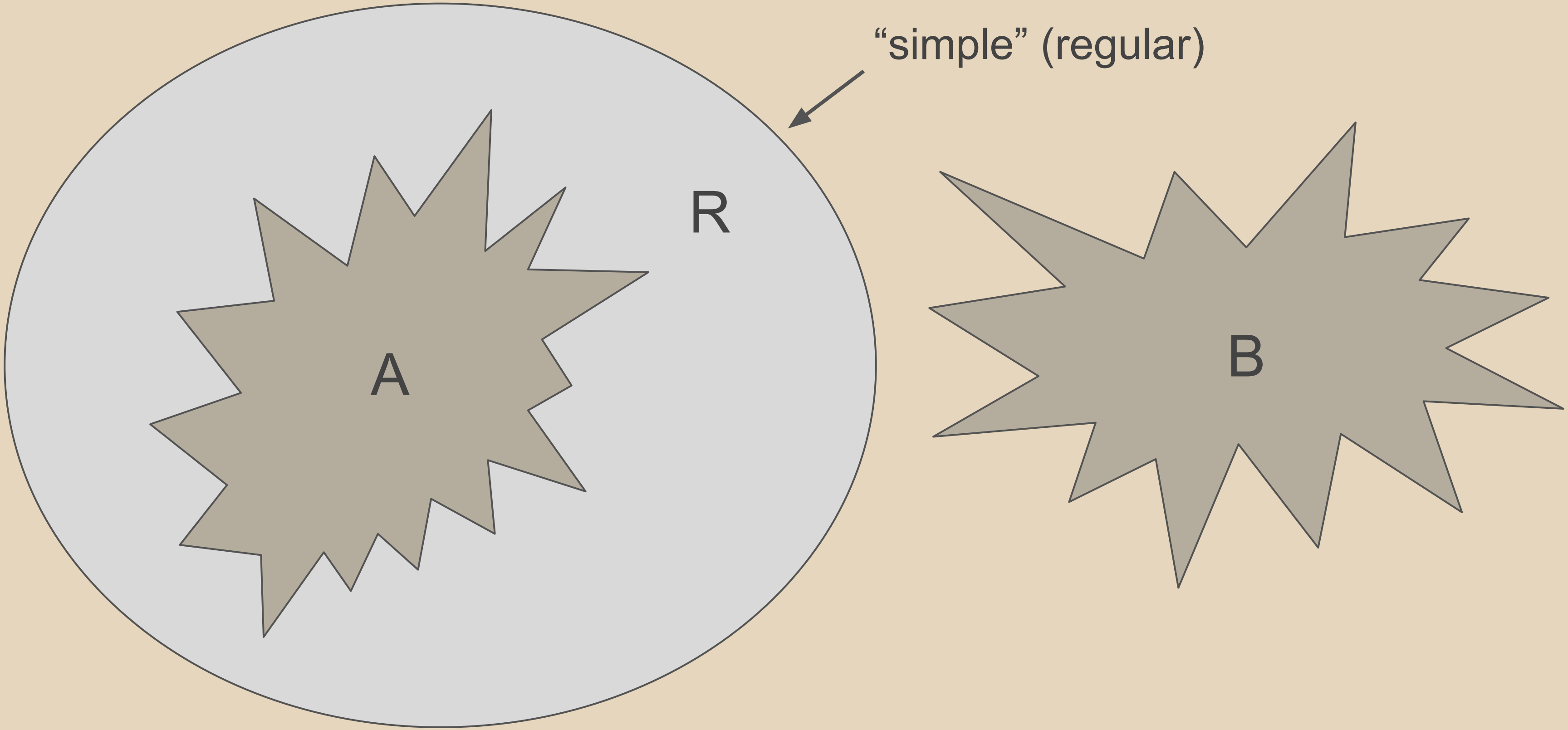

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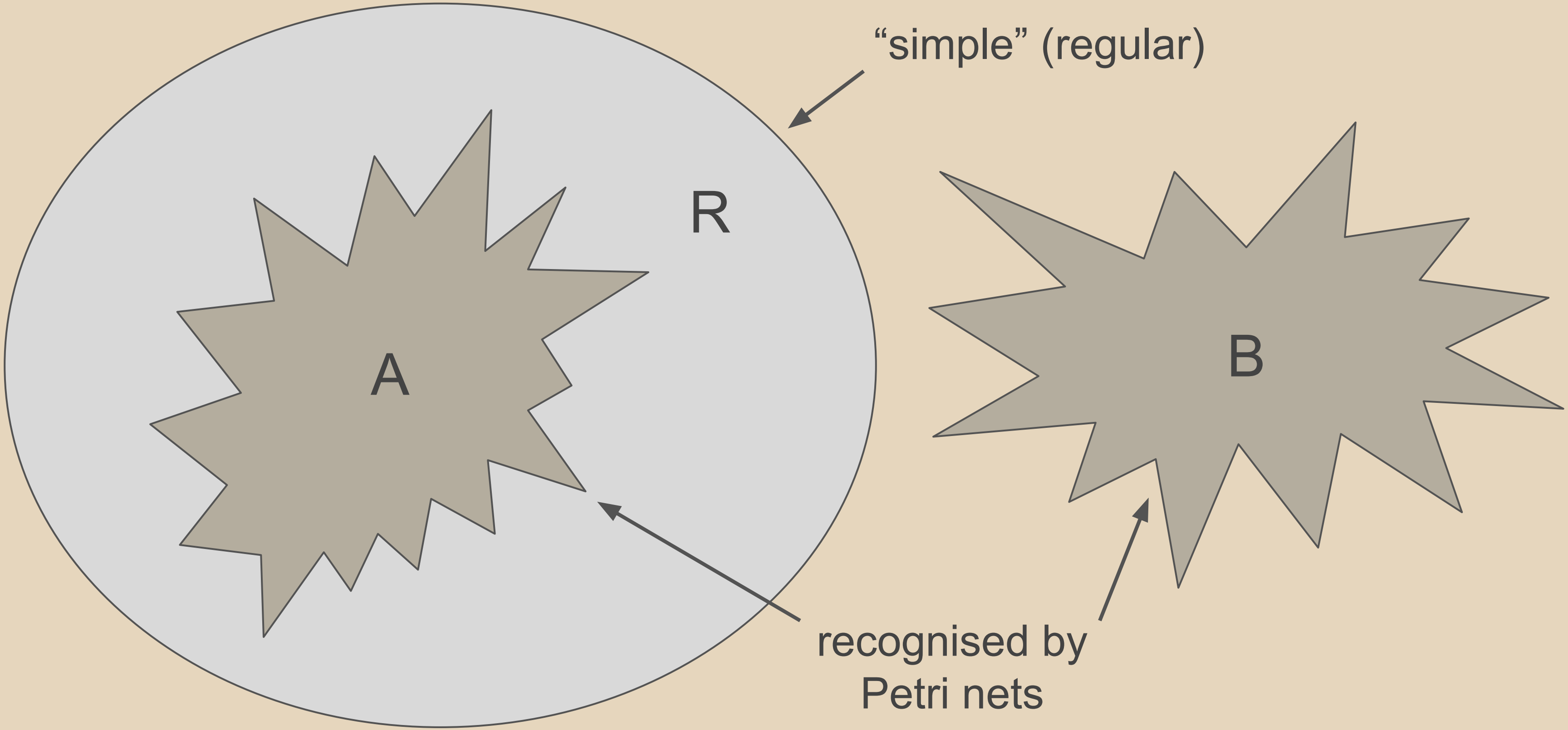

Decidable separability

1. 1CN [Czerwiński, Lasota LICS’17]
   (one $\mathbb{N}$-counter, with zero test).

   Via the *Regular Overapproximation* technique
   $\rightarrow$ **next talk by Wojtek Czerwiński.**

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Regular separability of $\mathbb{Z}$-PN

What is a $\mathbb{Z}$-Petri net?

Nondeterministic finite automaton A
+ finitely many $\mathbb{Z}$-counters (no zero test).

Accepting runs: counters start and end with value zero.

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Theorem. Regular separability is decidable for $\mathbb{Z}$-PN.

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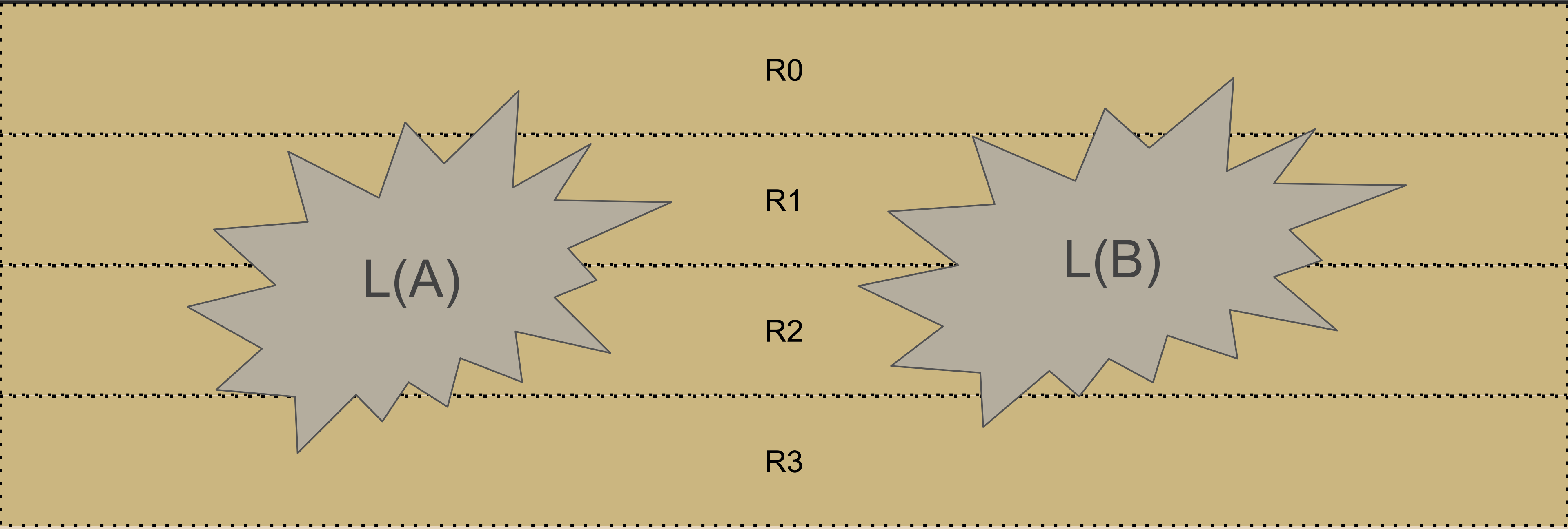
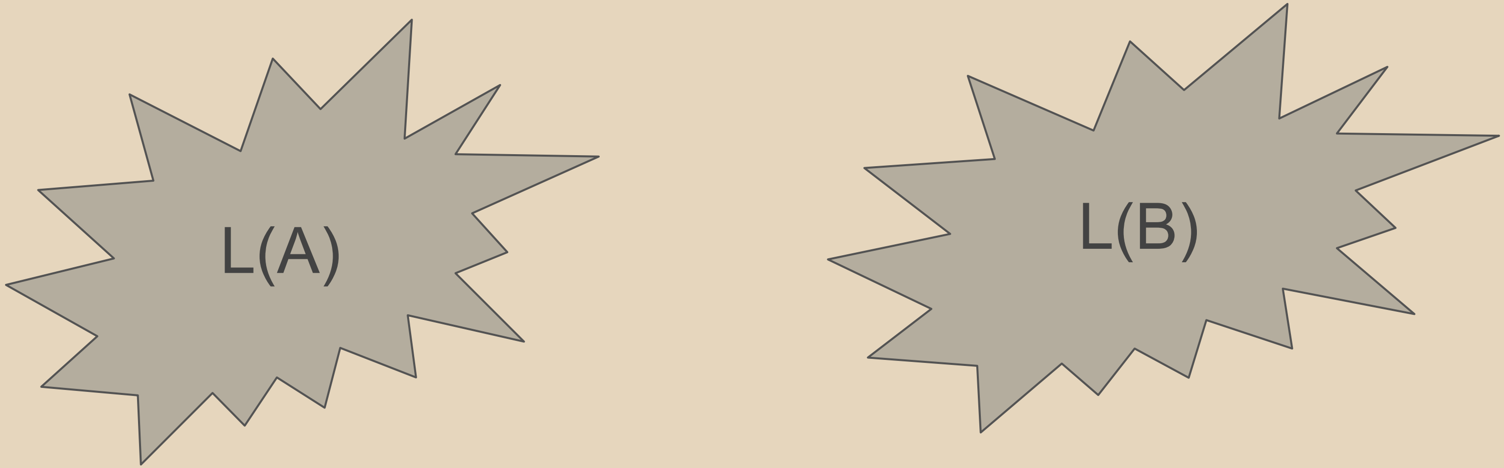


A, B separable iff, for every i, A ∩ Ri, B ∩ Ri separable

Regular partitioning: Application



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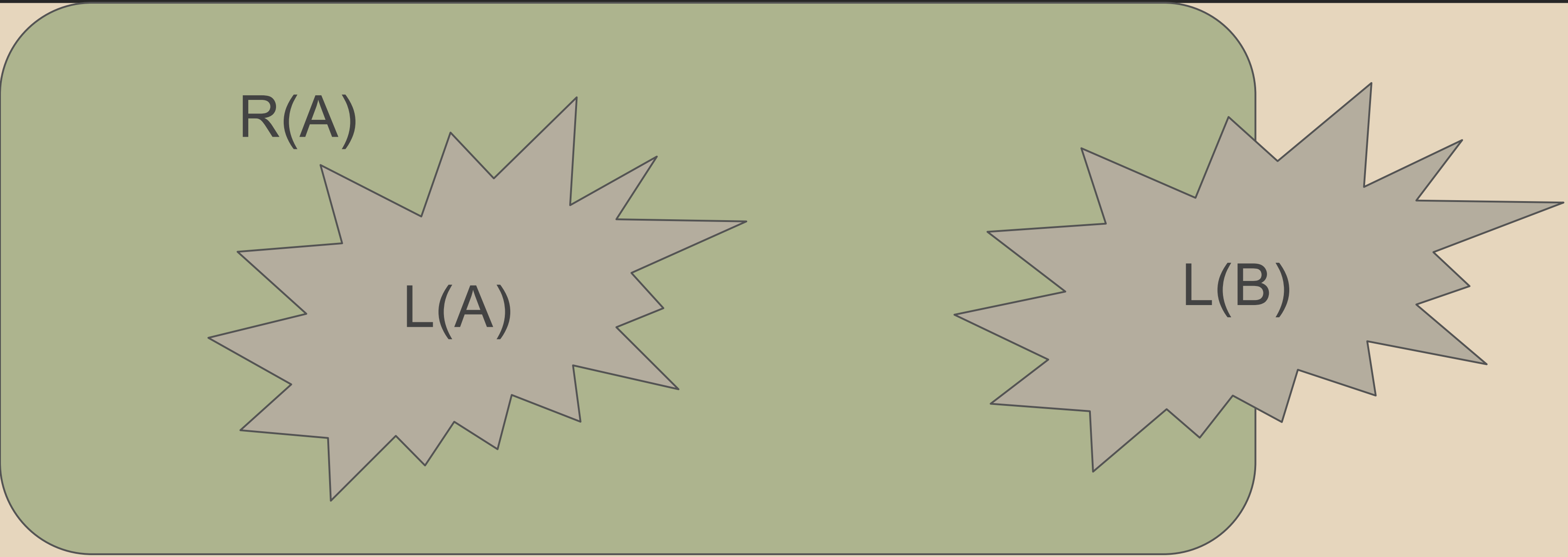

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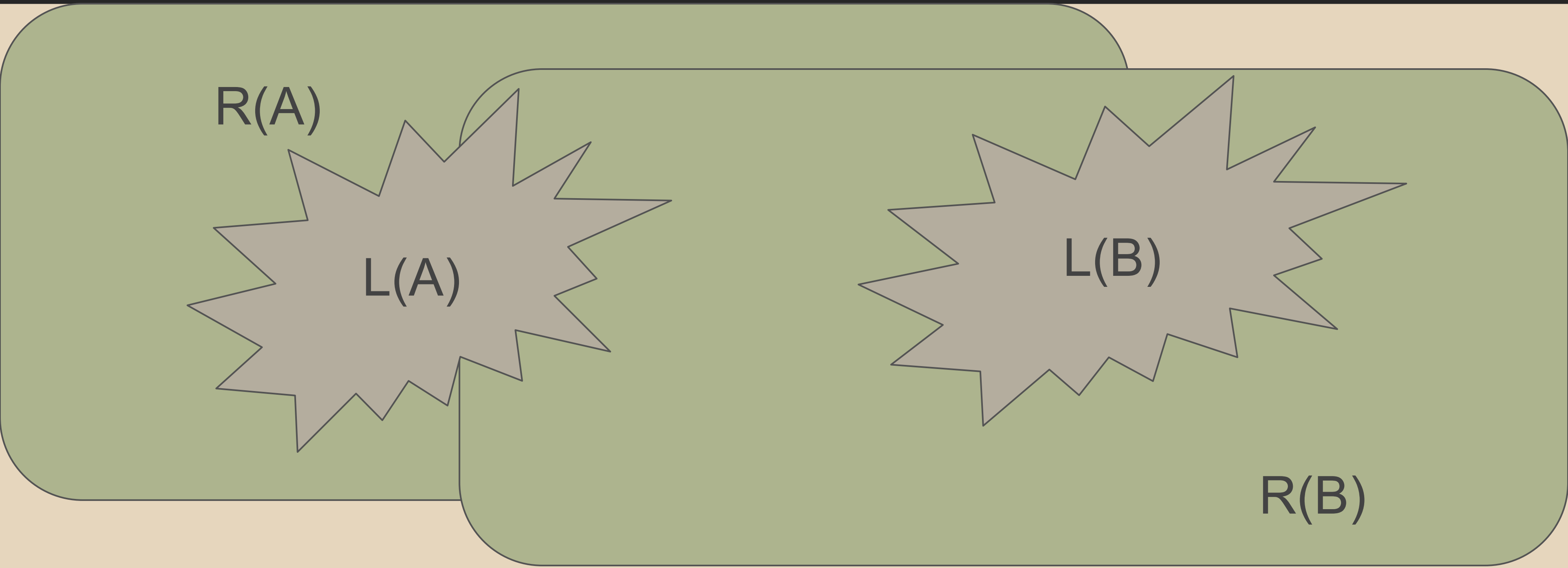

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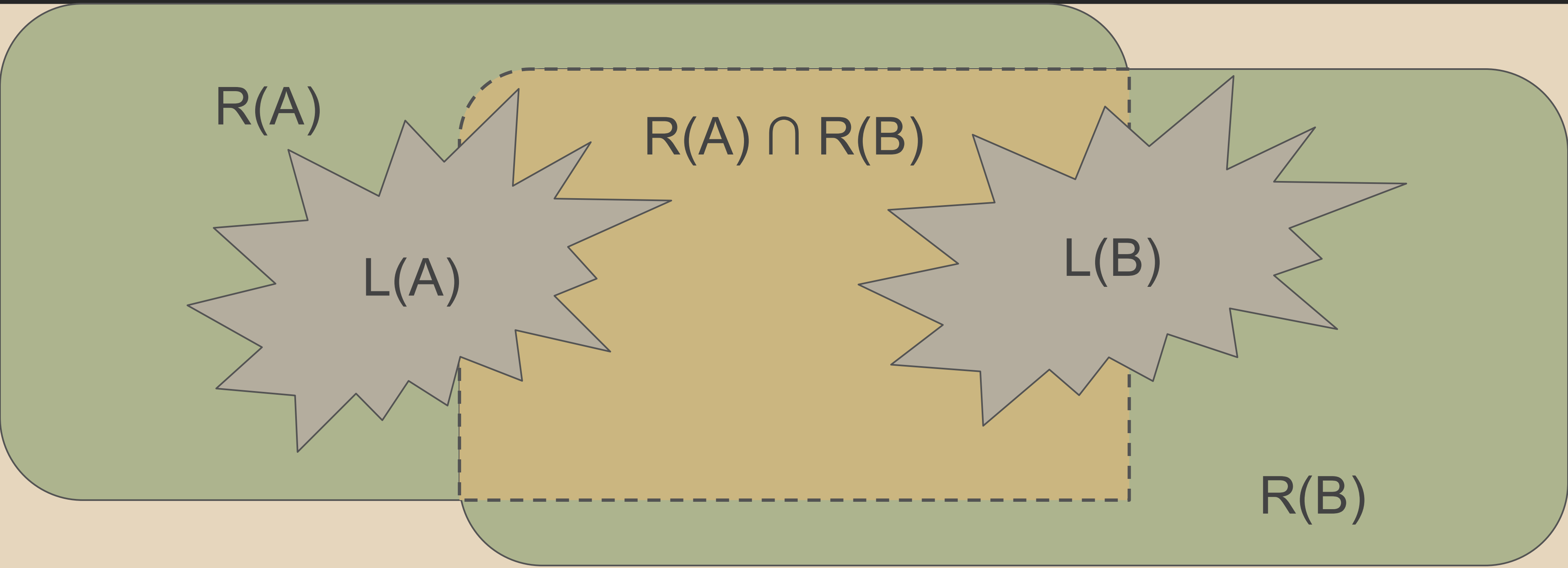

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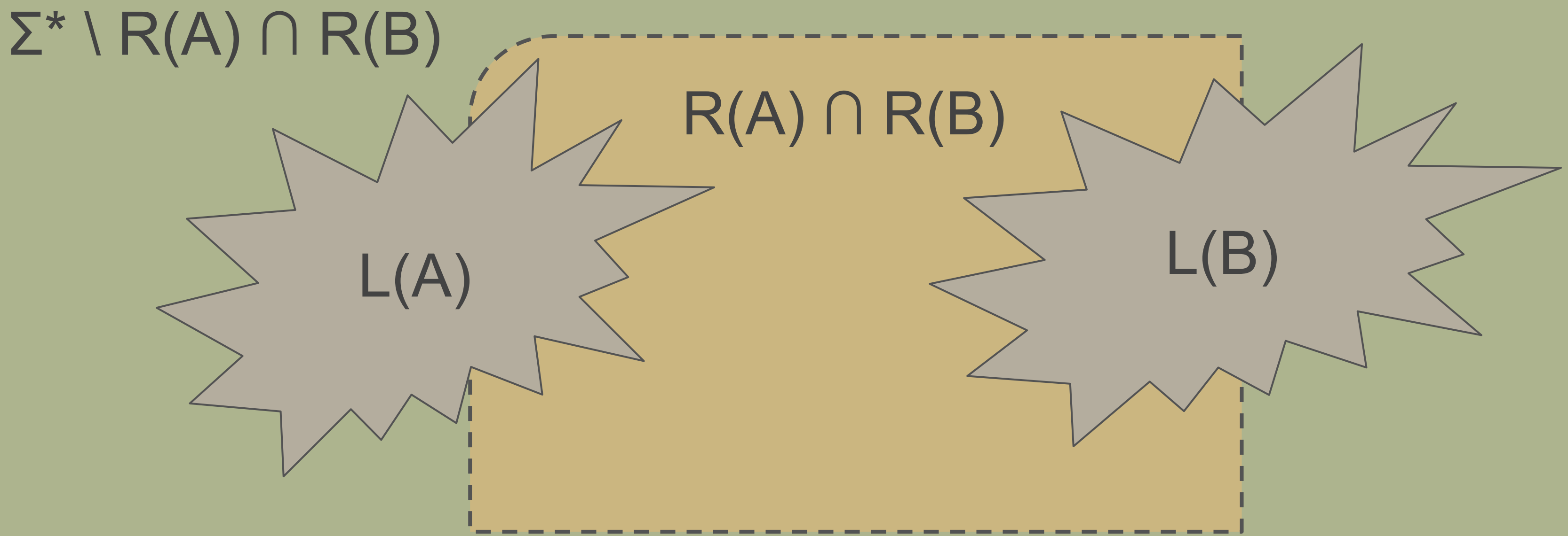

We can assume the *same underlying DFA* for the two ℤ-PNs.

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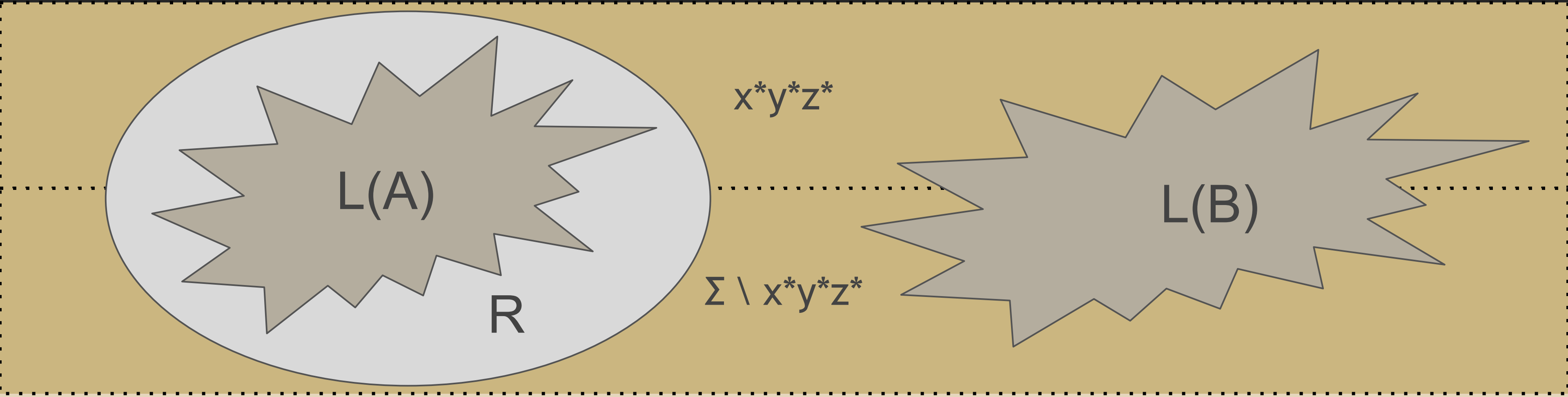
Reduction to bounded languages

Consider *simple cycles* instead of single transitions:

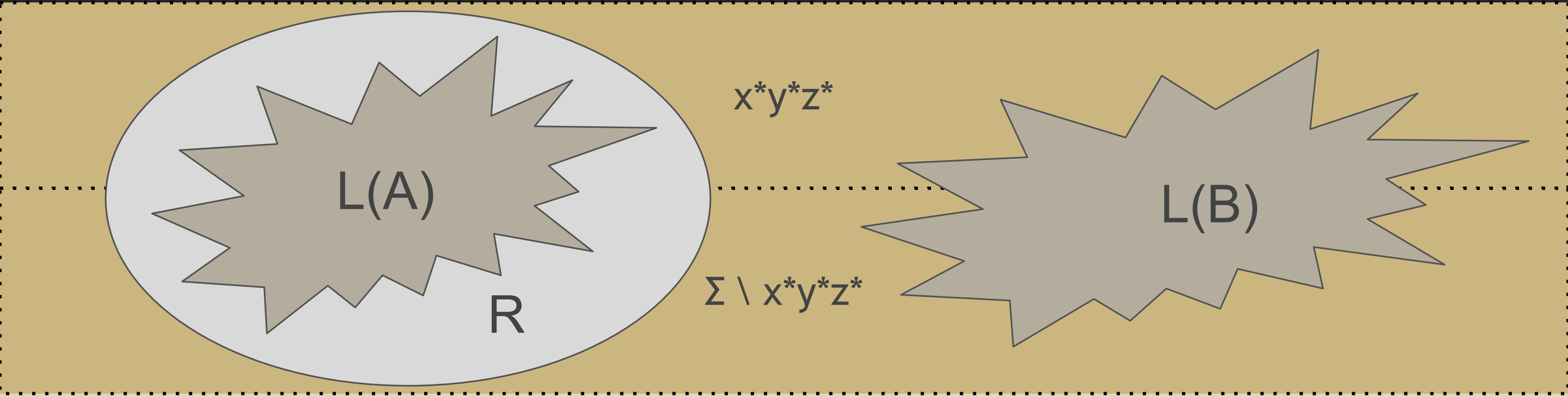
- Cycles can be rearranged (once enough states have been visited).
- We can fix an order for cycles → bounded language.

Reduction to bounded languages



Lemma. L(A), L(B) are regular separable iff L(A), L(B) are regular separable in x*y*z*

Reduction to bounded languages



regular separability of $\mathbb{Z}$-PN languages →

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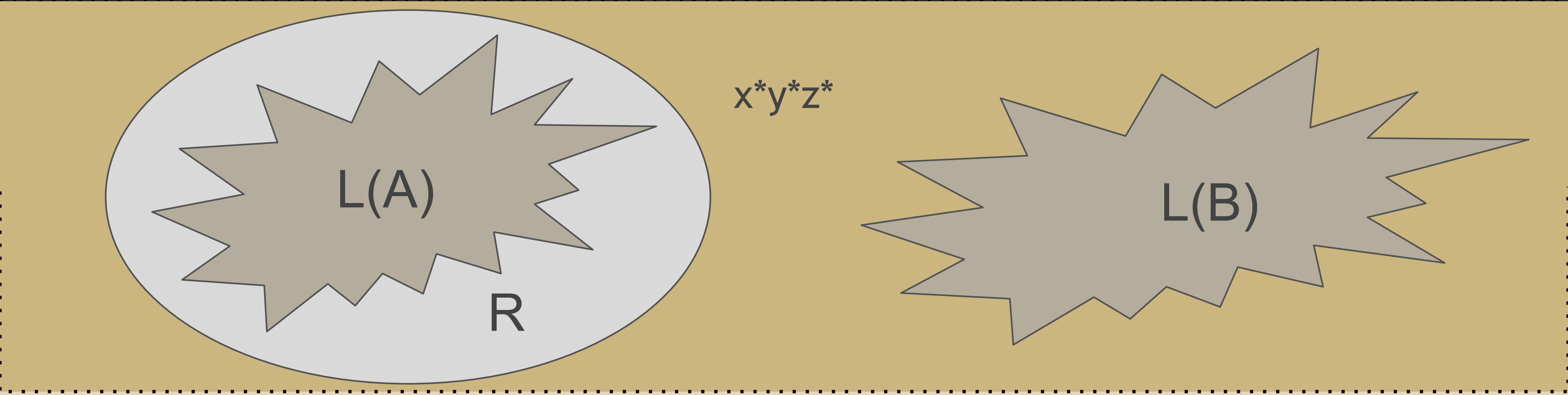

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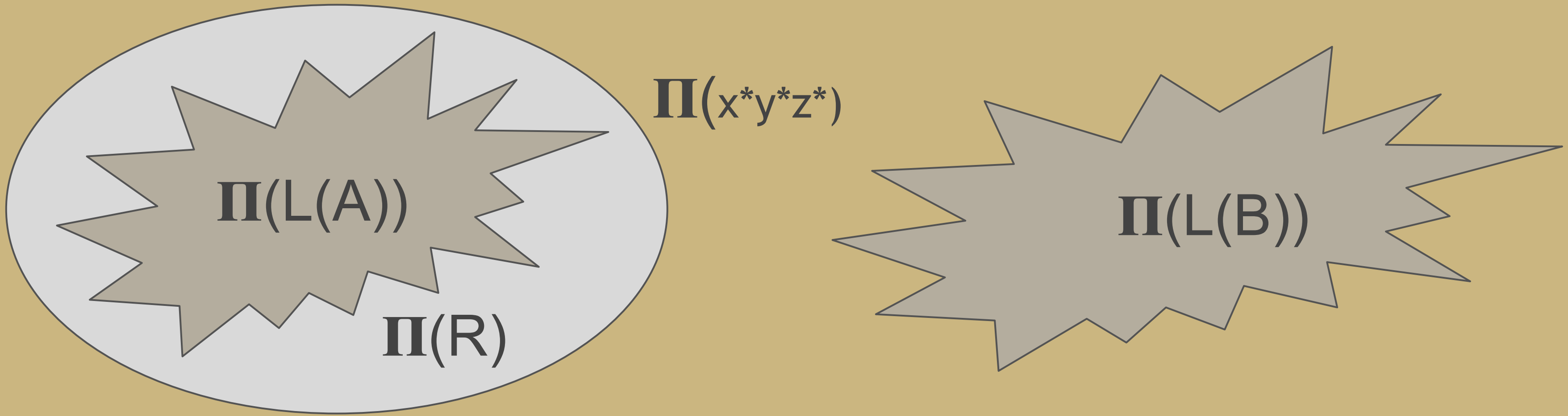

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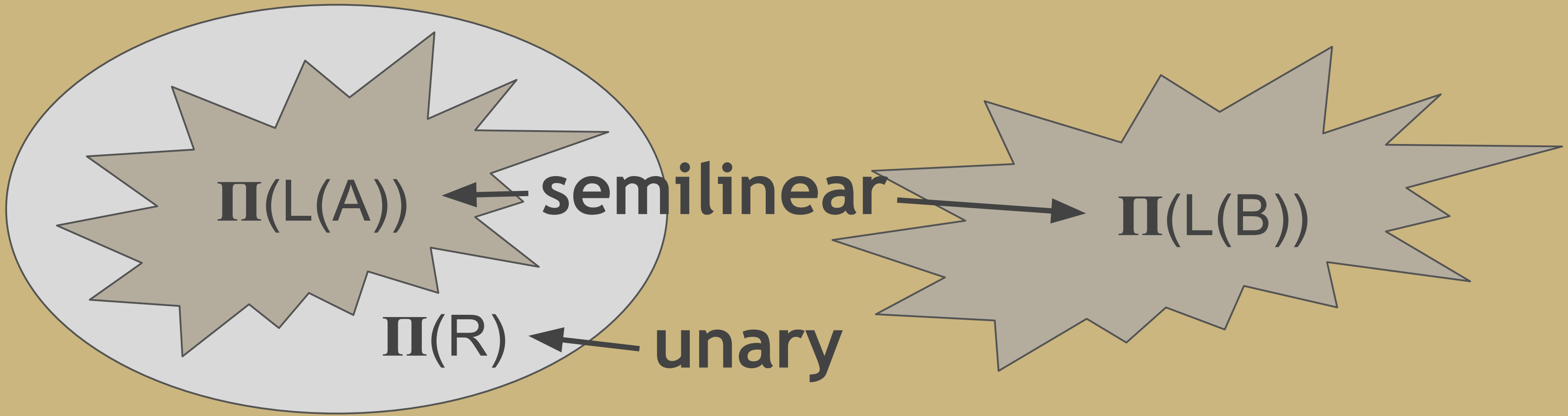
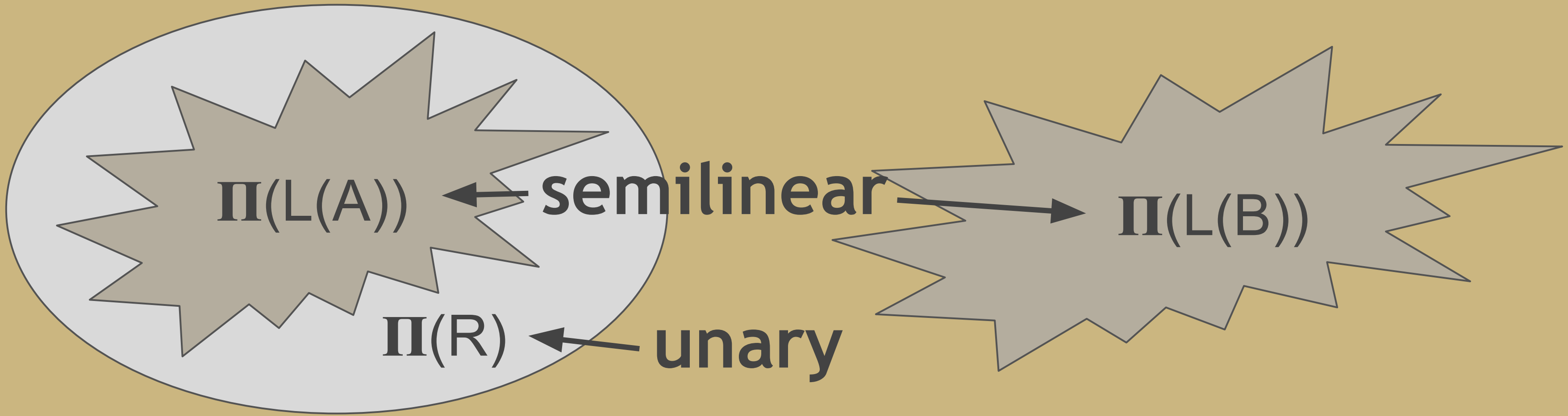


regular separability of ℤ-PN languages → *bounded* regular separability of *bounded* ℤ-PN languages

Reduction to bounded languages



regular separability of $\mathbb{Z}$-PN languages → *bounded* regular separability of *bounded* $\mathbb{Z}$-PN languages → unary separability of semilinear sets

[Choffrut, Grigorieff ILP'06]

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Conclusions

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