Bounded-Regret Determinization of Max-Plus Automata

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Abstract

We give tight complexity bounds for the problem of determining if a given max-plus automaton is bounded-regret determinizable. This generalizes results of Aminof et al. regarding determinization of max-plus automata by pruning.

More precisely, we study strategies to resolve the non-determinism of a max-plus automaton on the fly: for every word — given letter by letter — the strategy yields a run of the automaton. The regret of the strategy is then defined as the maximal difference between the value of a constructed run for a word and the value assigned to that word by the automaton. We establish that, given a max-plus automaton and a regret bound, determining if there is a strategy with regret of at most that bound, is EXPTIME-complete.

Max-plus automata. *Weighted automata* (WA, for short) correspond to the quantitative generalization of finite automata with weights on transitions: instead of a mapping from (finite) words to $\{0, 1\}$ — that is, the indicator function of the machine's *language* — they realize functions from words to values of a semiring.

A *max-plus automaton* $\mathcal{N}$ is a WA over the semiring $(\mathbb{Z} \cup \{-\infty\}, \max, +)$. The value of a run is the sum of the weights occurring on its transitions, and the value of a word is the maximal value of all its accepting runs. Absent transitions have a weight of $-\infty$ and runs of value $-\infty$ are considered non-accepting. This defines a partial function denoted $[\![\mathcal{N}]\!] : \Sigma^* \to \mathbb{Z}$ whose domain is denoted by $\mathcal{L}_{\mathcal{N}}$.

Determinization. In contrast with unweighted automata, it is known that functions definable by non-deterministic max-plus automata strictly contain those definable by deterministic max-plus automata. This motivates the search for an algorithm which decides, given a max-plus automaton, whether a deterministic max-plus automaton defining the same function exists. To this day, the largest class for which this problem is known to be decidable is the class of polynomially ambiguous max-plus automata [KL09].

Bounded-regret determinization. Non-deterministic max-plus automata can be used to formalize competitive analysis of online algorithms [AKL10]. Intuitively, the "unbounded look-ahead" of an offline algorithm is captured by the non-determinism. A competitive online algorithm thus corresponds to a strategy to resolve, for every input α, the non-determinism of automaton $\mathcal{N}$ on the fly, while ensuring the value of the constructed run has *minimal regret* with respect to $[\![\mathcal{N}]\!](\alpha)$. If there exists such a strategy with regret bounded by r, we say $\mathcal{N}$ is *bounded-regret determinizable.*

It transpires that if a max-plus automaton is bounded-regret determinizable, then it is also determinizable.

Contributions. We show that determining if a given max-plus automaton is bounded-regret determinizable is decidable in exponential time. Furthermore, if the regret bound is given as input — instead of existentially quantified — the problem is EXPTIME-complete. Interestingly, the upper bound is obtained using a quantitative version of the *Joker game* defined by Kuperberg and Skrzypczak [KS15]. This quantitative Joker game is a type of energy game with resets.

This talk is based on results from [FJL$^+$17].

References

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