# Bounded-Delay Determinization of Max-Plus Automata

Ismaël Jecker

Université libre de Bruxelles

**Abstract**

Determining whether a weighted automaton $\mathcal{N}$ over the semiring $(\mathbb{Z} \cup \{-\infty\}, \max, +)$ is determinizable is a long-standing open problem. We propose to approach it with the bounded delay determinization problem: instead of asking whether there exists a deterministic weighted automaton $\mathcal{D}$ equivalent to $\mathcal{N}$, we add the restriction that $\mathcal{D}$ needs to be close to $\mathcal{N}$ with respect to the *delay distance*.

We prove that $\mathcal{N}$ is determinizable if and only if there exists an integer $k$ such that $\mathcal{N}$ is $k$-delay determinizable. Hence the bounded delay determinization problem is as difficult as the general determinization problem. When $k$ is given as input, we show that deciding whether $\mathcal{N}$ is $k$-delay determinizable is EXPTIME-complete.

**Max-plus automata.** *Weighted automata* (WA, for short) correspond to the quantitative generalization of finite automata with weights on transitions: instead of a mapping from (finite) words to $\{0, 1\}$ — that is, the indicator function of the machine's *language* — they realize functions from words to values of a semiring.

A *max-plus automaton* $\mathcal{N}$ is a WA over the semiring $(\mathbb{Z} \cup \{-\infty\}, \max, +)$. The value of a run is the sum of the weights occurring on its transitions, and the value of a word is the maximal value of all its accepting runs. Absent transitions have a weight of $-\infty$ and runs of value $-\infty$ are considered non-accepting. This defines a partial function denoted $[\![\mathcal{N}]\!] : \Sigma^* \to \mathbb{Z}$ whose domain is denoted by $\mathcal{L}_{\mathcal{N}}$.

**Determinization.** In contrast with unweighted automata, it is known that functions definable by non-deterministic max-plus automata strictly contain those definable by deterministic max-plus automata. This motivates the search for an algorithm which decides, given a max-plus automaton, whether a deterministic max-plus automaton defining the same function exists. To this day, the largest class for which this problem is known to be decidable is the class of polynomially ambiguous max-plus.

**Bounded-delay determinization.** Let $k$ be a positive integer. A WA $\mathcal{N}$ is *$k$-delay determinizable* if there exists a deterministic automaton $\mathcal{D}$ that defines the same function as $\mathcal{N}$, and such that for every word $\alpha$ in $\mathcal{L}_{\mathcal{N}}$, there exists a maximal accepting run $\rho$ of $\mathcal{N}$ on $\alpha$ such that the accepting run of $\mathcal{D}$ on $\alpha$ is always at most $k$-away from $\rho$. That is, along all prefixes of the same length, the absolute difference between the running sums of weights of the two runs is at most $k$. If there exists a $k$ such that $\mathcal{N}$ is $k$-delay determinizable, we say that $\mathcal{N}$ is bounded-delay determinizable.

**Contributions.** We show that for every integer $k$, determining whether a given max-plus automaton is $k$-delay determinizable is EXPTIME-complete. Moreover, we prove that a max-plus automaton is determinizable if and only if it is also bounded-delay determinizable.

This talk is based on results from the paper "On Delay and Regret Determinization of Max-Plus Automata" (ICALP 2017), written in collaboration with Emmanuel Filiot, Nathan Lhote, Guillermo A. Pérez, and Jean-François Raskin.