Bounded-Delay Determinization of Max-Plus Automata

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Abstract

Determining whether a weighted automaton $\mathcal{N}$ over the semiring $(\mathbb{Z} \cup \{-\infty\}, \max, +)$ is determinizable is a long-standing open problem. We propose to approach it with the bounded delay determinization problem: instead of asking whether there exists a deterministic weighted automaton $\mathcal{D}$ equivalent to $\mathcal{N}$, we add the restriction that $\mathcal{D}$ needs to be close to $\mathcal{N}$ with respect to the *delay distance*.

We prove that $\mathcal{N}$ is determinizable if and only if there exists an integer k such that $\mathcal{N}$ is k-delay determinizable. Hence the bounded delay determinization problem is as difficult as the general determinization problem. When k is given as input, we show that deciding whether $\mathcal{N}$ is k-delay determinizable is EXPTIME-complete.

Max-plus automata. *Weighted automata* (WA, for short) correspond to the quantitative generalization of finite automata with weights on transitions: instead of a mapping from (finite) words to $\{0, 1\}$ — that is, the indicator function of the machine's *language* — they realize functions from words to values of a semiring.

A *max-plus automaton* $\mathcal{N}$ is a WA over the semiring $(\mathbb{Z} \cup \{-\infty\}, \max, +)$. The value of a run is the sum of the weights occurring on its transitions, and the value of a word is the maximal value of all its accepting runs. Absent transitions have a weight of $-\infty$ and runs of value $-\infty$ are considered non-accepting. This defines a partial function denoted $[\![\mathcal{N}]\!] : \Sigma^* \to \mathbb{Z}$ whose domain is denoted by $\mathcal{L}_{\mathcal{N}}$.

Determinization. In contrast with unweighted automata, it is known that functions definable by non-deterministic max-plus automata strictly contain those definable by deterministic max-plus automata. This motivates the search for an algorithm which decides, given a max-plus automaton, whether a deterministic max-plus automaton defining the same function exists. To this day, the largest class for which this problem is known to be decidable is the class of polynomially ambiguous max-plus.

Bounded-delay determinization. Let k be a positive integer. A WA $\mathcal{N}$ is *k-delay determinizable* if there exists a deterministic automaton $\mathcal{D}$ that defines the same function as $\mathcal{N}$, and such that for every word α in $\mathcal{L}_{\mathcal{N}}$, there exists a maximal accepting run ρ of $\mathcal{N}$ on α such that the accepting run of $\mathcal{D}$ on α is always at most k-away from ρ. That is, along all prefixes of the same length, the absolute difference between the running sums of weights of the two runs is at most k. If there exists a k such that $\mathcal{N}$ is k-delay determinizable, we say that $\mathcal{N}$ is bounded-delay determinizable.

Contributions. We show that for every integer k, determining whether a given max-plus automaton is k-delay determinizable is EXPTIME-complete. Moreover, we prove that a max-plus automaton is determinizable if and only if it is also bounded-delay determinizable.

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