From LTL and Limit-Deterministic Büchi Automata to Deterministic Parity Automata

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Abstract. Controller synthesis for general linear temporal logic (LTL) objectives is a challenging task. The standard approach involves translating the LTL objective into a deterministic parity automaton (DPA) by means of the Safra-Piterman construction. One of the challenges is the size of the DPA, which often grows very fast in practice, and can reach double exponential size in the length of the LTL formula. In this talk I describe a single exponential translation from limit-deterministic Büchi automata (LDBA) to DPA, and show that it can be concatenated with a recent efficient translation from LTL to LDBA to yield a double exponential, "Safraless" LTL-to-DPA construction. Lastly I report on an implementation, a comparison with the SPOT library, and performance on several sets of formulas, including instances from the 2016 SyntComp competition. This work has been published in [EKRS17] and is joint work with Javier Esparza (Technische Universität München), Jan Křetínský (Technische Universität München), and Jean-François Raskin (Université libre de Bruxelles).

Limit-deterministic Büchi automata (LDBA, also known as semi-deterministic Büchi automata) were introduced by Vardi to solve the qualitative probabilistic model-checking problem: Decide if the executions of a Markov chain or Markov Decision Process satisfy a given LTL formula with probability 1 [Var85,VW86,CY95]. The problem faced by these authors was that fully nondeterministic Büchi automata (NBAs), which are as expressible as LTL, and more, cannot be used for probabilistic model checking, and deterministic Büchi automata (DBA) are less expressive than LTL. The solution was to introduce LDBAs as a model in-between: as expressive as NBAs, but deterministic enough.

After these papers, LDBAs received little attention. The alternative path of translating the LTL formula into an equivalent fully deterministic Rabin automaton using Safra's construction [Saf88] was considered a better option, mostly because it also solves the quantitative probabilistic model-checking problem (computing the probability of the executions that satisfy a formula). However, recent papers have shown that LDBAs were unjustly forgotten. Blahoudek *et al.* have shown that LDBAs are easy to complement [BHS+16]. Kini and Viswanathan have given a single exponential translation of $\mathrm{LTL}_{\setminus \mathbf{GU}}$ to LDBA [KV15]. Finally, Sickert *et al.* describe in [SEJK16] a double exponential translation for full LTL that can also be applied to the quantitative case, and behaves better than Safra's construction in practice.

In this talk I'll add to this trend by showing that LDBAs as an intermediate step are also attractive for synthesis. The standard solution to the synthesis problem with LTL objectives consists of translating the LTL formula into a deterministic parity automaton (DPA) with the help of the Safra-Piterman construction [Pit07]. While limit-determinism is not "deterministic enough" for the synthesis problem, we introduce a conceptually simple and worst-case optimal translation LDBA→DPA. Our translation exploits the fact that after seeing an accepting state runs cannot branch anymore and thus bears some similarities with that of [Fin15] where, however, a Muller acceptance condition is used. This condition can also be phrased as a Rabin condition, but not as a parity condition.

Together with the translation LTL→LDBA of [SEJK16], the construction provides a "Safraless", procedure to obtain a DPA from an LTL formula. However, the direct concatenation of the two constructions does not yield an algorithm of optimal complexity: the LTL→LDBA translation is double exponential (and there is a double-exponential lower bound), and so for the LTL→DPA translation we only obtain a triple exponential bound. This problem is solved by showing that the LDBAs derived from LTL formulas satisfy a special property, and prove that for such automata the concatenation of the two constructions remains double exponential. To the best of our knowledge, this is the first double exponential "Safraless" LTL→DPA procedure. (Another asymptotically optimal "Safraless" procedure for determinization of Büchi automata with Rabin automata as target has been presented in [FKVW15].)

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