Automata in the category of gueings of vector spaces*

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joint-work with Daniela Petrişan

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In this paper, we introduce a novel form of automata: "hybrid-set-vector automata" that accepts maps from words to a field. These automata possess a finite set of states, and to each state is attached a vector space of finite dimension over this field. Upon reading a letter, the automaton updates deterministically the finite state, and applies a linear map to the content of the vector space. In this respect, this is a natural joint generalization of deterministic finite state automata together with Schützenberger's automata weighted aver a field [3]. Our automata come with an extra feature: the vector spaces corresponding to the different states are glued together in a way inspired by the free co-completion of a category. Thanks to this last characteristic, the "hybrid-set-vector automata" enjoy the existence of a minimal automata for a language. These minimal automata may trade states for dimensions in the vector spaces, and as a consequence save resources compared to the minimal automata weighted over a field.

Concretely, the development of this work is performed at a categorical level. Given a category, we construct the category of its *glueings*, which is a restriction of its free co-completion. Under suitable assumptions on the category, the automata that use its glueings as configuration spaces admit minimal acceptors for a given language.

Though the existence of the minimal automaton is proved, we do not know yet how to effectivelly compute it. This question is a generalization of the open question of sequentialization of weighted automata as studied by Lombardy and Sakarovitch [2].

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References

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