Regular Separability of Parikh Automata

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Abstract

We investigate a subclass of languages recognised by vector addition systems, namely languages of nondeterministic Parikh automata. While the regularity problem (is the language of a given automaton regular?) is undecidable for this model, we surprisingly show decidability of the regular separability problem: given two Parikh automata, is there a regular language that contains one of them and is disjoint from the other? This is based on our recent work [4].

Context. We investigate regular separability for languages of finite words. We say that a language U is *separated from* a language V *by* S if $U \subseteq S$ and $V \cap S = \emptyset$. In the sequel we also often say that U *and* V are *separated by* S. For a family of languages $\mathcal{G}$, the *regular separability problem for* $\mathcal{G}$ asks for two given languages $U, V \in \mathcal{G}$ whether U is separated from V by some regular language.

While separability of regular languages by subclasses thereof has been extensively studied [7, 15, 9, 14, 17, 16], (regular) separability of *nonregular* languages attracted little attention till now. The reasons for this may be twofold. First, for regular languages one can use standard algebraic tools, like syntactic monoids, and indeed most of the results have been obtained with the help of such techniques. Second, some strong intractability results have been known already since 70's, when Szymanski and Williams proved that regular separability of context-free languages is undecidable [18]. Later Hunt [10] generalized this result for every class $\mathcal{F}$ closed under finite boolean combinations and containing all languages of the form $w\Sigma^*$ for $w \in \Sigma^*$. This is a very weak condition, so it seemed that nothing nontrivial can be done outside regular languages with respect to separability problems. Furthermore, Szymanski and Williams's negative result has recently been strengthened by considering two incomparable subclasses of pushdown automata. First, Kopczyński has shown that regular separability is undecidable for languages of visibly pushdown automata [13], and then Czerwiński and Lasota have shown that the same problem is undecidable for one counter automata [6].

On the positive side, piecewise testable separability has been shown decidable for context-free languages, languages of vector addition systems (VAS languages), and some other classes of languages [8]. Another surprising result has been recently obtained by Czerwiński and Lasota [6] who show that regular separability is decidable (and PSPACE-complete) for languages recognized by one counter nets (i.e., one counter automata without zero test). Notice that in all these examples regularity (resp. piecewise testability) is undecidable, but regular (resp.

piecewise testable) separability *is* decidable, and until recently there were not many results of this kind.

Finally, in [5] we have shown decidability of *unary separability* of reachability sets of vector addition systems (VASes). By *unary sets* we mean Parikh images of commutative regular languages, and thus the latter problem is equivalent to commutative regular separability of (commutative closures of) VAS languages. The decidability status of the regular separability problem for the whole class of VAS languages remains open.

Our result. We report a further progress towards solving the open problem above by providing a positive decidability result and a new negative undecidability result: As our first (positive) result, we show decidability of the regular separability problem for the subclass of VAS languages where we allow negative counter values during a run. This class of languages is also known as languages of *integer VASSes*, and it admits many different characterizations; for instance, it coincides with languages of *one-way reversal-bounded counter machines* [11], *Parikh automata* [12] (cf. also [1, Proposition 11]), which in turn are equivalent to the very similar model of *constrained automata* [2]. In this paper, we present our results in terms of constrained automata, but given the similarity with Parikh automata (and in light of their equivalence), we overload the name Parikh automata for both models.

Notice that PA languages are not closed under complement, and thus our decidability result about regular separability does not imply decidability of the regularity problem (is the language of a given Parikh automaton regular?). Moreover, the regularity problem for PA languages is actually *undecidable* [1][1], which makes our decidability result one of few instances where regularity is undecidable but regular separability is decidable.

Parikh automata are finite nondeterministic automata where accepting runs are further restricted to satisfy a semilinear condition on the multiset of transitions appearing in the run. Our decidability result is actually stated in the more general setting of $\mathcal{C}$*-Parikh automata*, where $\mathcal{C} \subseteq \bigcup_{d\in\mathbb{N}} \mathcal{P}(\mathbb{N}^d)$ is a class of sets of vectors used as an acceptance condition. We prove that the regular separability problem for languages of $\mathcal{C}$-Parikh automata effectively reduces to the *unary* separability problem for the class $\mathcal{C}$ itself, provided that $\mathcal{C}$ is effectively closed under inverse images of affine functions. Two prototypical classes $\mathcal{C}$ satisfying the latter closure condition are semilinear sets and VAS reachability sets. Moreover, unary separability of semilinear sets is known to be decidable [3], and as recalled before the same result has recently been extended to VAS reachability sets [5]. As a consequence of our reduction, we deduce decidability of regular separability of $\mathcal{C}$-Parikh automata languages where the acceptance condition $\mathcal{C}$ can be instantiated to either the semilinear sets, or the VAS reachability sets.

References

[1] Michaël Cadilhac, Alain Finkel, and Pierre McKenzie. On the Expressiveness of Parikh Automata and Related Models. In *Proc. of NCMA'11*, pages 103–119, 2011.

[1]Later shown decidable for unambiguous PA [2].

[2] Michaël Cadilhac, Alain Finkel, and Pierre McKenzie. Unambiguous constrained automata. *Int. J. Found. Comput. Sci.*, 24(7):1099–1116, 2013. URL: `http://dx.doi.org/10.1142/S0129054113400339`, `doi:10.1142/S0129054113400339`.

[3] Christian Choffrut and Serge Grigorieff. Separability of rational relations in $A^* \times \mathbb{N}^m$ by recognizable relations is decidable. *Inf. Process. Lett.*, 99(1):27–32, 2006.

[4] Lorenzo Clemente, Wojciech Czerwiński, Sławomir Lasota, and Charles Paperman. Regular separability of parikh automata. In *In Proc. of ICALP'17*, 2017.

[5] Lorenzo Clemente, Wojciech Czerwinski, Slawomir Lasota, and Charles Paperman. Separability of Reachability Sets of Vector Addition Systems. In *Proc. of STACS'17*, volume 66 of *LIPICs*, pages 24:1–24:14, 2017. URL: `http://drops.dagstuhl.de/opus/volltexte/2017/7009`, `doi:10.4230/LIPIcs.STACS.2017.24`.

[6] Wojciech Czerwinski and Slawomir Lasota. Regular separability of one counter automata. In *Proc. of LICS'17*. To appear.

[7] Wojciech Czerwiński, Wim Martens, and Tomás Masopust. Efficient separability of regular languages by subsequences and suffixes. In *Proc. of ICALP'13*, pages 150–161, 2013.

[8] Wojciech Czerwiński, Wim Martens, Lorijn van Rooijen, and Marc Zeitoun. A note on decidable separability by piecewise testable languages. In *Proc. of FCT'15*, pages 173–185, 2015.

[9] Jean Goubault-Larrecq and Sylvain Schmitz. Deciding piecewise testable separability for regular tree languages. In *Proc. of ICALP'16*, pages 97:1–97:15, 2016. URL: `http://dx.doi.org/10.4230/LIPIcs.ICALP.2016.97`, `doi:10.4230/LIPIcs.ICALP.2016.97`.

[10] Harry B. Hunt III. On the decidability of grammar problems. *J. ACM*, 29(2):429–447, 1982.

[11] Oscar H. Ibarra. Reversal-bounded multicounter machines and their decision problems. *J. ACM*, 25(1):116–133, 1978.

[12] Felix Klaedtke and Harald Rueß. Monadic second-order logics with cardinalities. In *Proc. of ICALP'03*, pages 681–696, 2003. `doi:10.1007/3-540-45061-0_54`.

[13] Eryk Kopczynski. Invisible pushdown languages. In *Proc. of LICS'16*, pages 867–872, 2016. URL: `http://doi.acm.org/10.1145/2933575.2933579`, `doi:10.1145/2933575.2933579`.

[14] Thomas Place, Lorijn van Rooijen, and Marc Zeitoun. Separating regular languages by locally testable and locally threshold testable languages. In *Proc. of FSTTCS'13*, pages 363–375, 2013.

[15] Thomas Place, Lorijn van Rooijen, and Marc Zeitoun. Separating regular languages by piecewise testable and unambiguous languages. In *Proc. of MFCS'13*, pages 729–740, 2013.

[16] Thomas Place and Marc Zeitoun. Going higher in the first-order quantifier alternation hierarchy on words. In *Proc. of ICALP'14*, pages 342–353, 2014.

[17] Thomas Place and Marc Zeitoun. Separating regular languages with first-order logic. *Log. Methods Comput. Sci.*, 12(1), 2016.

[18] Thomas G. Szymanski and John H. Williams. Noncanonical extensions of bottom-up parsing techniques. *SIAM Journal on Computing*, 5(2):231–250, 1976.