Proof Complexity of Constraint Satisfaction Problems

Albert Atserias   Joanna Ochremiak

We analyse how the standard reductions between constraint satisfaction problems affect their proof complexity. We show that, for the most studied propositional and semi-algebraic proof systems, the classical constructions of pp-interpretability, homomorphic equivalence and addition of constants to a core preserve the proof complexity of the CSP. As a result, for those proof systems, the classes of constraint languages for which small unsatisfiability certificates exist can be characterised algebraically. We illustrate our results by a gap theorem saying that a constraint language either has resolution refutations of bounded width, or does not have bounded-depth Frege refutations of subexponential size. The former holds exactly for the widely studied class of constraint languages of bounded width. This class is also known to coincide with the class of languages with Sums-of-Squares refutations of sublinear degree, a fact for which we provide an alternative proof. We hence ask for the existence of a natural proof system with good behaviour with respect to reductions and simultaneously small size refutations beyond bounded width. We give an example of such a proof system by showing that bounded-degree Lovász-Schrijver satisfies both requirements.

The talk is based on a paper "Proof Complexity Meets Algebra" presented at ICALP 2017.