On Reversible Transducers *

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Automata and transducers   Automata theory is a prominent domain of theoretical computer science, initiated in the 60s [**?**] and still very active nowadays. Many extensions of finite automata have been studied such as automata over more complex structures (infinite words, trees, *etc*) or transducers which can be seen as automata with an additional write-only output tape and which will be the focus of our study in the remainder of this article.

Transducers have been studied for almost as long as automata [**?**] and important results have been obtained, however the theory of transducers is not as advanced as automata theory. One of the reasons for this is that many descriptions which are equivalent for automata become different in expressiveness in the case of transducers. For instance, deterministic and non-deterministic automata recognize the same class of languages, the *regular languages*. However this is not the case for transducers since in particular a deterministic transducer must realize a function while a non-deterministic one may realize a relation. Similarly, by allowing the reading head to move left and right, one gets a two-way model of automata and it is known that two-way automata are as expressive as one-way automata [**?**]. However two-way transducers can model relations and functions that are unobtainable in the one-way case, such as the function mirror which reverses its input. Recently, two-way transducers were also proven to be equivalent to the one-way deterministic model of *streaming string transducers*, which can be thought of as transducers with write-only registers.

Reversible transducers   A transition system is called *reversible* when for every input, the directed graph of configurations is composed of nodes of in-degree and out-degree at most one. This property is stronger than the more studied notion of determinism since it allows to navigate back and forth between the steps of a computation. In this article, we study the class of transducers that are simultaneously deterministic and codeterministic, *i.e.* reversible. The main motivation for the definition of this class is its good properties with respect to composition. When we consider one-way transducers, runs only go forward and thus determinism gives good properties for composition: the next step of a run is computed in constant time. However, when considering composition of two-way transducers, the second machine can move to the left, which corresponds to rewinding the run of the first machine. Then the stronger property of reversibility allows for this back and forth navigation over runs of transducers, and we recover the property of reaching the next (or previous here) step of a computation in a constant time. This leads to the recovery of the polynomial state complexity of composition which exists for deterministic one-way transducers.

Let us now discuss the expressiveness of reversible transducers. Regarding automata in the one-way case, it is well-known that any regular language can be recognized by a

� **Figure 1** The language A^*aA^* can be recognized by a deterministic (left) or codeterministic (right) automaton, but not by a reversible one.

deterministic one-way automaton and symmetrically by a codeterministic one-way automaton, since the mirror of a regular language is still regular. However, the class of one-way reversible automata is very restrictive (see Figure 1 for an example). It turns out however, that if we allow bidirectionality then any regular language can be recognized by a reversible automaton. In fact, a two-way reversible automaton can be constructed from either a one-way or two-way automaton using only a linear number of states. We prove, as a consequence of our main theorem, that reversible transducers are as expressive as functional two-way transducers, and exactly capture the class of regular functions. As stated earlier, regular functions are also characterized by streaming string transducers (SST). As a byproduct, we also give a quadratic construction from copyless SST to reversible transducers, improving known results.

Synthesis problem and uniformization of transducers   In the bigger picture of verification, two-way transducers can be used to model transformations of programs or non-reactive systems. If we consider the synthesis problem, where the specification is given as a relation of admissible input-output pairs, an implementation is then given as a function, with the same domain, relating a unique output to a given input. The uniformization problem asks if given a relation, we can extract a function that has the same domain, and is included in the relation. We argue that the synthesis problem can be instantiated in the setting of transformations as the problem of uniformization of a non-deterministic two-way transducer by a functional transducer. Our main result states that we can uniformize any non-deterministic two-way transducer by a reversible transducer with a single exponential blow-up.