Liveness in L/U Parametric Timed Automata*

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Properties of systems are often characterized as safety properties ("something bad will never happen") and liveness properties ("something good will eventually happen"). Safety generally reduces to reachability, while liveness is more complex. The "good" behavior may not be reached for two main reasons: either there is a deadlock, a state in which the system cannot evolve anymore, or there is a livelock, an infinite path never reaching the "good" behavior. Both situations are captured by the CTL operator EG.

We study here those behaviors in the context of parametric timed systems, in which some timing features (e. g., the duration of a task, a transmission delay in a network, the delay to trigger a watchdog, etc.) are not known and replaced by symbolic constants, called *parameters*. The objective of verification on such partially defined systems, is then to synthesize the possible valuations of parameters such that some properties are satisfied.

Context Parametric timed automata (PTAs) [AHV93] have been introduced to deal with such parametric timed systems. They consist in finite automata equipped with real-valued clocks that can be compared with constants or parameters in constraints restricting if and when the edges can be taken.

The simple problem of whether there exists a valuation for each parameter such that some control location is reachable in the timed automaton obtained by replacing the parameters with those valuations (also called EF-emptiness) is undecidable for PTAs for both integer- and rational-valued parameters. Several alternative proofs refine this result in terms of the number of parameters, of clocks compared to parameters, etc. (see, e. g., [Mil00,Doy07,BO14,BBLS15,And15]).

In order to overcome these disappointing results, lower-bound/upper-bound parametric timed automata (L/U-PTAs) are introduced as a subclass of PTAs where each parameter either always appears as an upper bound when compared to a clock, or always as a lower bound [HRSV02]. The EF-emptiness problem, and also the EF-universality problem ("Can we reach a given location, regardless of what valuations we give to the parameters?") are decidable for L/U-PTAs.

In [BL09], infinite acceptance properties are considered: the emptiness and the universality of the valuation set for which a given location is infinitely often traversed are decidable for integer-valued parameters.

In [JLR15], it is shown that the AF-emptiness problem ("Is the set of parameter valuations such that the system reaches a given location for all runs, empty?") is undecidable for L/U-PTAs with integer- and rational-valued parameters.

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Contribution With the notable exception of [JLR15], and to some extent of [BL09] which addresses the existence of cycles, all the works cited above focus on safety properties, through the basic problem of reachability. This is maybe not so surprising given that most results related to this simpler problem are already negative.

We address here the problem of liveness in PTAs, and more precisely, with the negative result of [JLR15] on AF-emptiness in mind, we start from L/U-PTAs with rational-valued parameters and further refine both the model and the properties. We prove that:

1. deciding the existence of at least one parameter valuation for which there exists an infinite run (discrete cycle) in the automaton is PSPACE-complete;
2. deciding the existence of a parameter valuation such that the system has a deadlock is however undecidable;
3. the problem of the existence of a valuation for which a run remains in a given set of locations exhibits a very thin border between decidability and undecidability: while this problem is decidable for L/U-PTAs with a bounded parameter domain with closed bounds, it becomes undecidable if either the assumption of boundedness or of closed bounds is lifted. This result confirms that L/U-PTAs stand at the border between decidability and undecidability.

Joint work This work is a joint work with Didier Lime [AL17].

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