Dynamic descriptive complexity of FO-definable modifications

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Joint work with
Thomas Schwentick Thomas Zeume

technische universität
dortmund

Highlights, September 2016

Social networks

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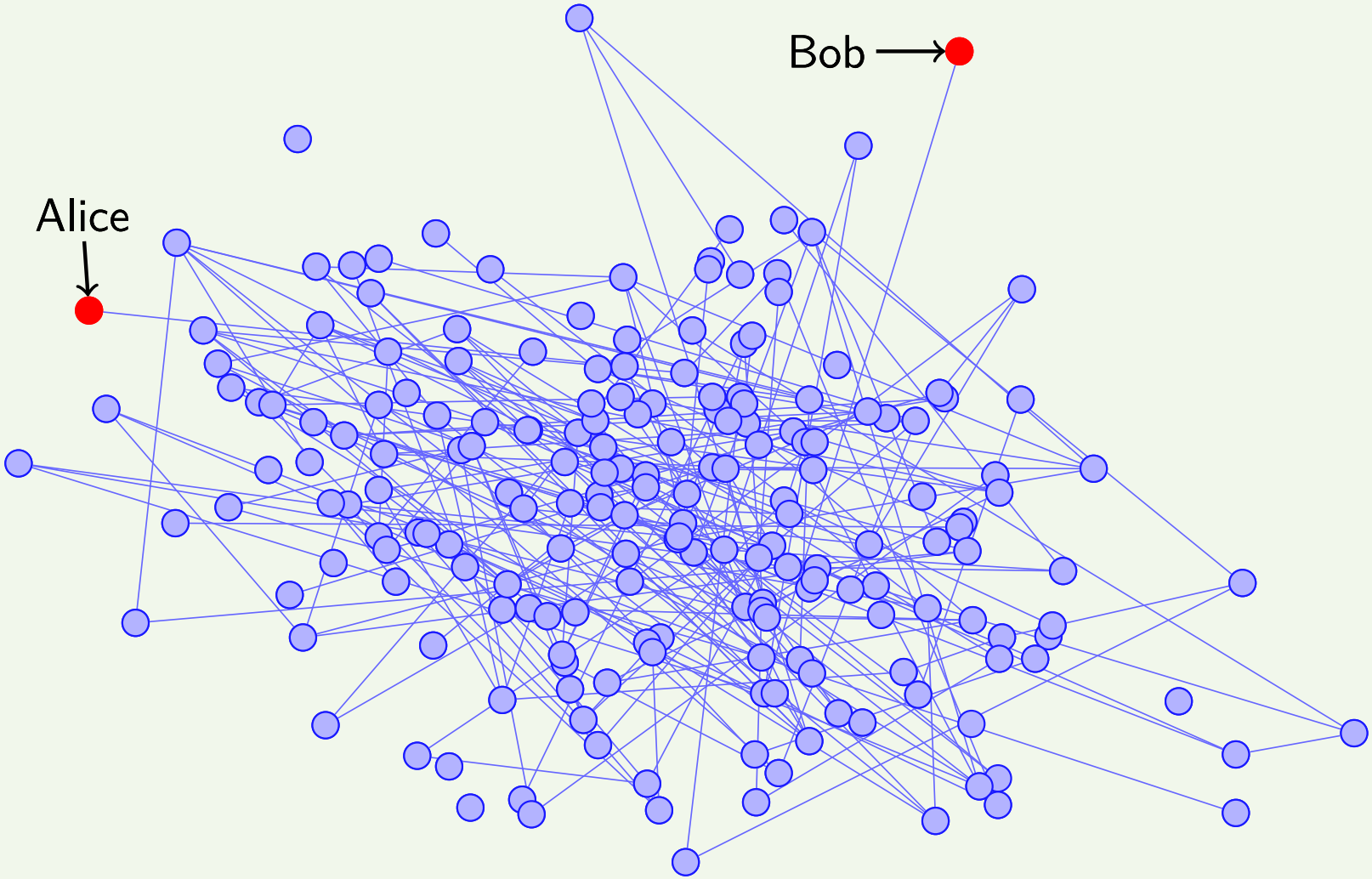

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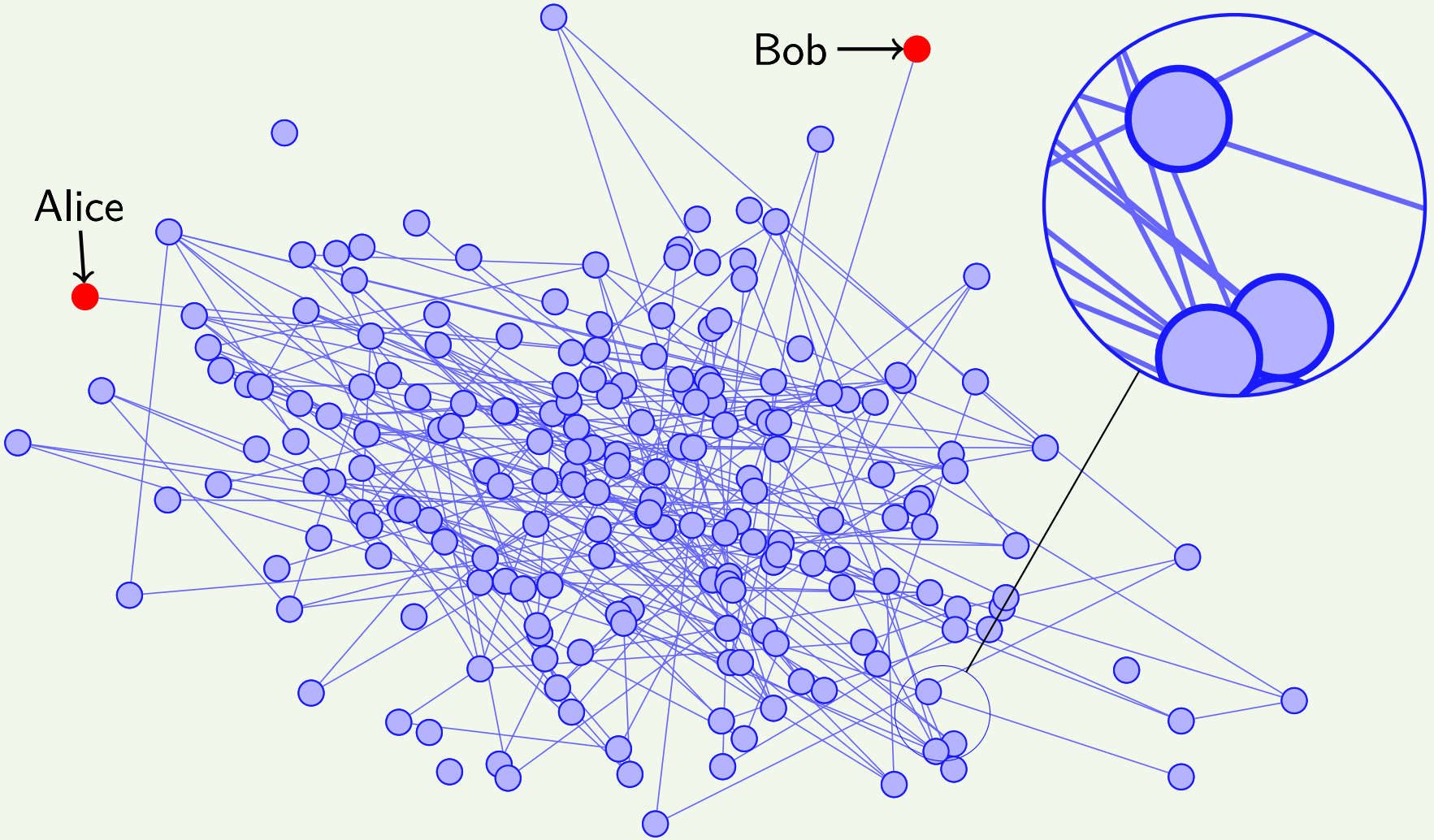

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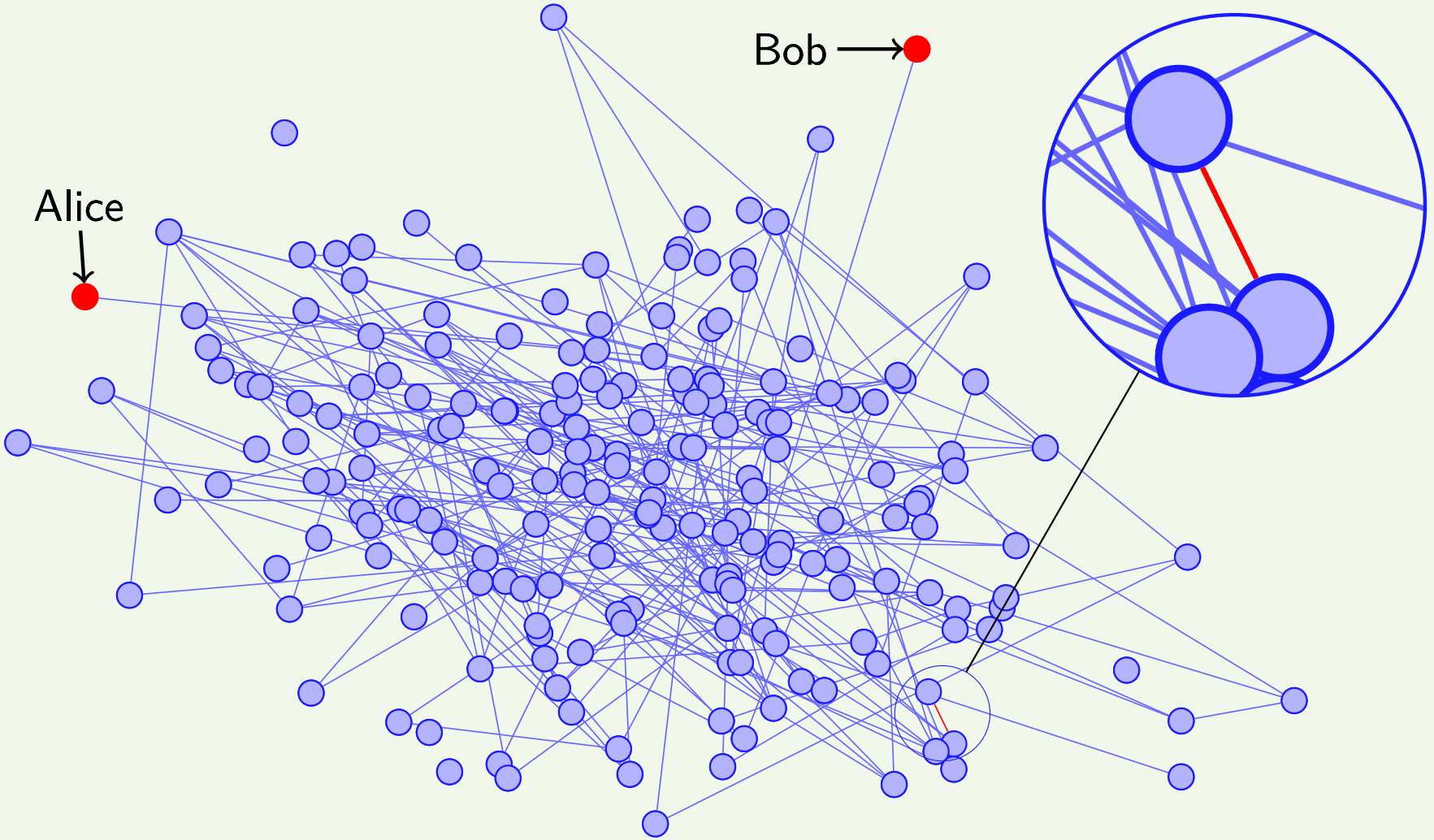

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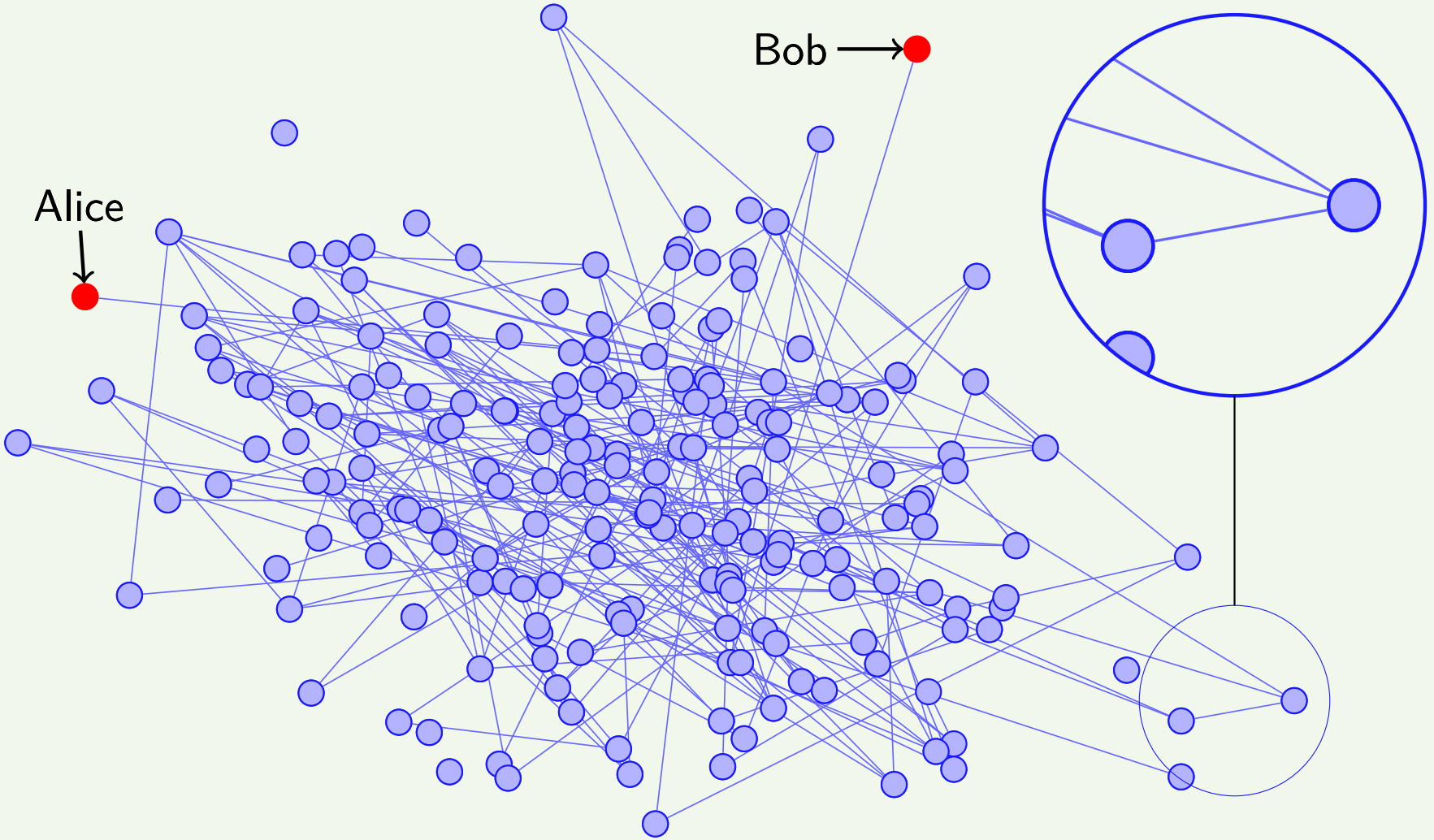

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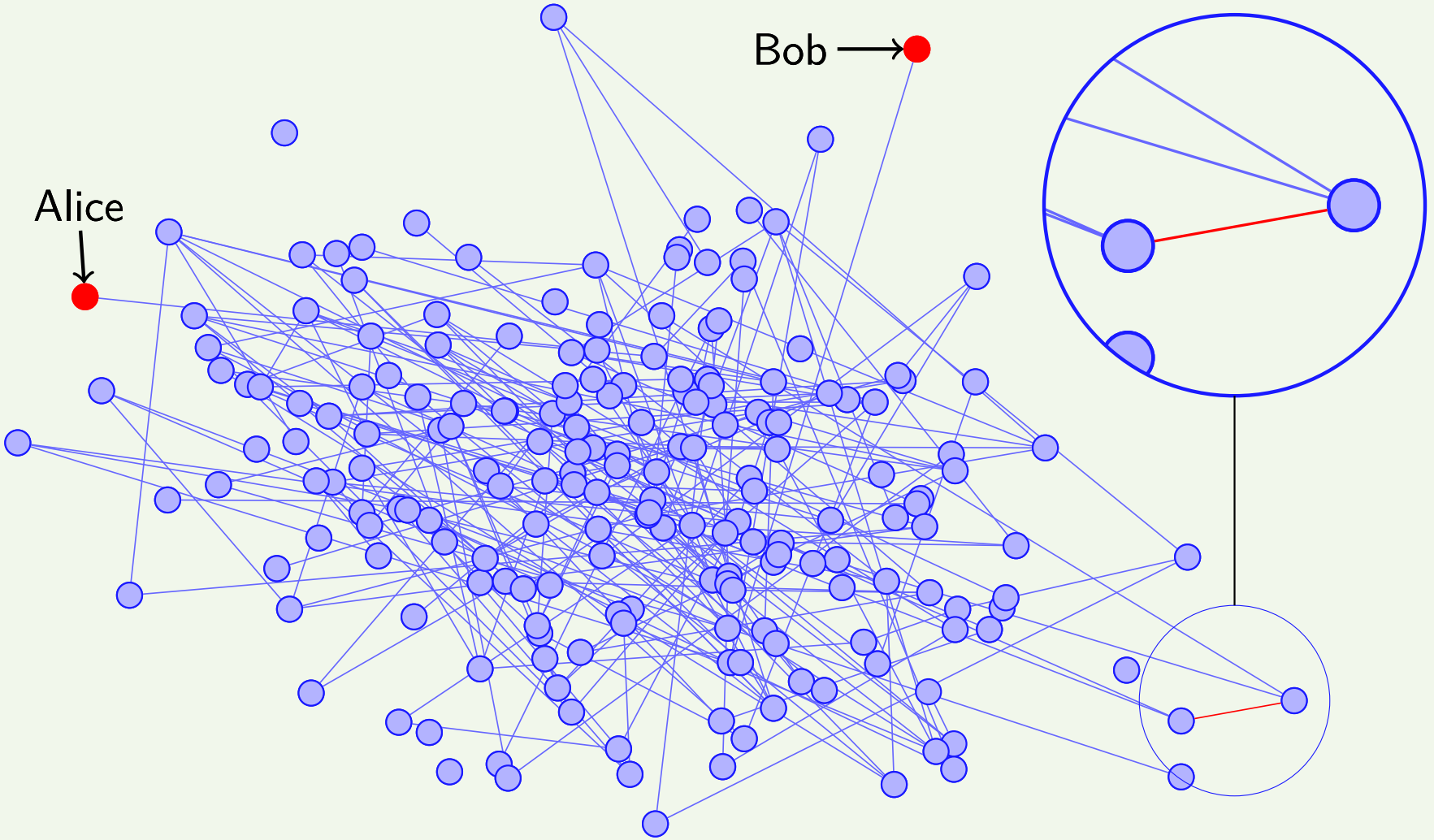

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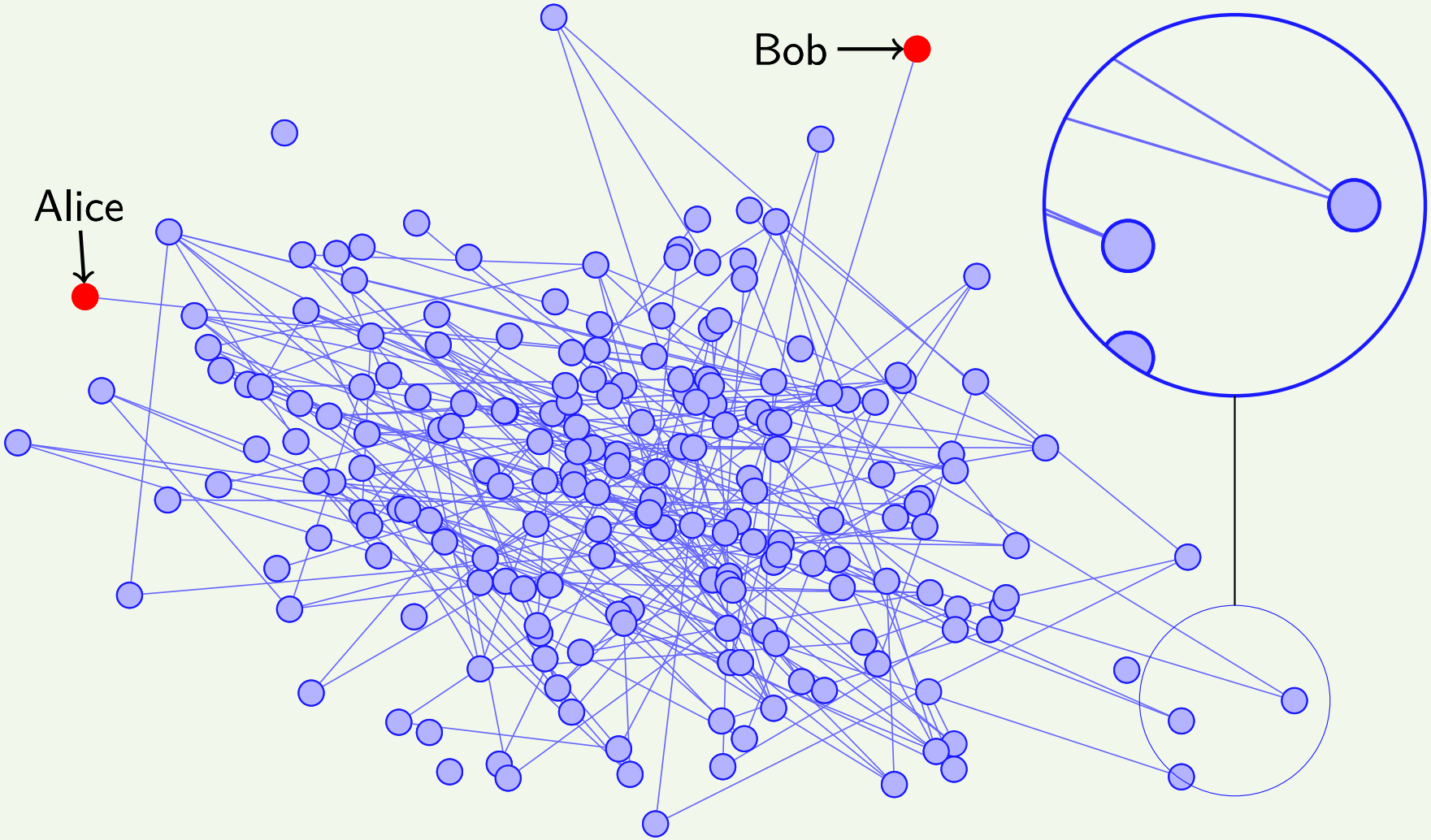

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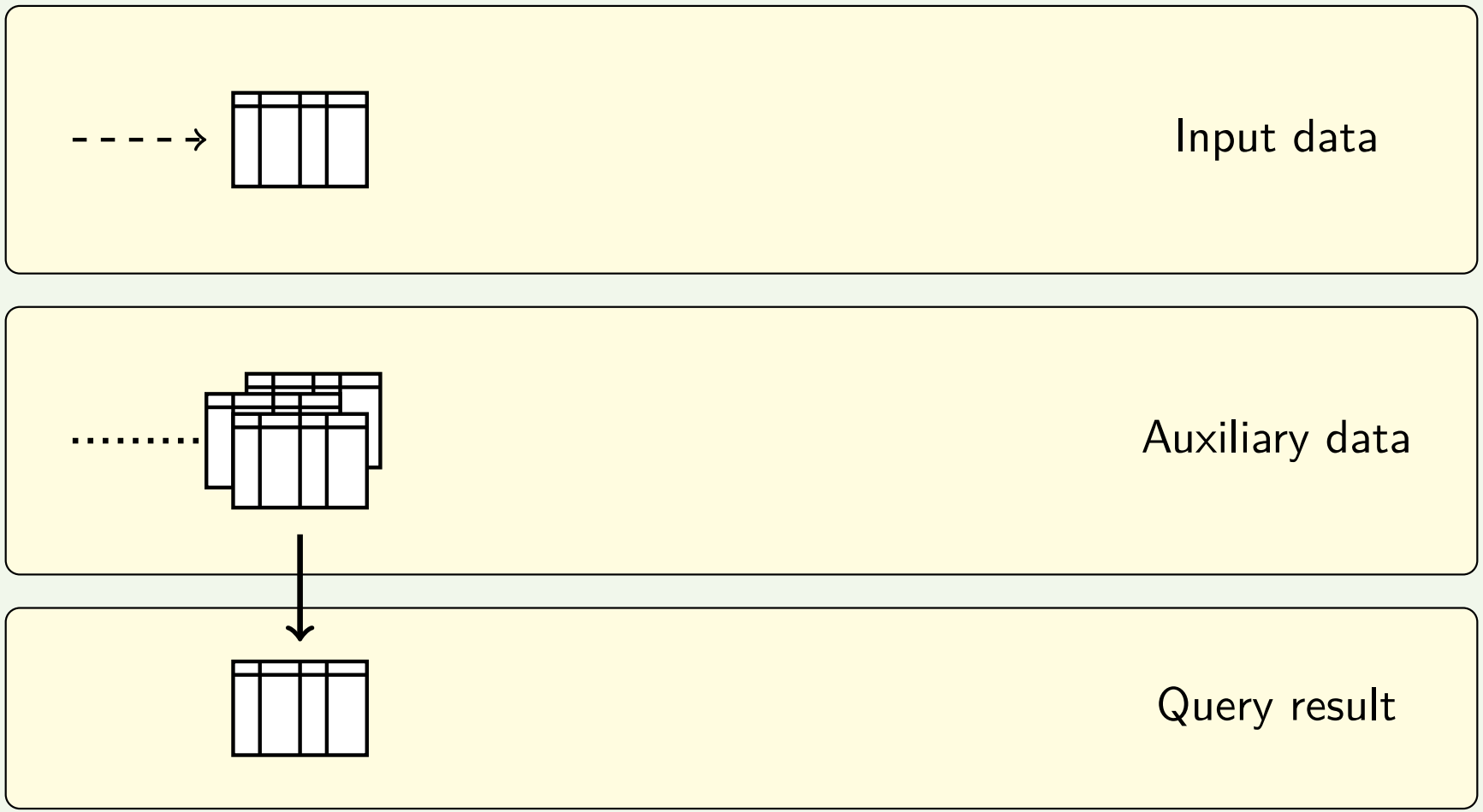

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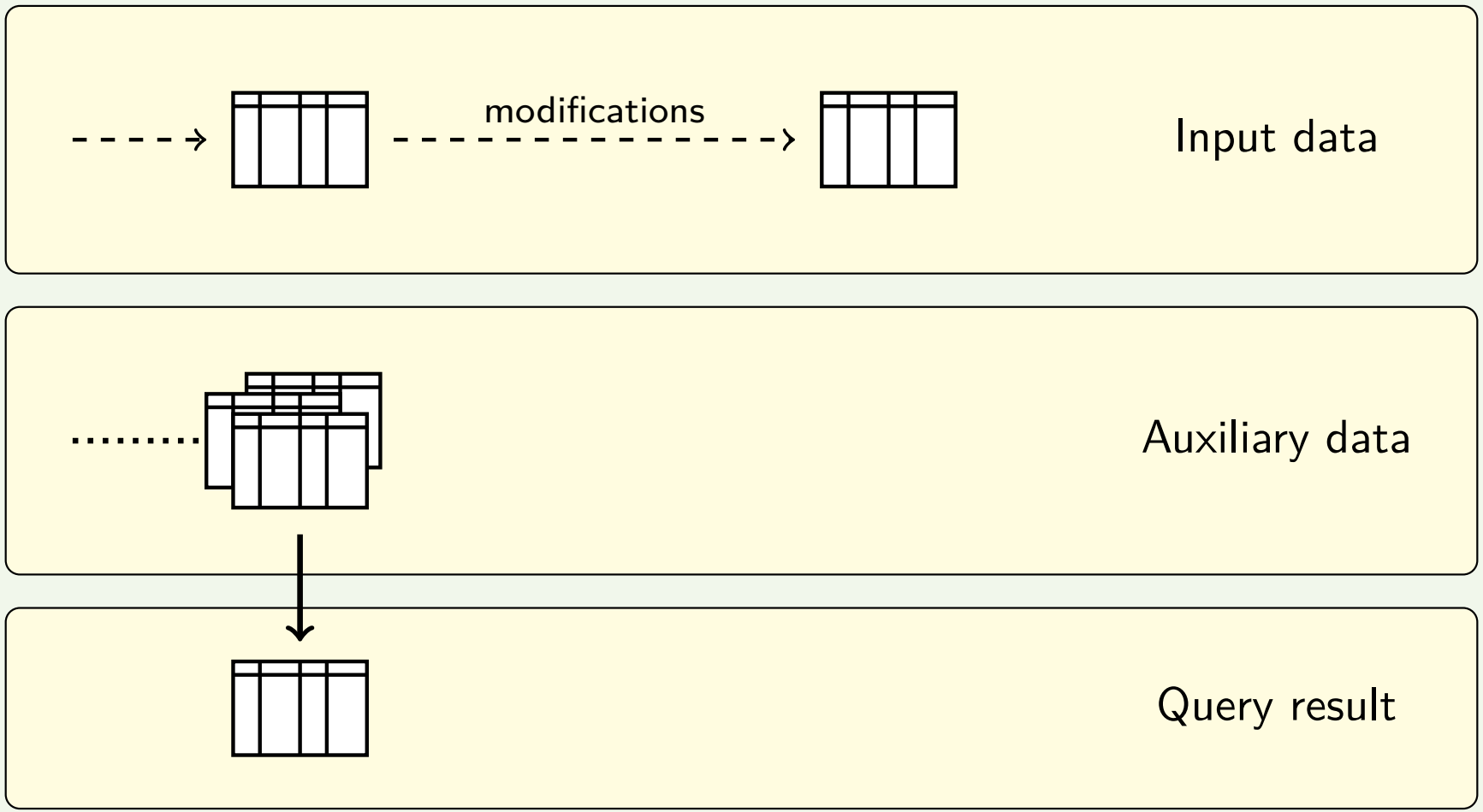

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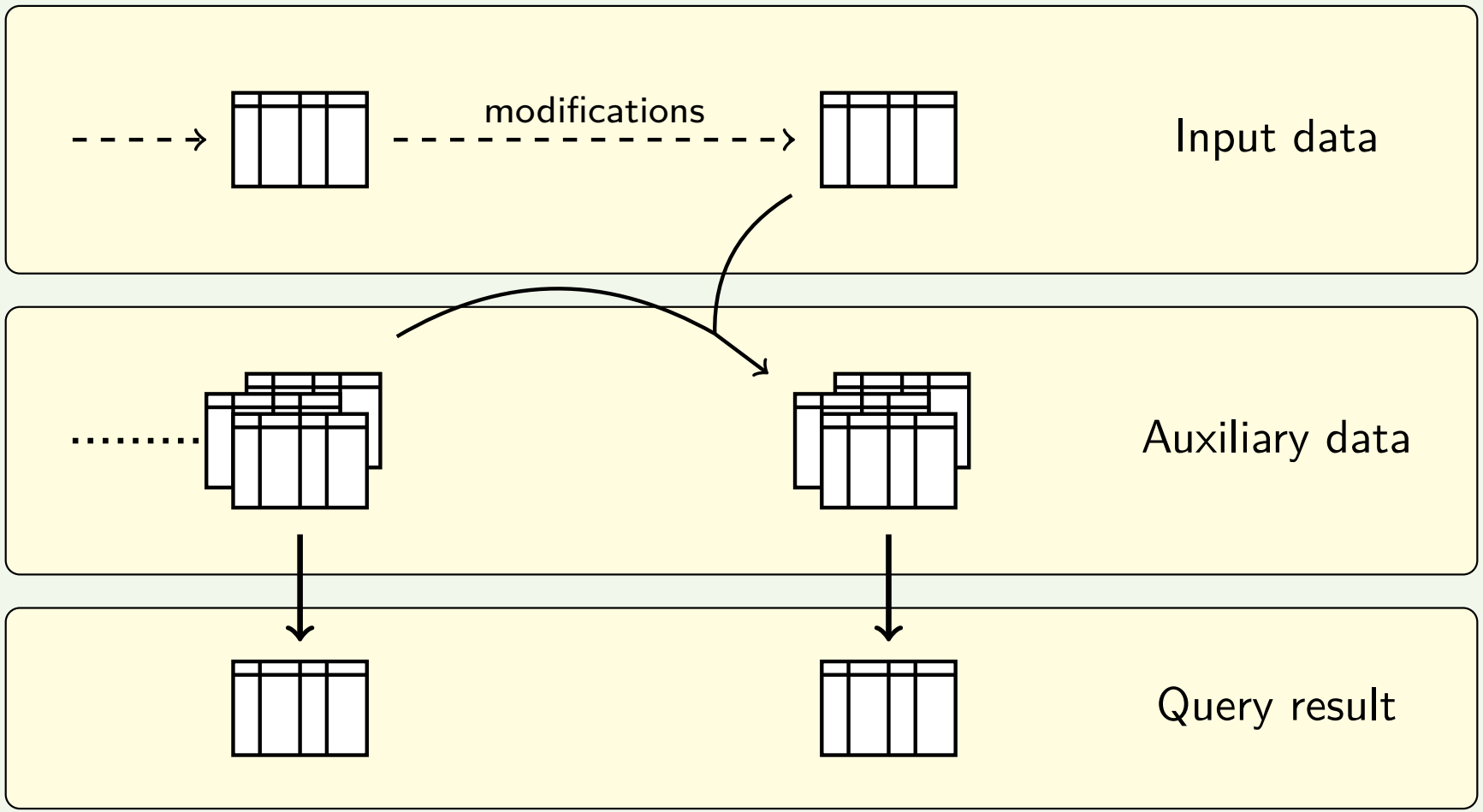

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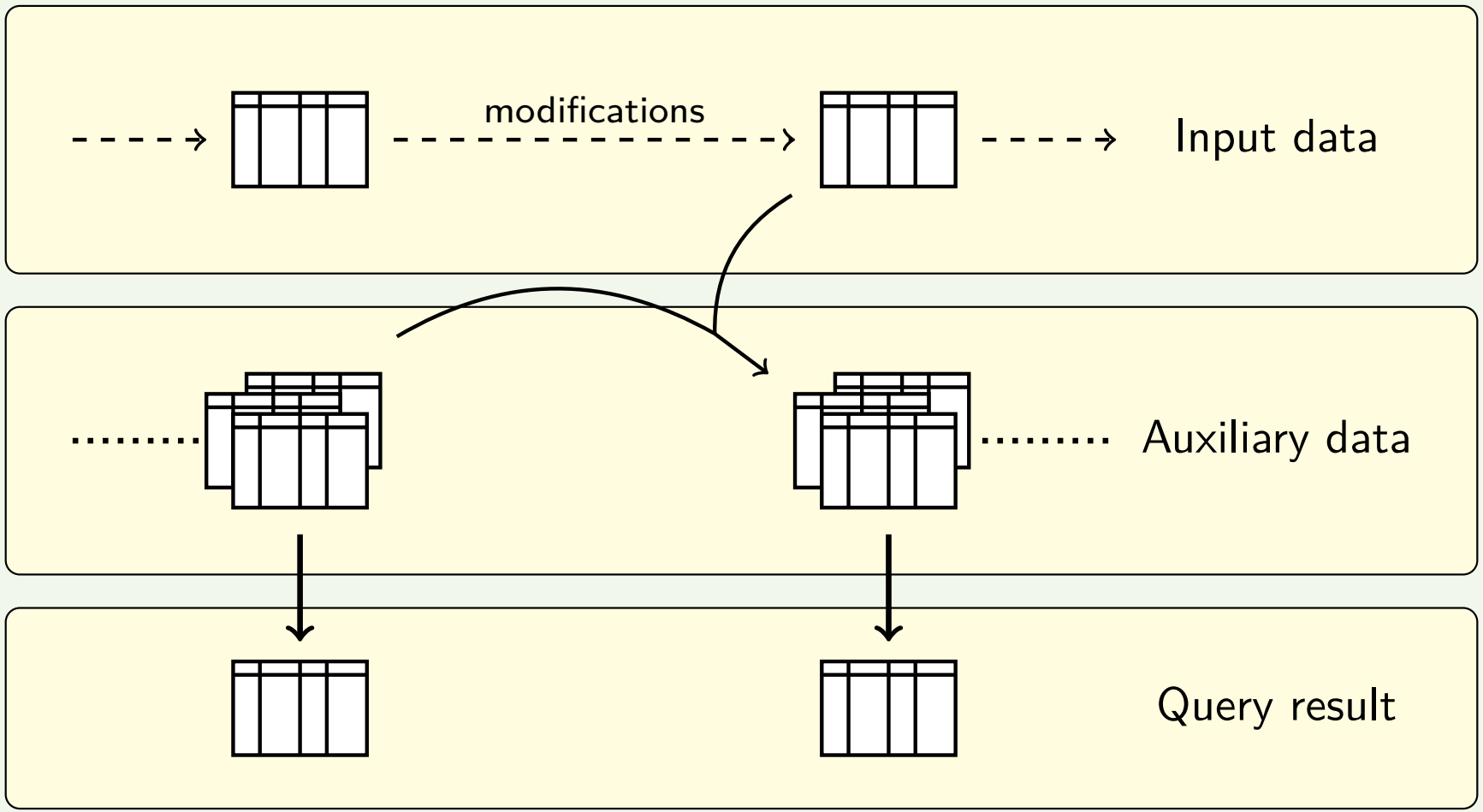

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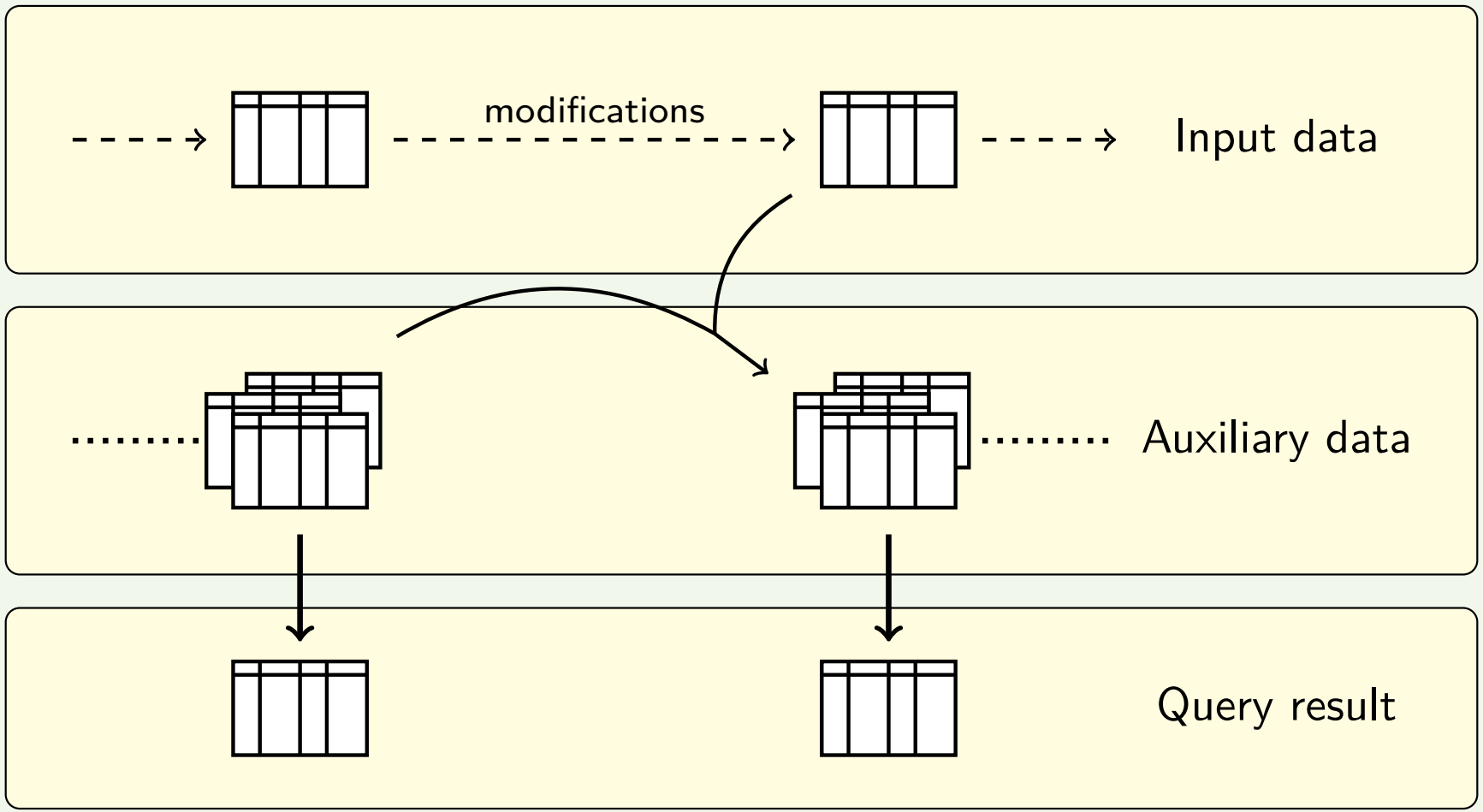
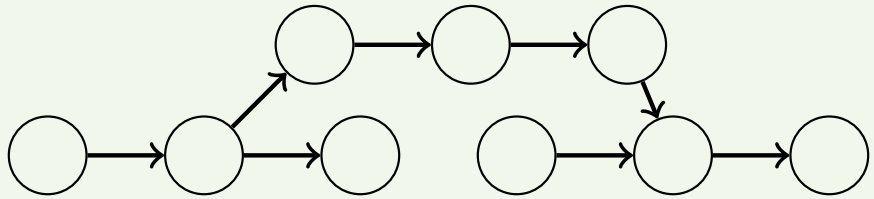


- Descriptive approach: Update auxiliary data by logical formulas

Example ([Patnaik and Immerman 1997])

Reachability on acyclic graphs: single edge insertion



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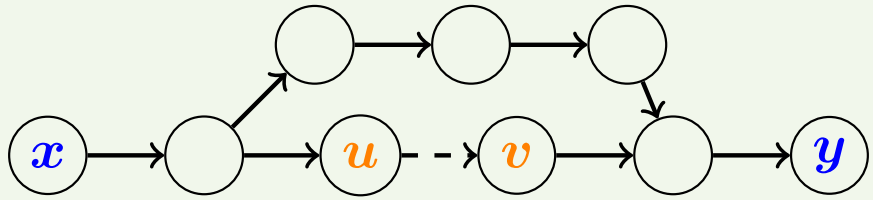

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- Is $(x, y) \in T$ after an edge (u, v) is inserted?

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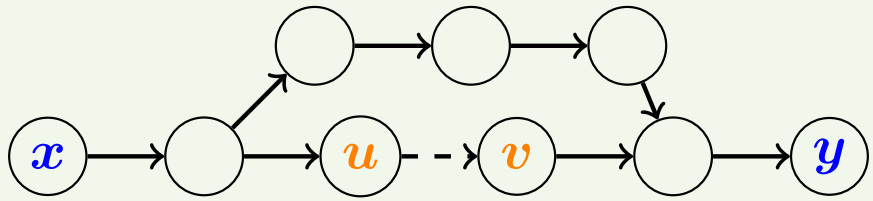

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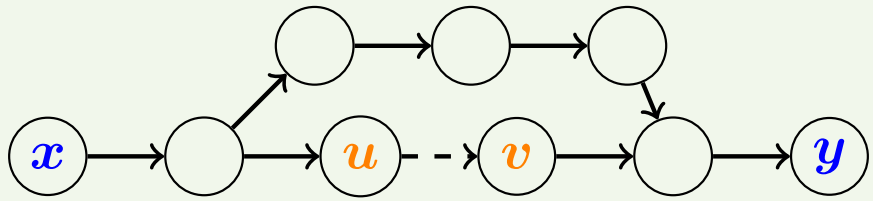

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- Dynamic Program: Collection of first-order formulas defining auxiliary relations after modifications

How much can change in one step?

- Arbitrary modifications
  - often used in View Maintenance (for example [Gupta et al. 1993])
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- Insertion or deletion of sets with some structural property
  - appears in [Dong et al. 1995]: insertion of sets closed under some operation
  - in [Dong and Pang 1997]: Reachability under deletions of "anti-chains" of edges/nodes

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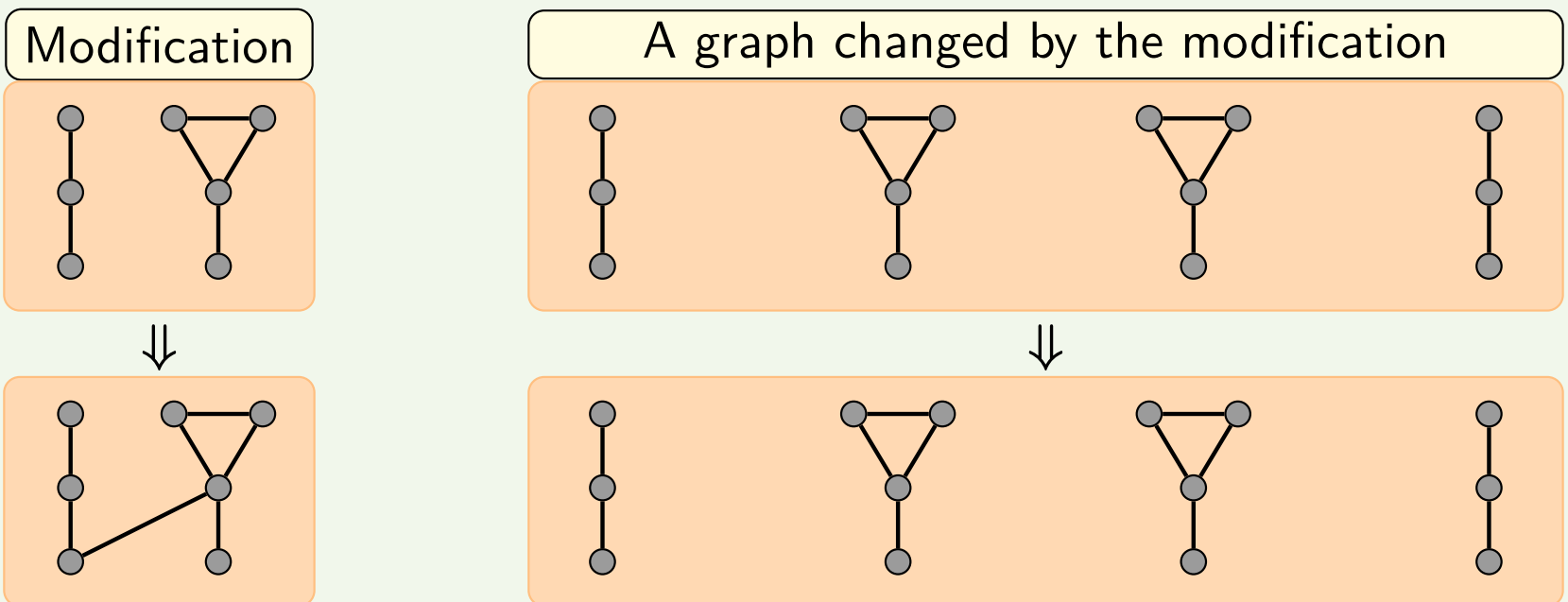

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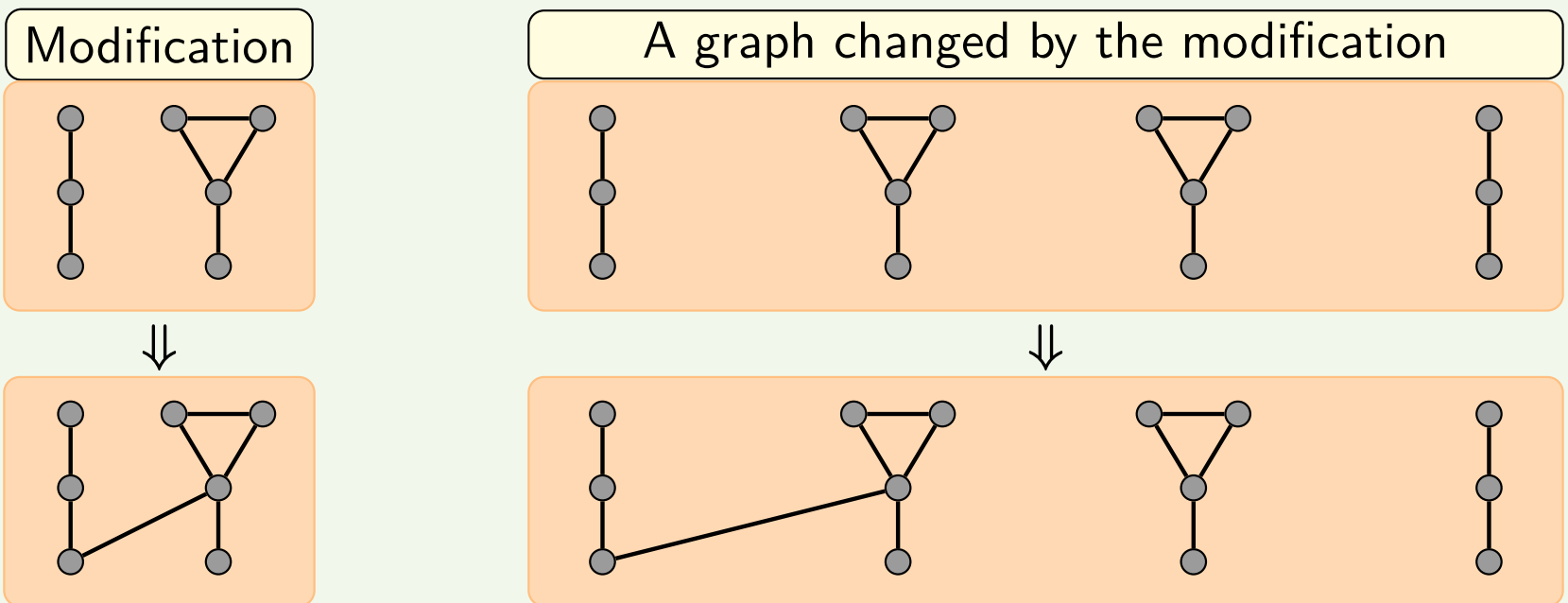

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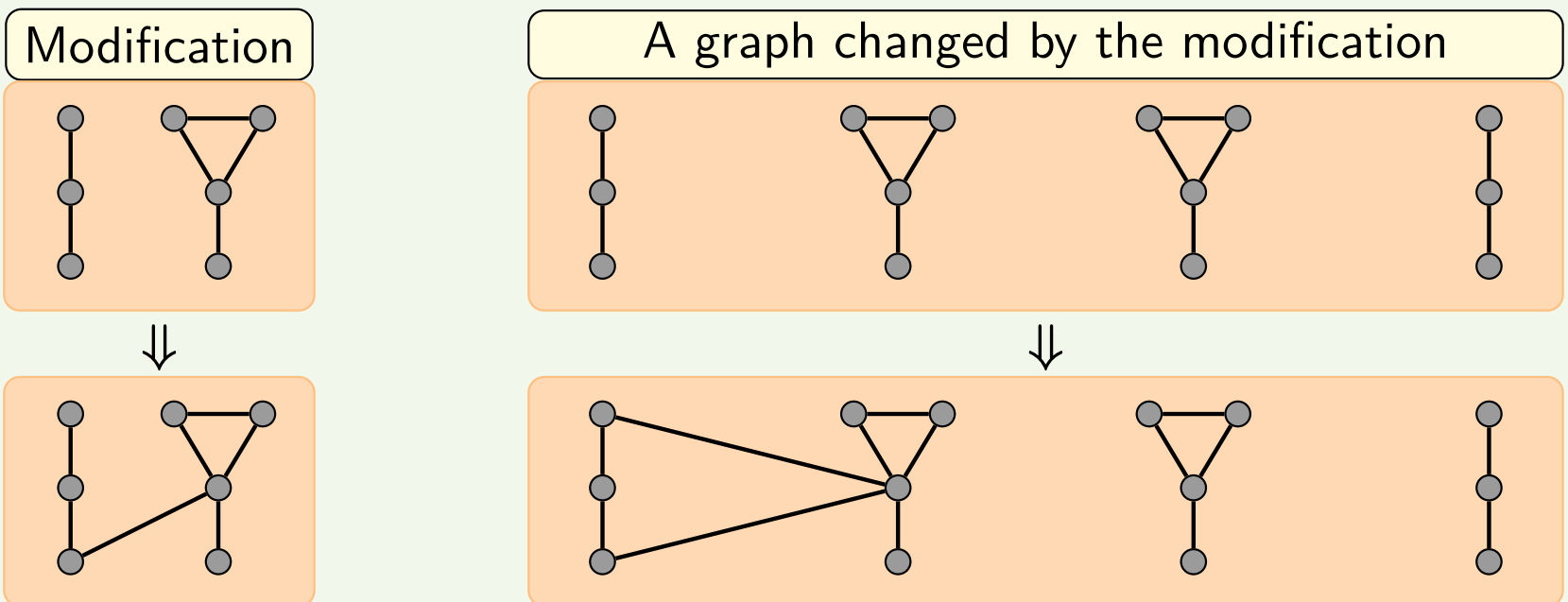

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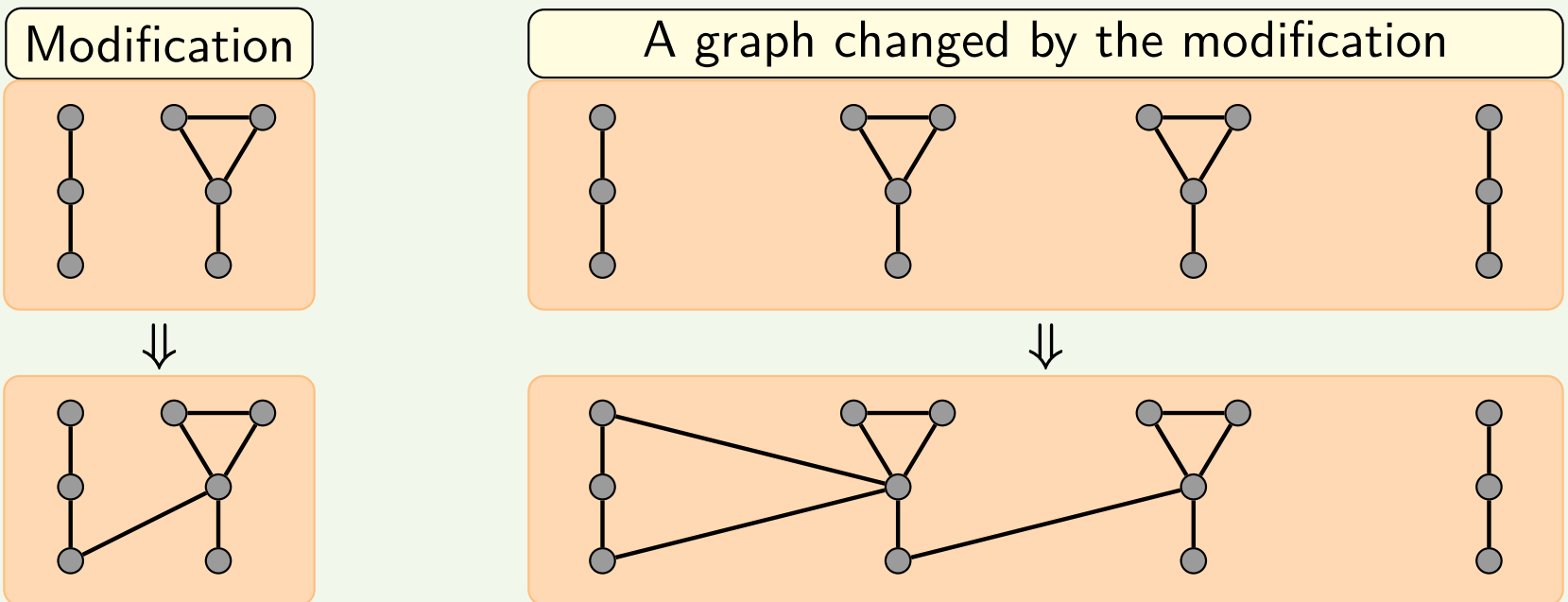

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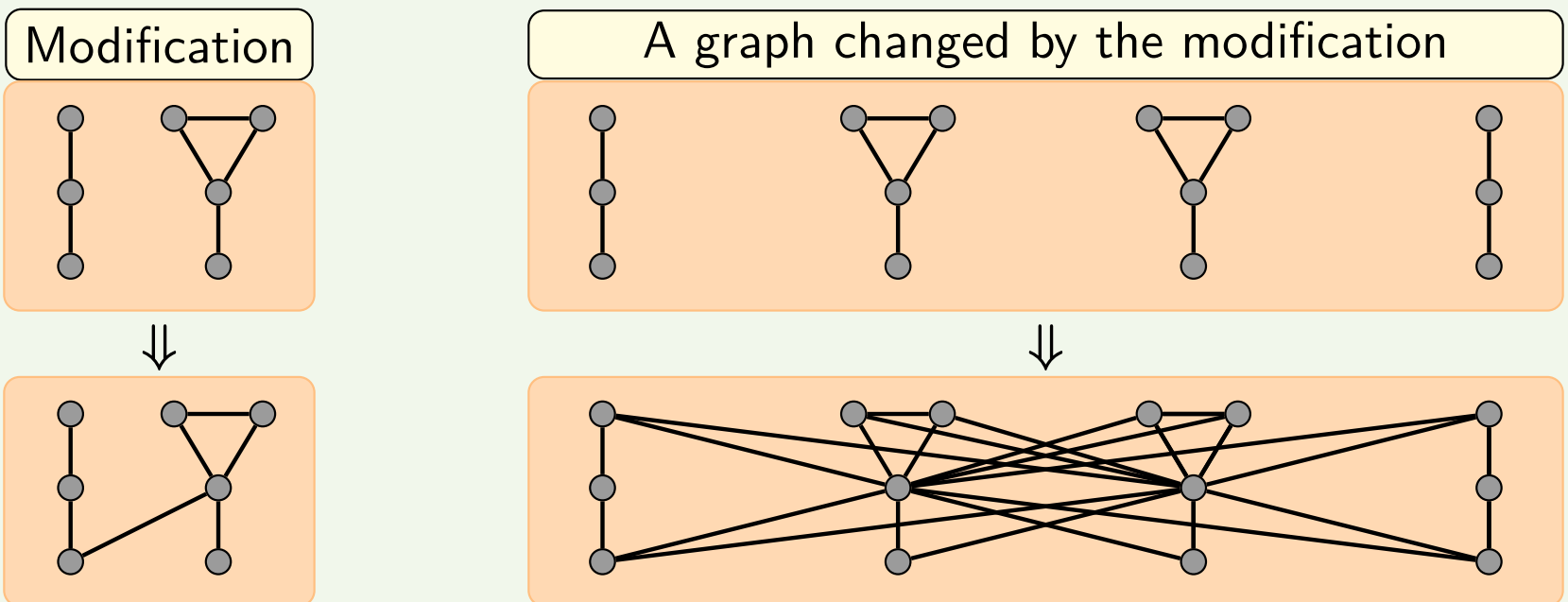

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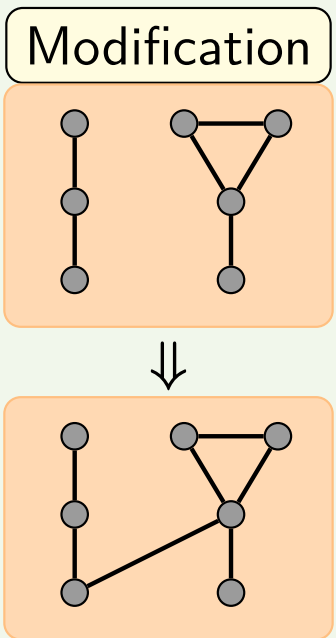
Parameter-free modifications

- All AC^1 queries on ordered databases under parameter-free first-order modifications

Maintaining Reachability in undirected graphs

- Proof idea: paths only need boundedly many new edges

Maintaining Reachability for the example modification



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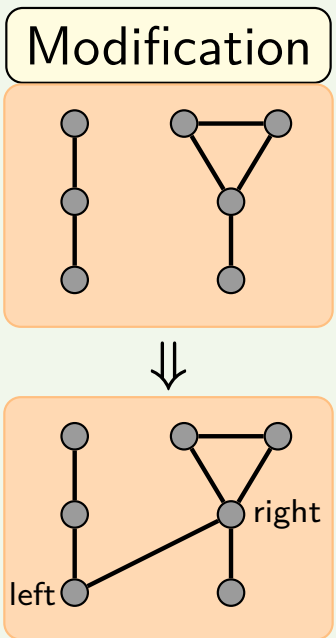

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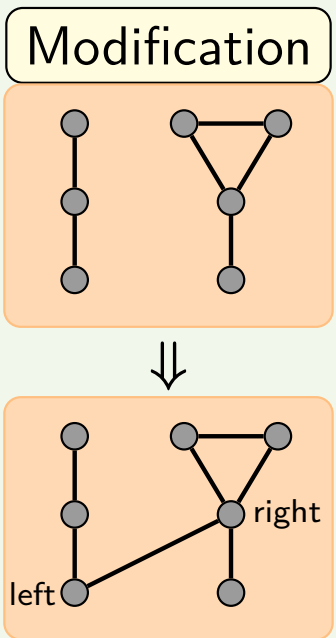

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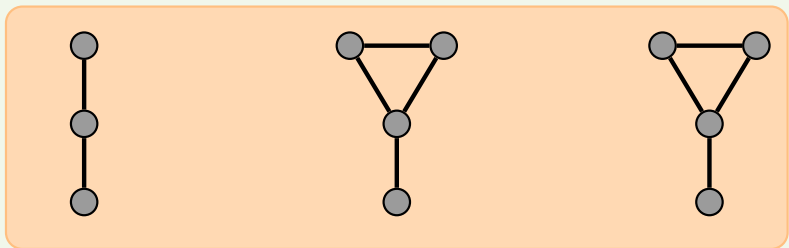

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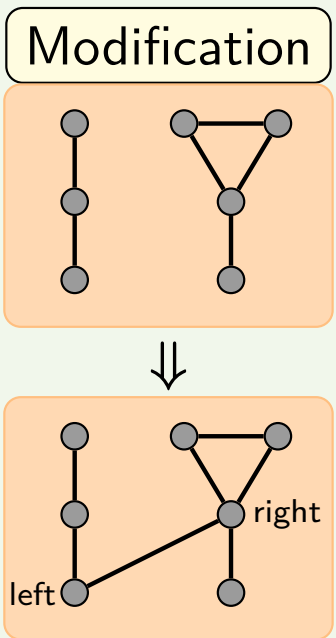

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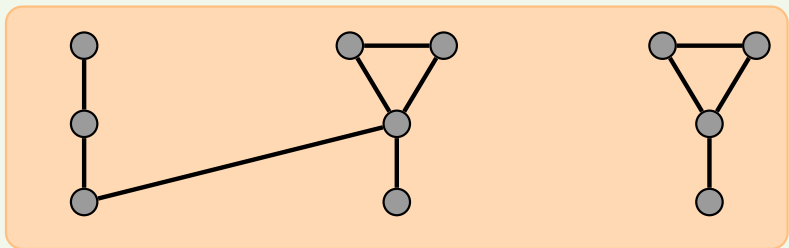

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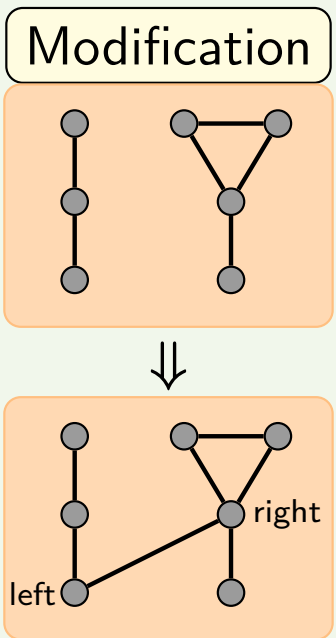

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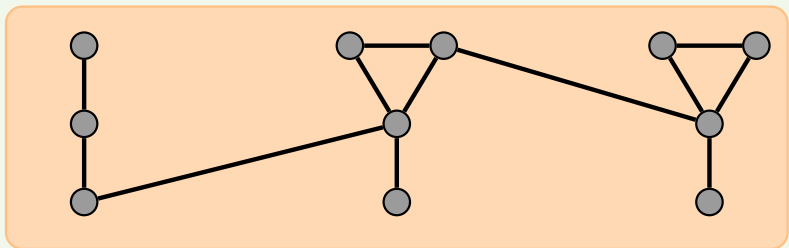

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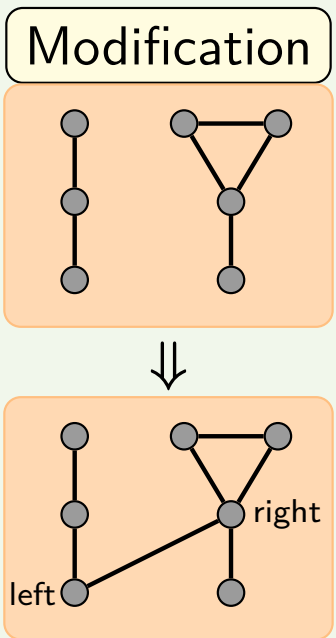

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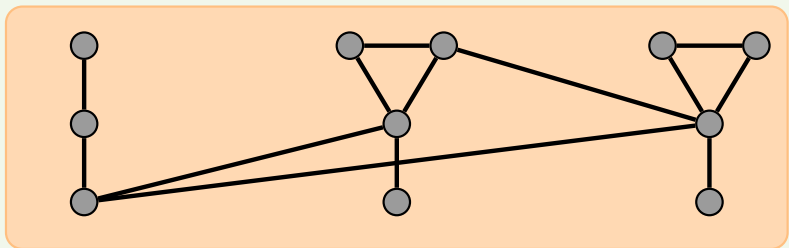

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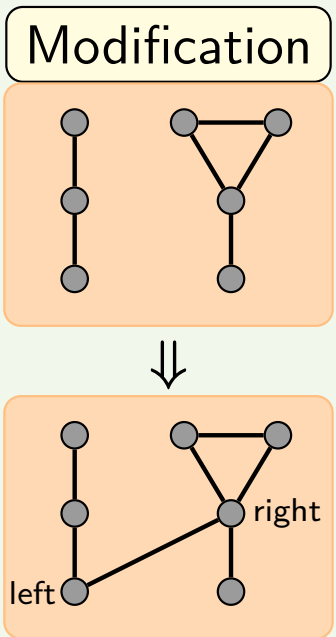

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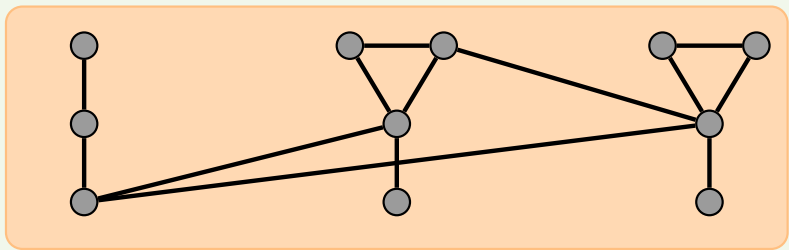

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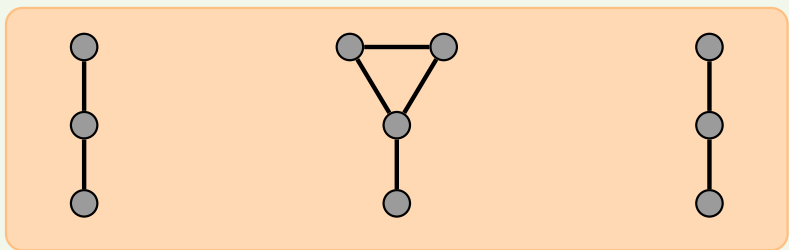

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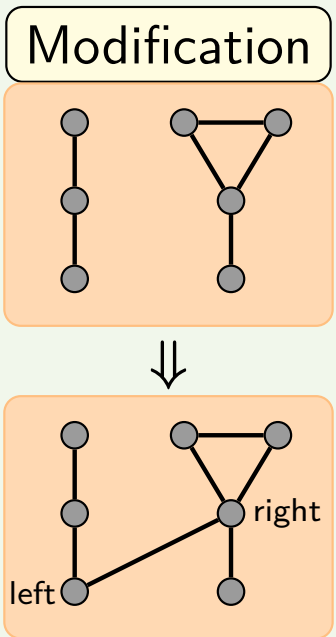

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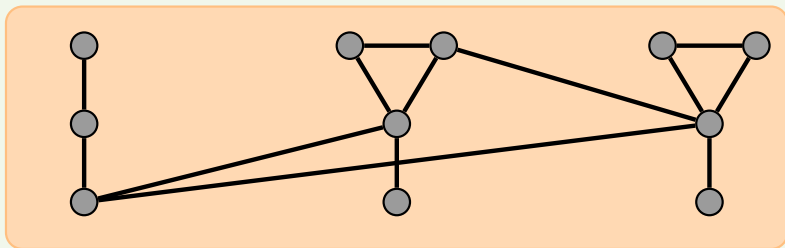

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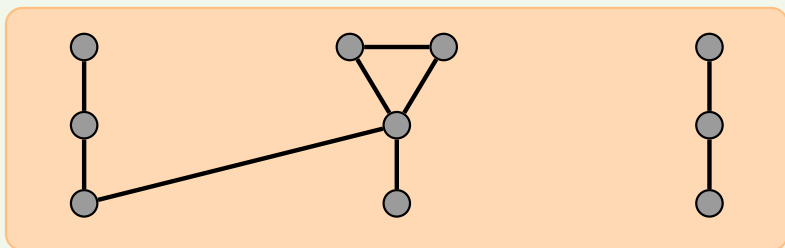

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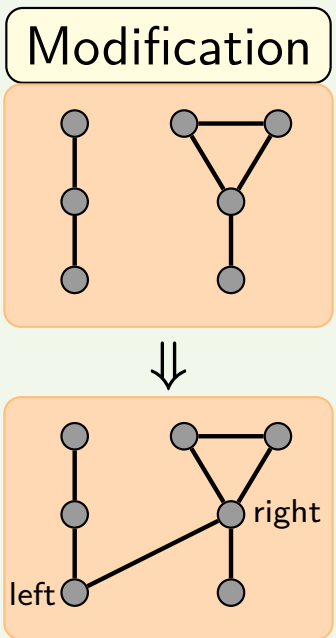

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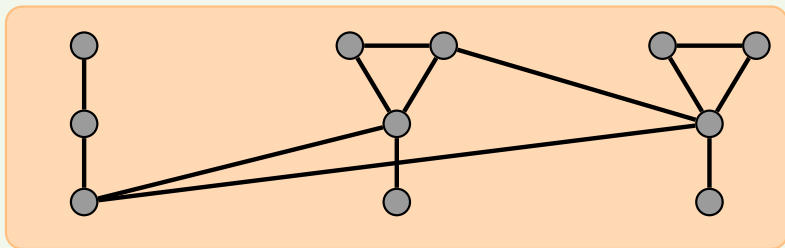

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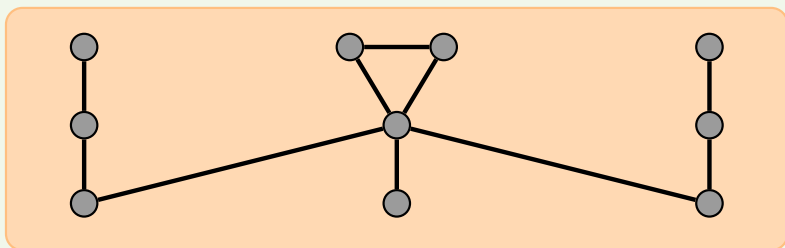

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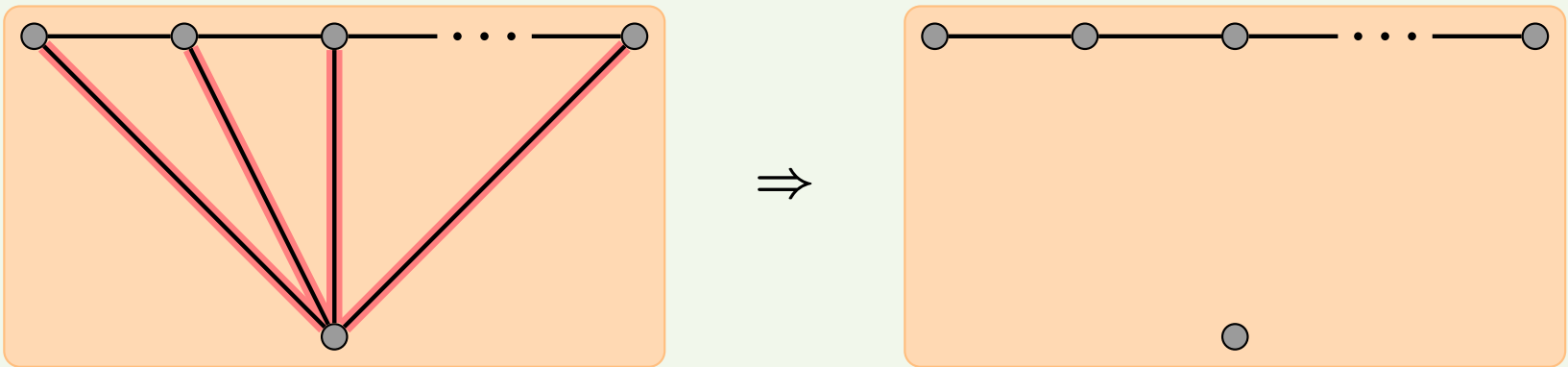
First-order deletions

How to maintain a spanning tree in the following situation?



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- For stronger modifications the techniques do not suffice
- To do: Show unmaintainability results for first-order modifications
  - Reachability under deletions
  - Reachability with quantifier-free update formulas

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