Real-time Synthesis is Hard!

@ Highlights 2016

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7 September 2016

Reactive synthesis

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- controllable actions owned by controller C:
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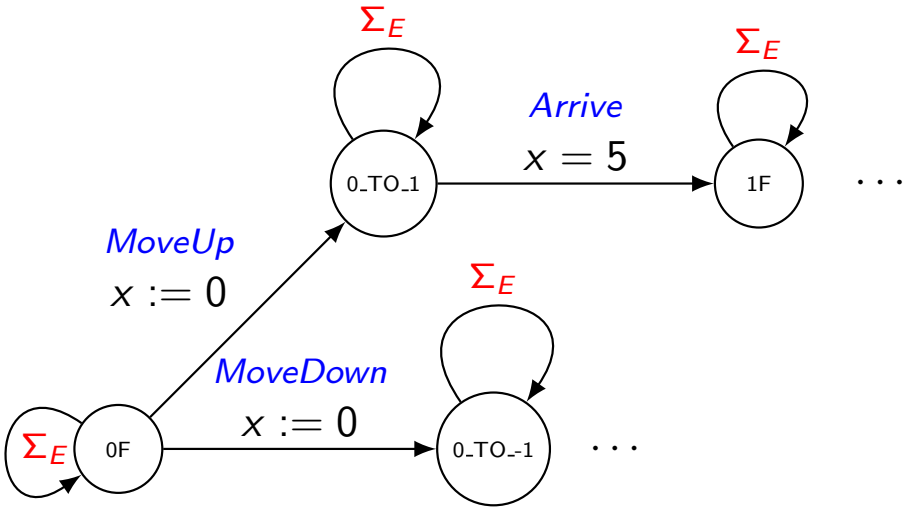

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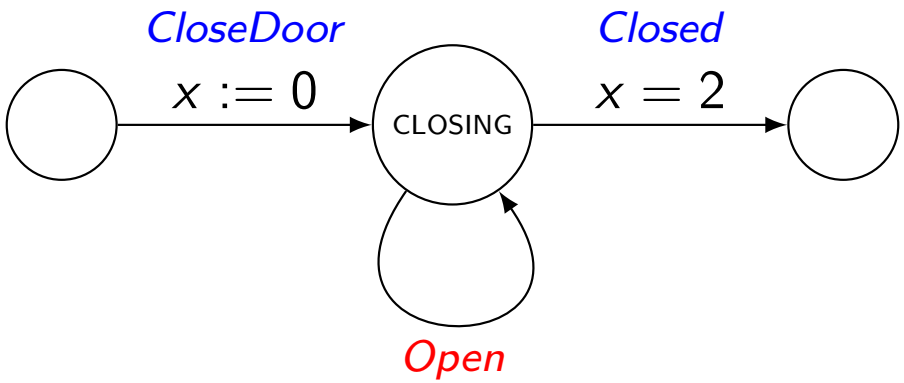

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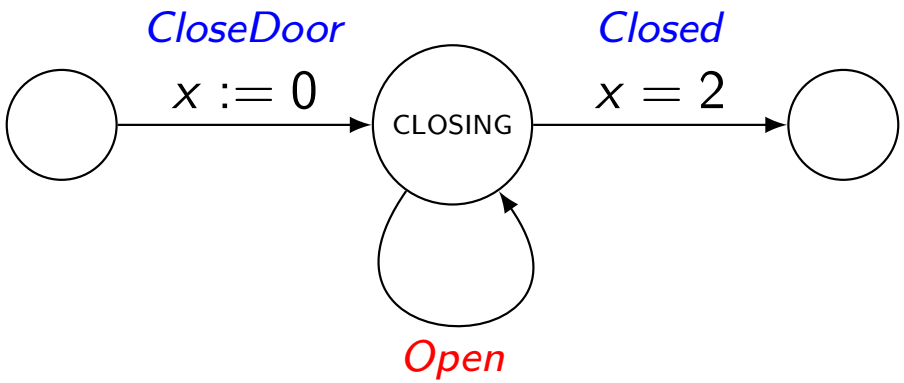

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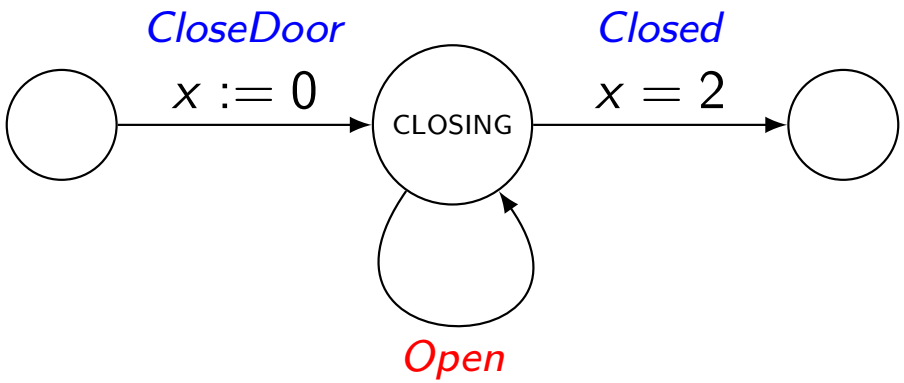

$(\Delta_C, \mathit{Closed})$ $(\Delta_E, \mathit{Open})$

Only action(s) with the shortest delay $\min(\Delta_C, \Delta_E)$ may be played.

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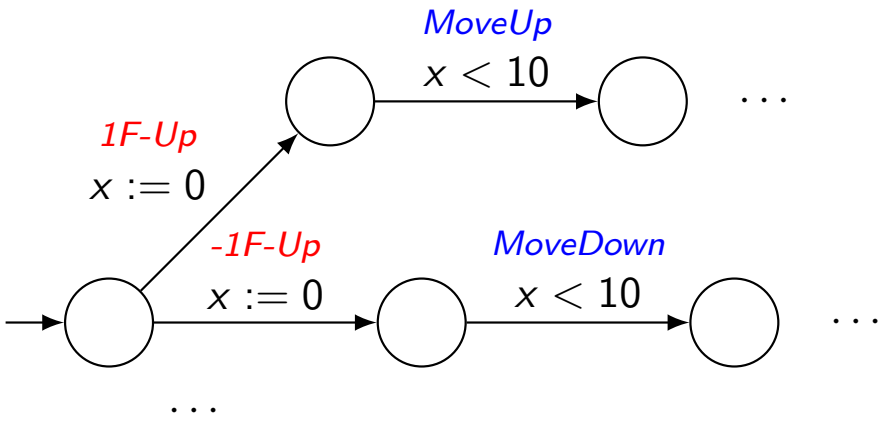

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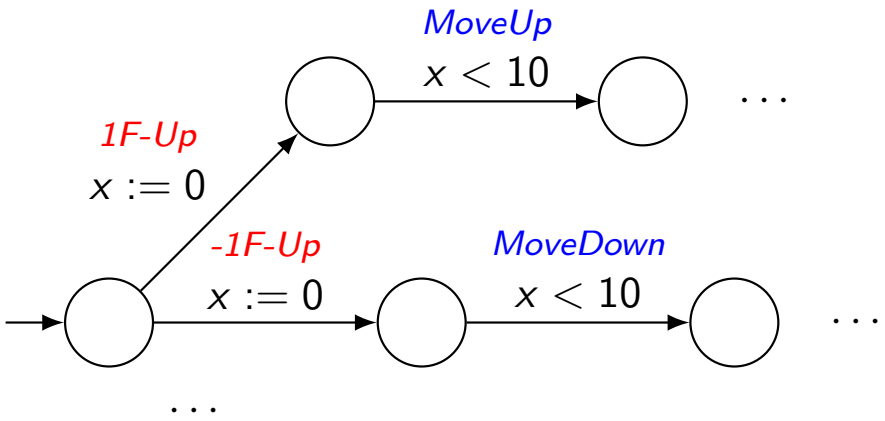

Specification $\mathcal{L}$: a set of timed words over Σ

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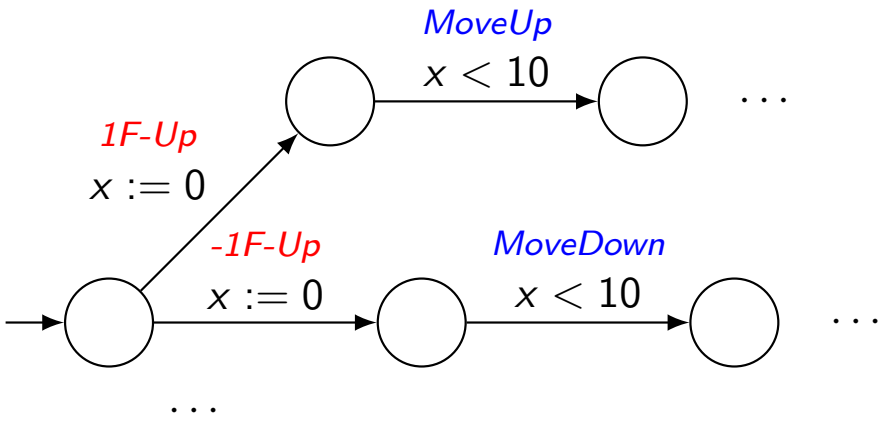


Specification $\mathcal{L}$: a set of timed words over Σ

Reactive synthesis problem (RS)

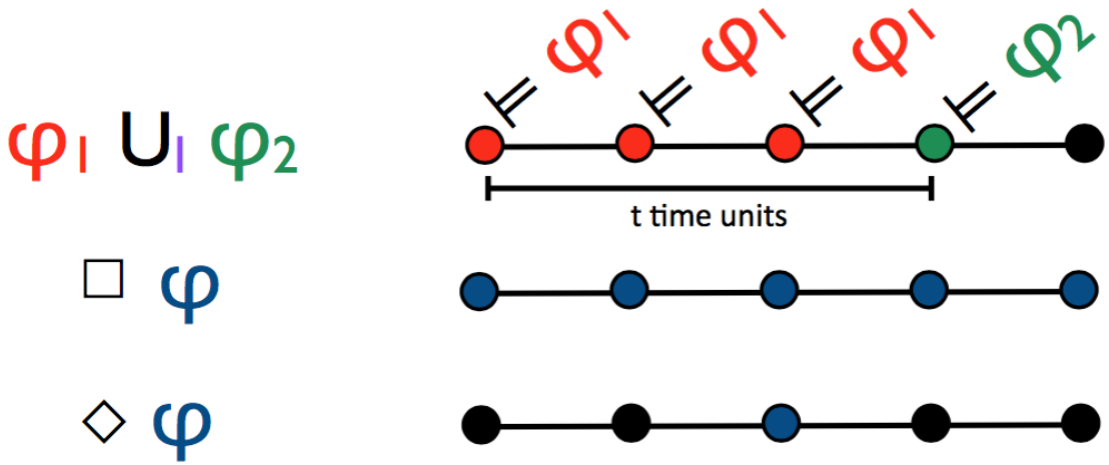
Given plant $\mathcal{P}$ and specification $\mathcal{L}$, find a strategy of Controller such that no matter what Environment does, every play satisfies the specification.

Metric Temporal Logic (MTL)

$$\varphi ::= \top \mid a \mid \neg\varphi \mid \varphi \wedge \varphi \mid \varphi \, \mathsf{U}_I \, \varphi$$

with $a \in \Sigma$, $I \subseteq [0, \infty)$ with bounds in $\mathbb{Q} \cup \{+\infty\}$

Models: finite (or infinite) timed words $\sigma = (a_1, t_1)(a_2, t_2) \cdots$



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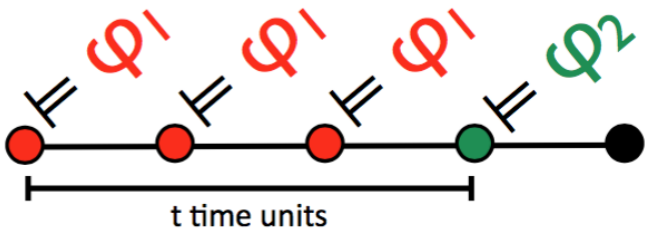


$\Box \, \varphi$

$\Diamond \, \varphi$

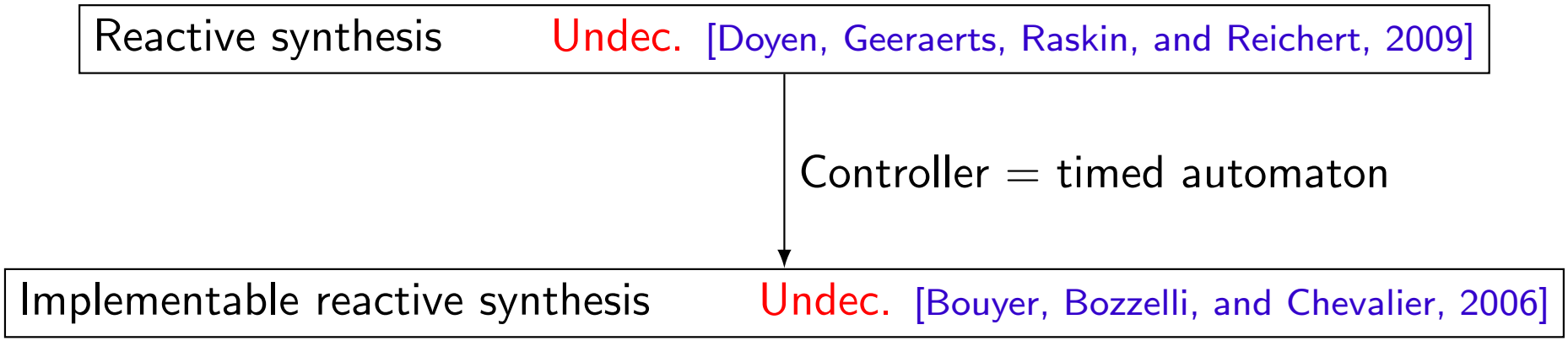
Theorem: [Doyen, Geeraerts, Raskin, and Reichert, 2009]

Reactive synthesis problem is undecidable for ECL (hence, MTL) specifications, even without plant.

Reactive synthesis for MTL

Reactive synthesis	Undec. [Doyen, Geeraerts, Raskin, and Reichert, 2009]

Reactive synthesis for MTL



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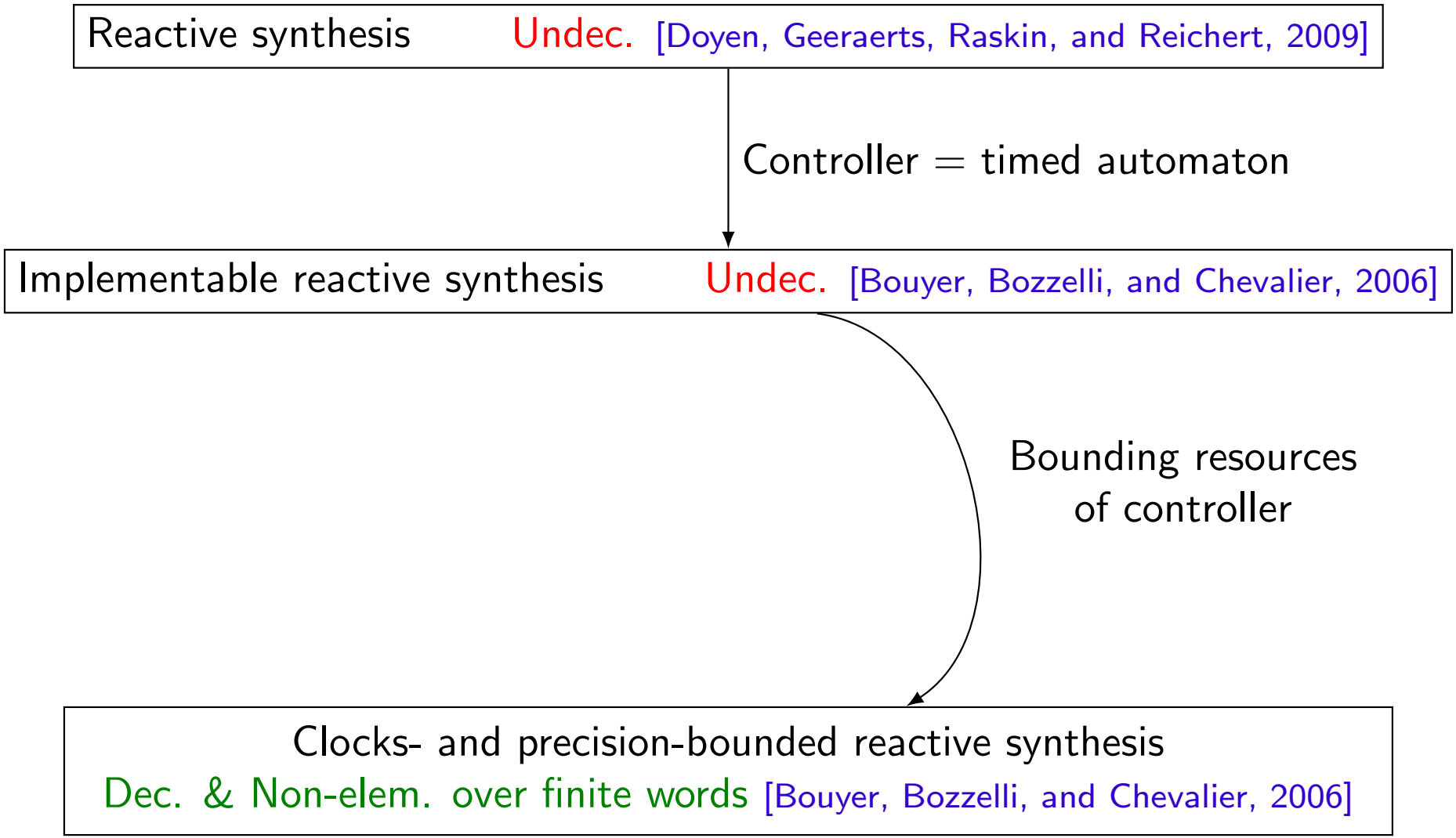

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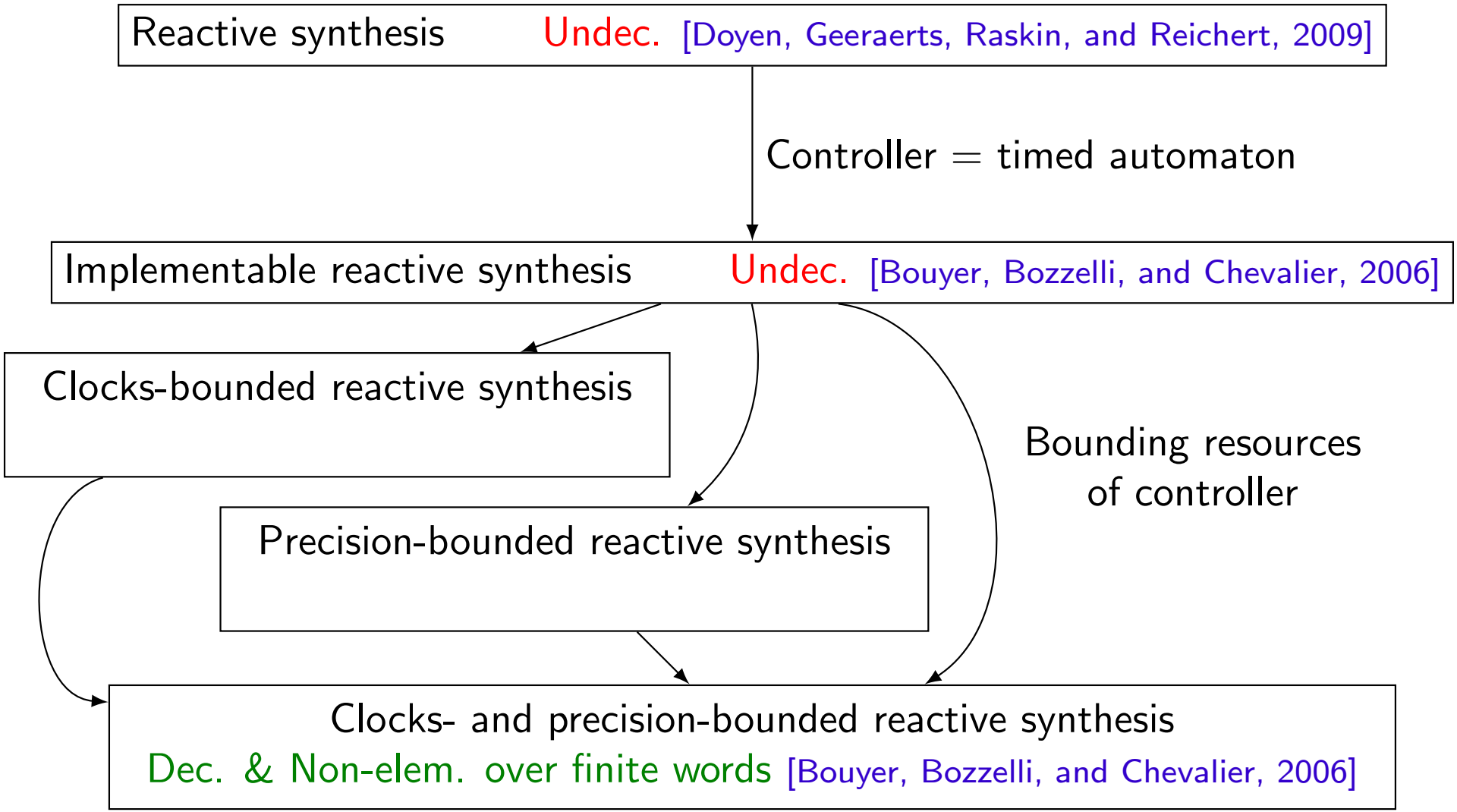

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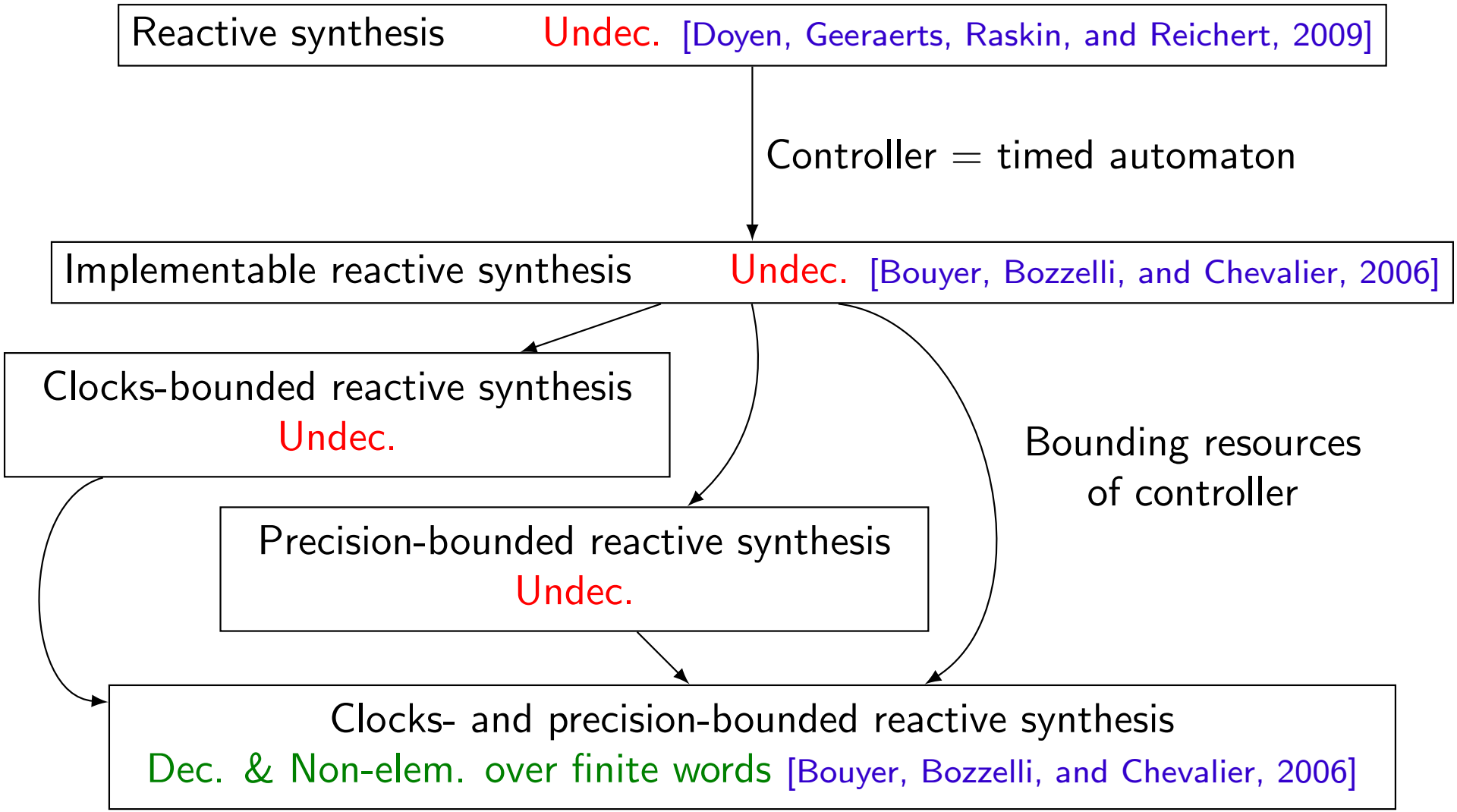

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MITL = non-punctual fragment of MTL:

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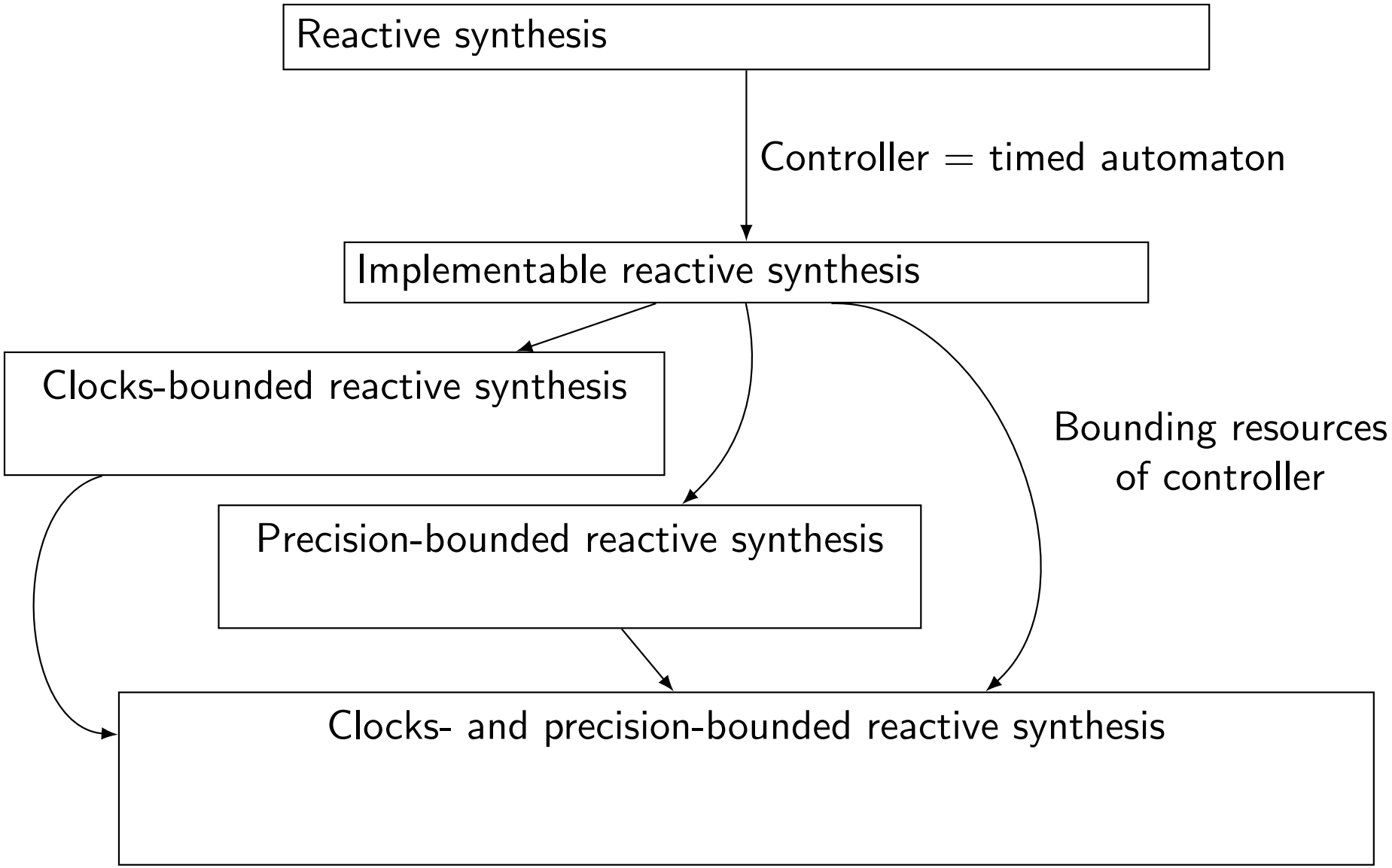
with $a \in \Sigma$, $I \subseteq [0, \infty)$ is a **non-singular** with bounds in $\mathbb{Q} \cup \{+\infty\}$

Contribution: reactive synthesis for MITL



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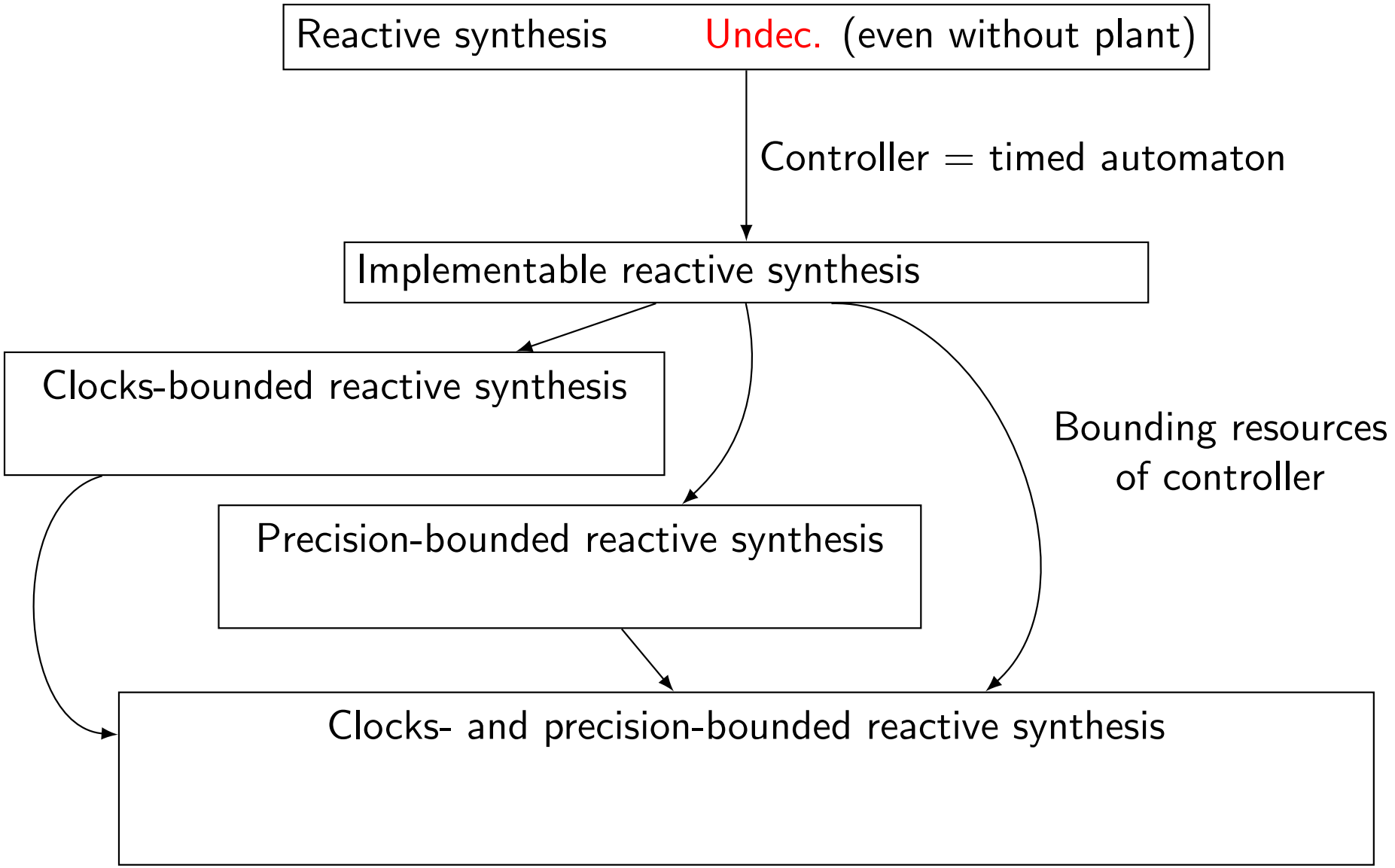

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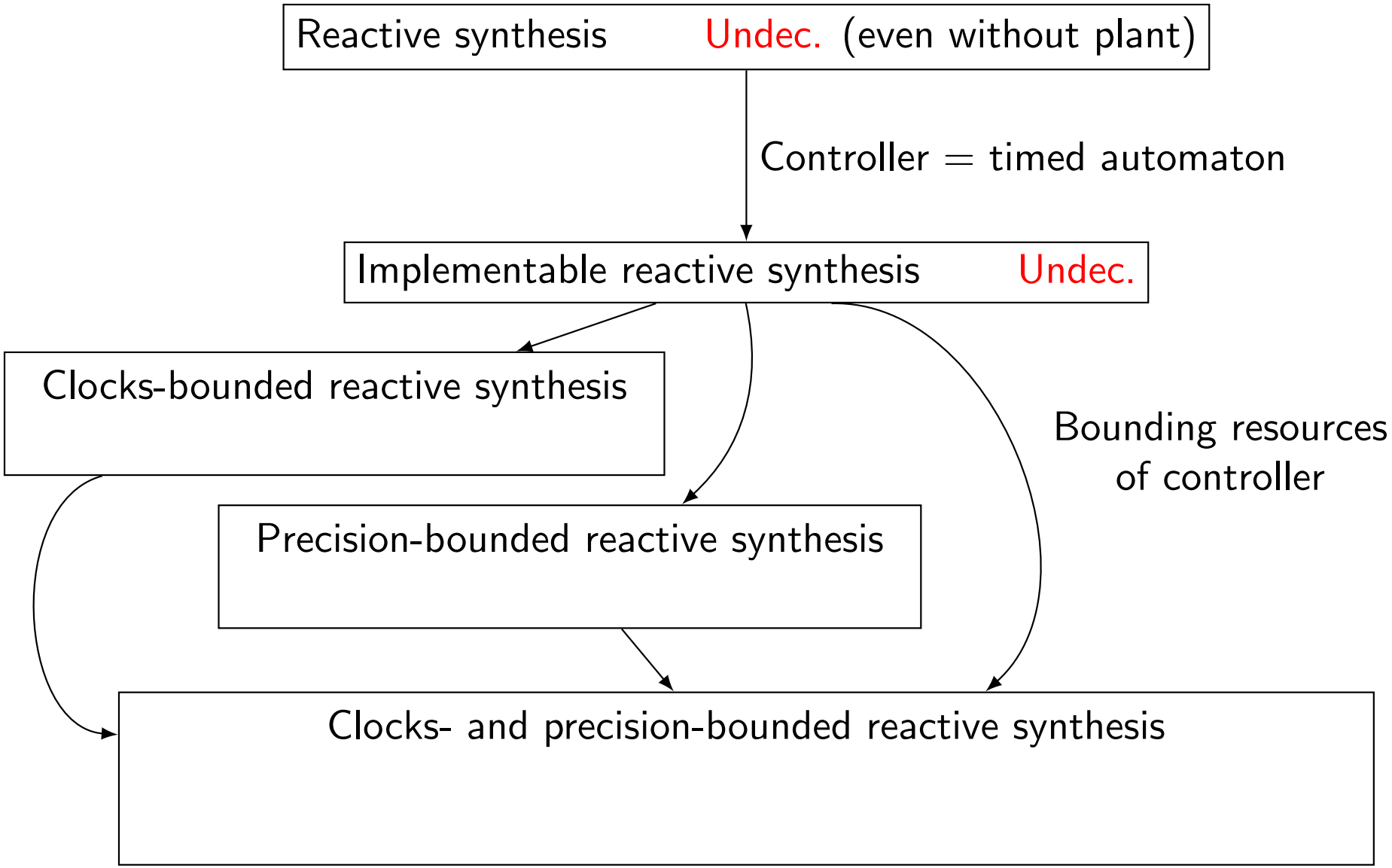

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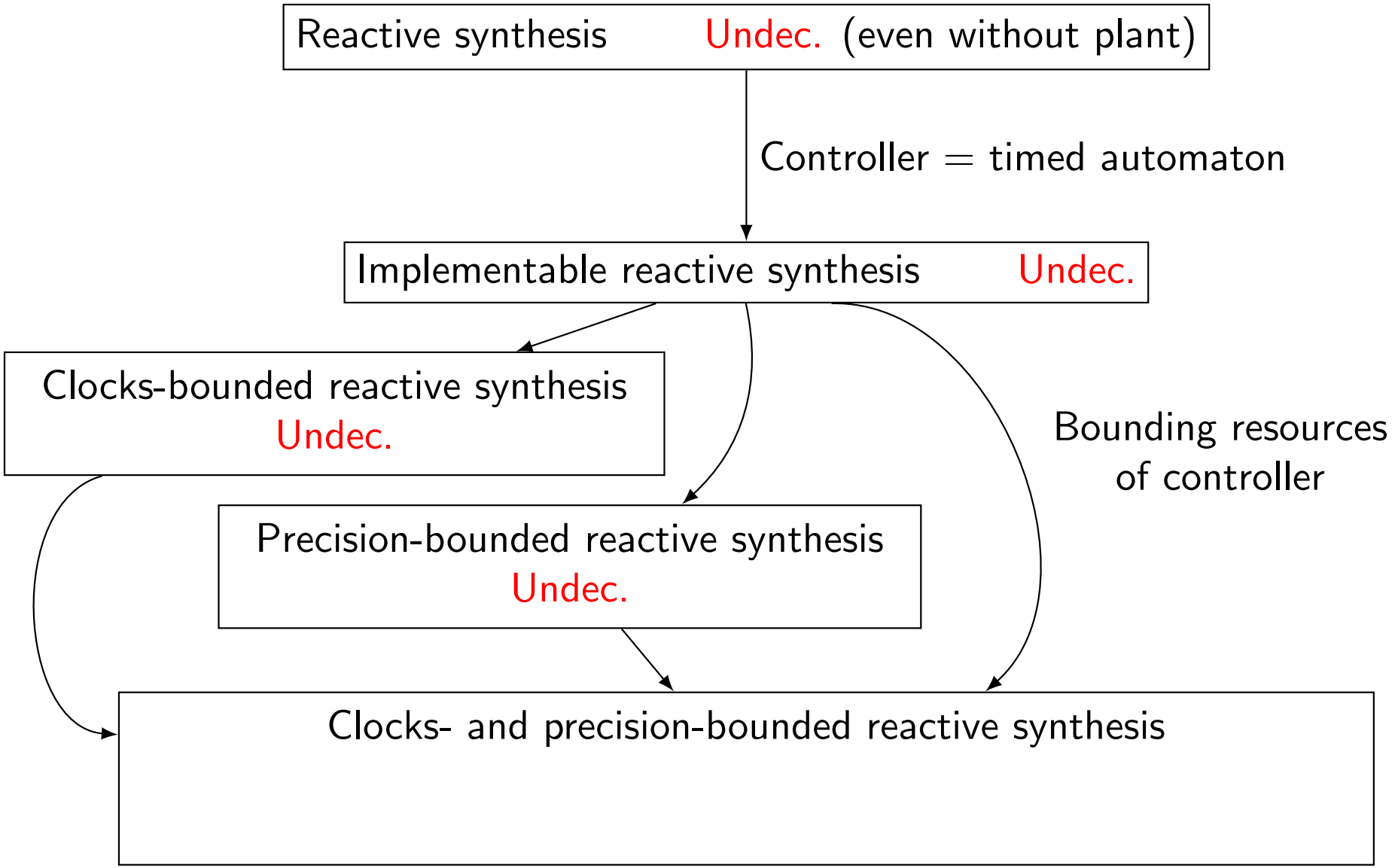

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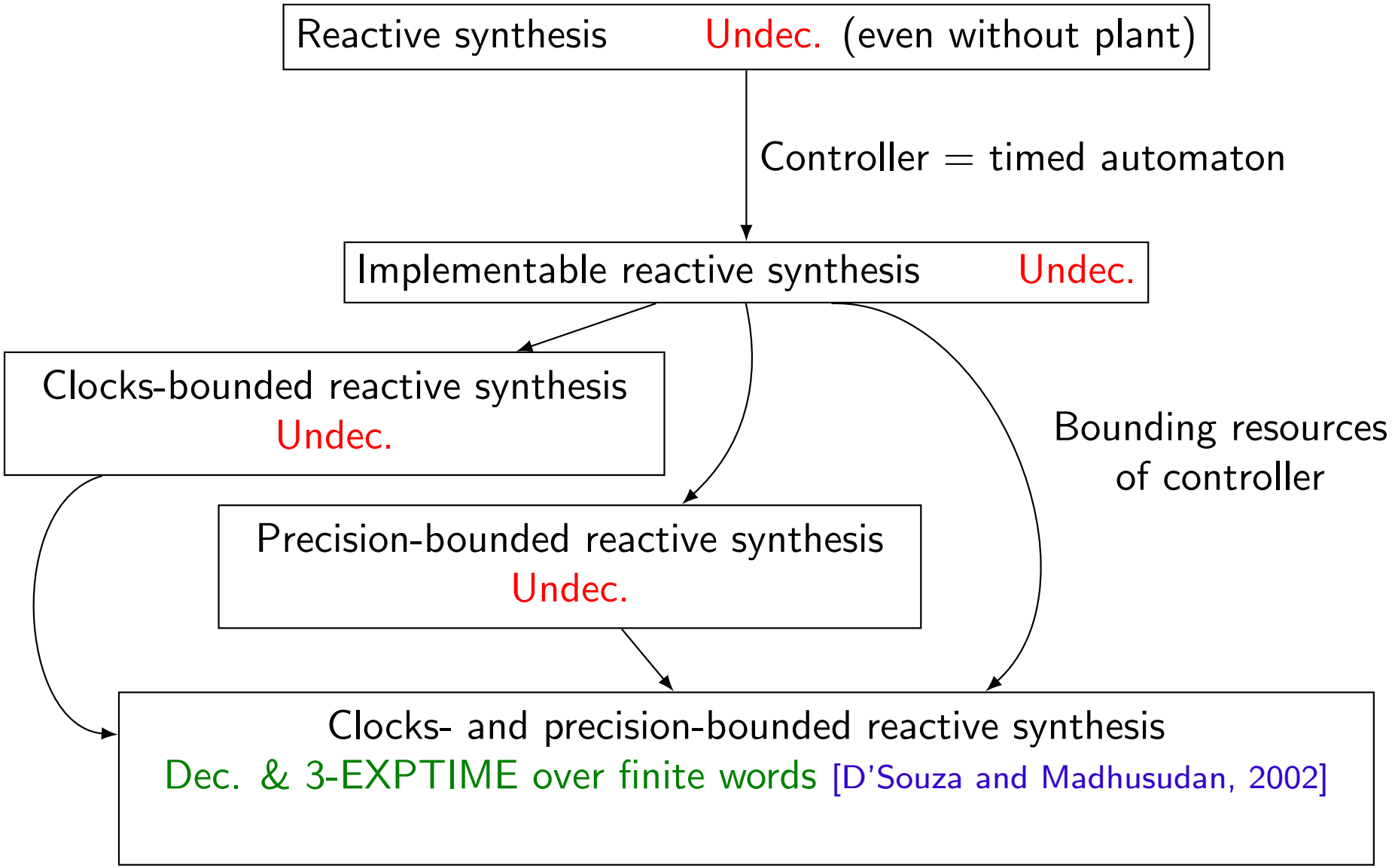

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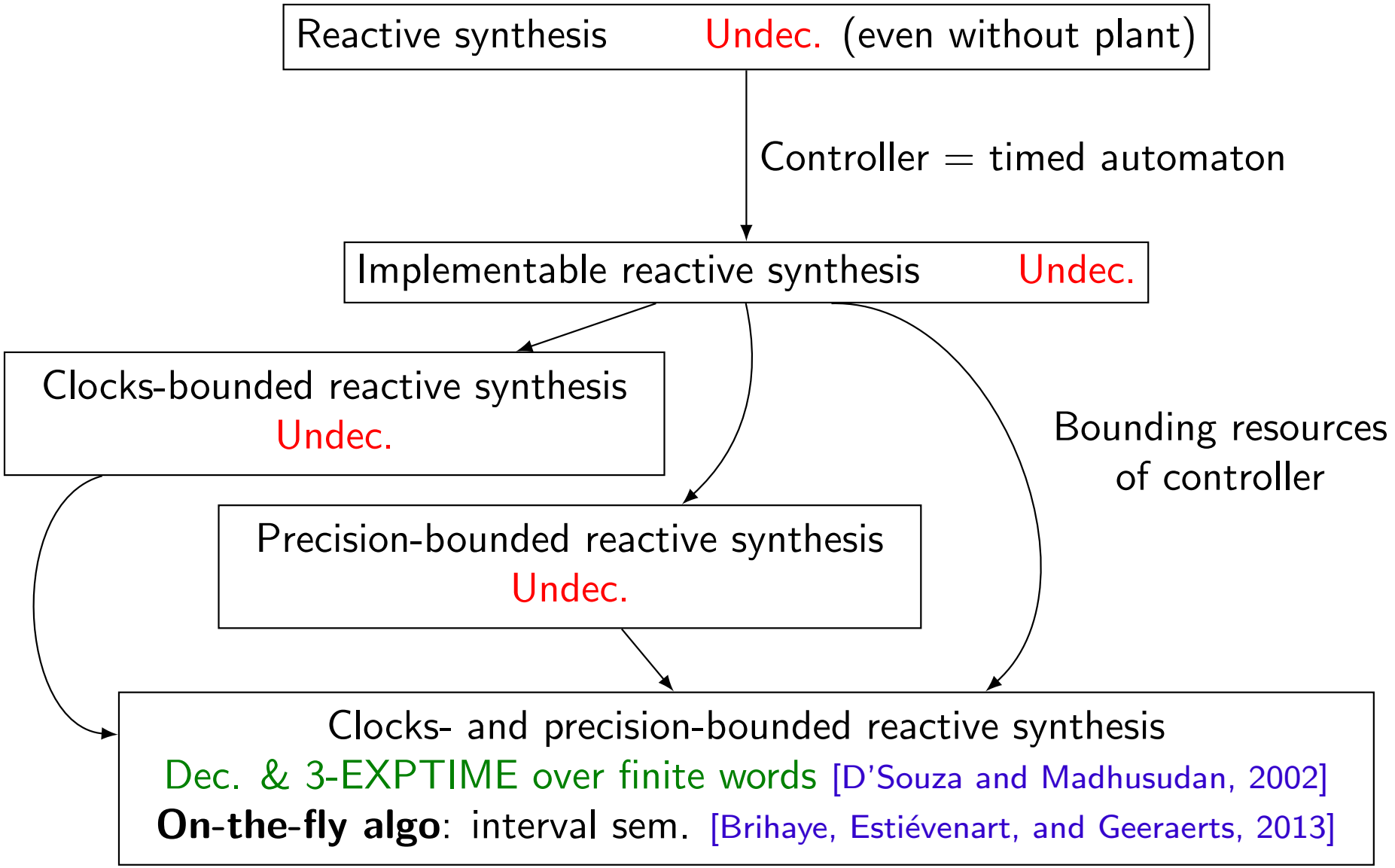

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Thank you for your attention! Questions?

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Implementable reactive synthesis (IRS)

C = deterministic symbolic transition system $\mathcal{T}$

- set of locations; if finite $\rightarrow \mathcal{T}$ is a DTA
- finite set of clocks X
- finite set of possible clock constraints *precision* (m, K):

$$g ::= \top \mid g \wedge g \mid x < \alpha/m \mid x \leqslant \alpha/m \mid x = \alpha/m \mid x \geqslant \alpha/m \mid x > \alpha/m$$

with $x \in X$, $m \in \mathbb{N}$ and $0 \leqslant \alpha \leqslant K$.

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Implementable reactive synthesis problem (IRS)

Given $\mathcal{P}$ and $\mathcal{L}$, find such a $\mathcal{T}$ that no matter what E does, every play satisfies the specification.

Recovering decidability...

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Bounded-resources reactive synthesis problem (BResRS)

Given $\mathcal{P}$, $\mathcal{L}$, and **a set of clocks X and precision (m, K)**, *find such a resource-bounded $\mathcal{T}$ that no matter what E does, every play satisfies the specification.*

Other fragments of MTL

IRS/BPrecRS/BClockRS for MITL are undecidable.
What about other fragments of MTL?

Theorem: [Ouaknine and Worrell, 2006]

Satisfiability and model-checking for Safety-MTL are decidable.

Theorem: [Bouyer, Markey, Ouaknine, and Worrell, 2007]

Model-checking for coFlat-MTL is decidable.

However with some more efforts, we can rewrite the formula in these fragments.

IRS/BPrecRS/BClockRS over infinite timed words:

	Safety-MTL	coFlat-MTL
desired	undec.	**undec.**
undesired	**undec.**	undec.

On the decidable side

Theorem: [D'Souza and Madhusudan, 2002]

BResRS is decidable for **undesired** specifications given in TA.

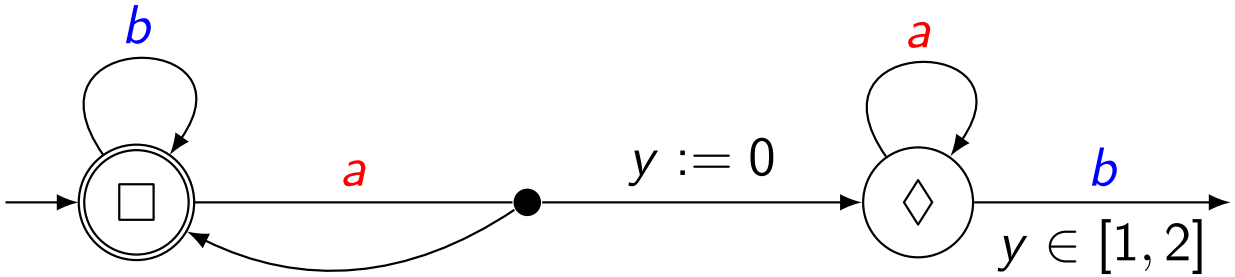
Theorem: [Bouyer, Bozzelli, and Chevalier, 2006]

BResRS is decidable for MTL specifications with non-primitive recursive complexity (over finite timed words).

We propose an **on-the-fly** algorithm that solves BResRS for MITL specifications with 3-EXPTIME complexity (over finite words).

From MTL to OCATA

$$\varphi = \Box(a \Rightarrow \Diamond_{[1,2]}\, b)$$



Execution on the timed word $(a, 0.5)(a, 0.6)(a, 1.2)(b, 2.3)$: