Synchronizing Data Words for Register Automata

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Data Words and Data Languages

Σ...a finite alphabet
D...an infinite data domain, e.g., $\mathbb{N}$, $\mathbb{R}_{\geq 0}$, `ASCII strings`

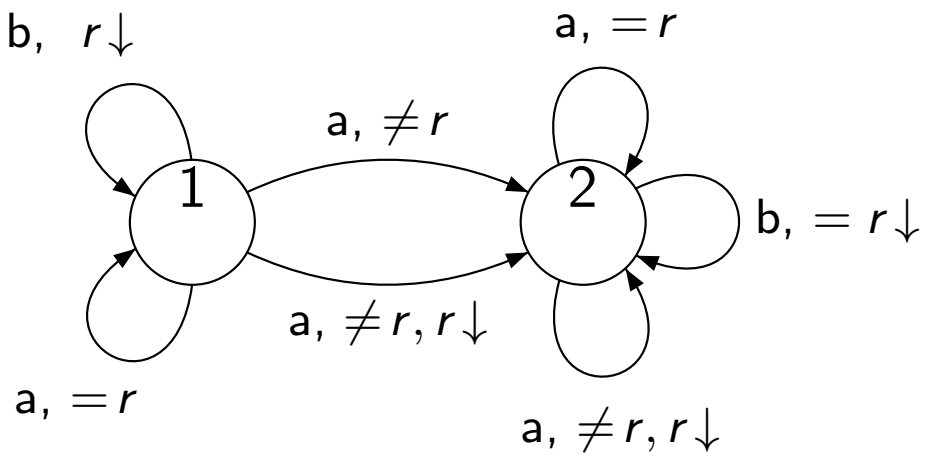
A **data word** is a finite sequence

$$(a_1, d_1)(a_2, d_2) \dots (a_n, d_n)$$

over $\Sigma \times D$.

A **data language** is a subset of $(\Sigma \times D)^*$.

Register Automata (RA)



- r register variable
- $r\downarrow$ store current datum into r
- $=r$ is the current datum equal to value stored in r?
- $\neq r$ is the current datum different from value stored in r?

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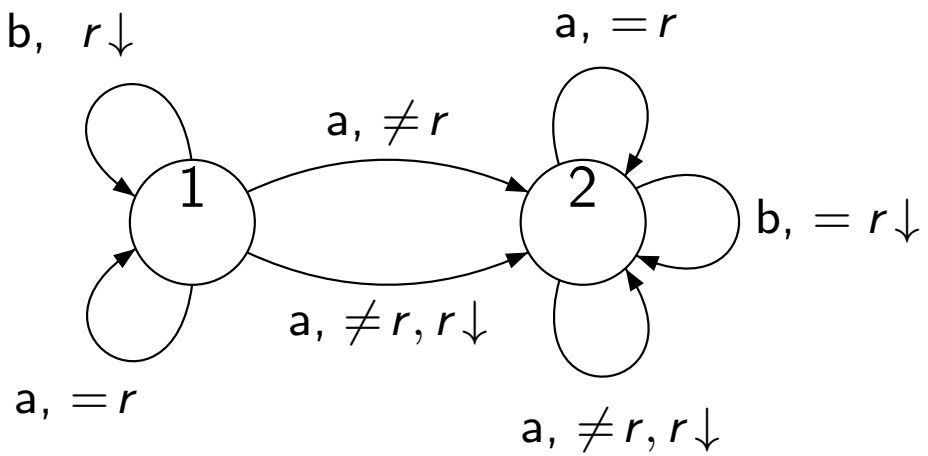

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$w = (a, d_1)(a, d_2)(a, d_3)$

$$(1, d_1) \xrightarrow{(a,d_1)} (1, d_1) \xrightarrow{(a,d_2)} (2, d_2) \xrightarrow{(a,d_3)} (2, d_3)$$

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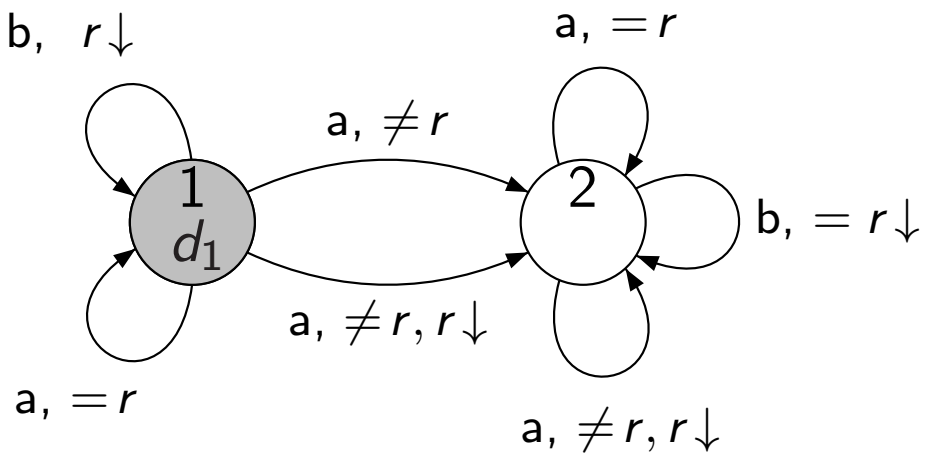

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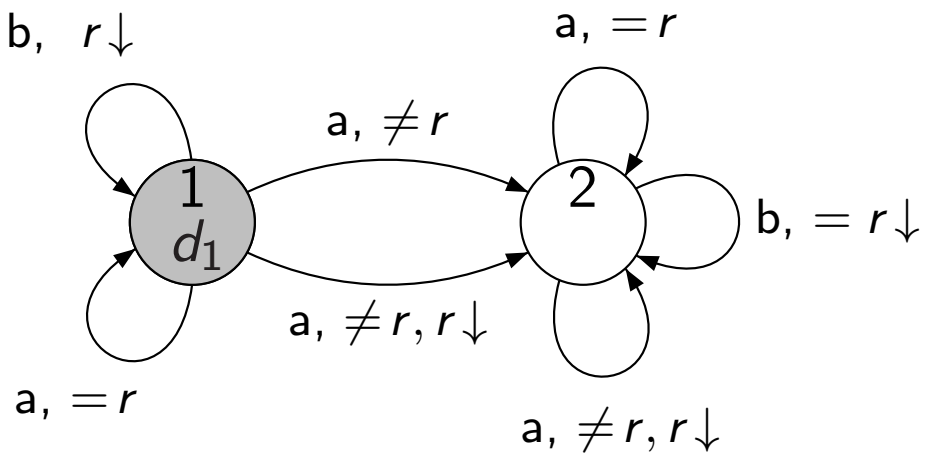

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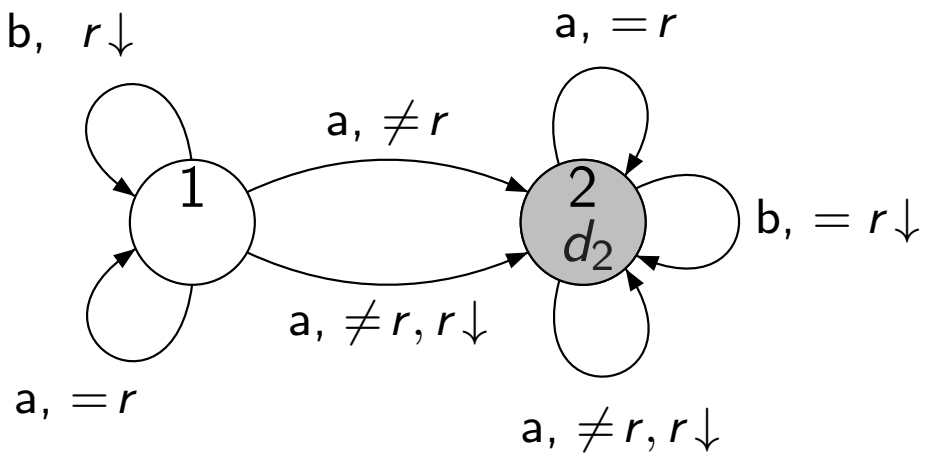

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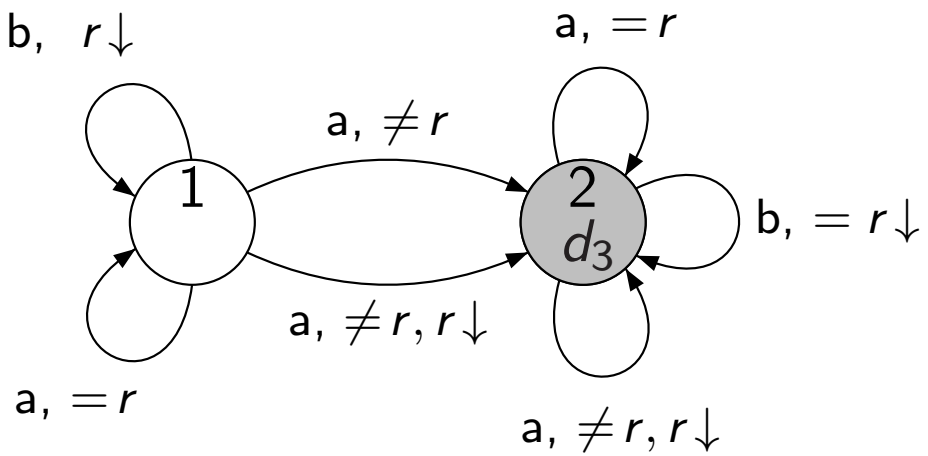

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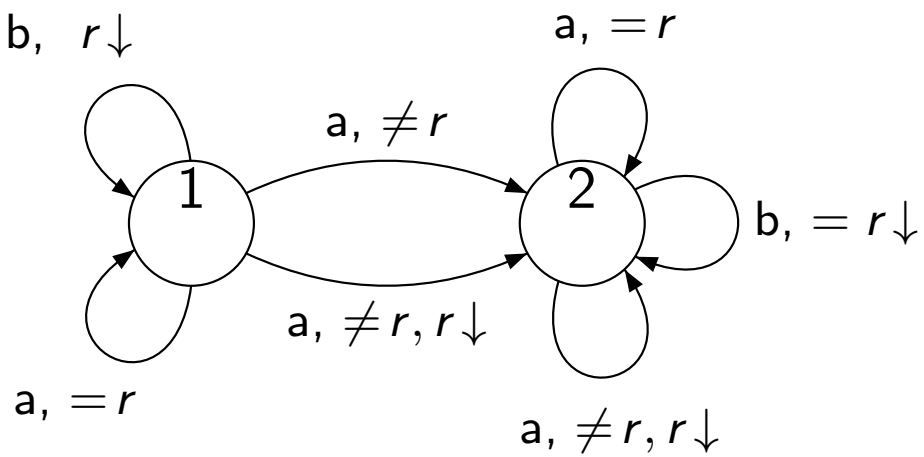

$$\{1,2\} \times D$$
$$\downarrow (a, d_1)$$
$$\{(1,d_1),(2,d_1)\} \cup (\{2\} \times D \backslash \{d_1\})$$
$$\downarrow (a, d_2)$$
$$\{(2,d_1),(2,d_2)\}$$
$$\downarrow (a, d_3)$$
$$\{(2,d_3)\}$$

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DRA	PSPACE-complete
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Instance:	Register automaton $\mathcal{A}$	Register automaton $\mathcal{A}$
Question:	Does there exist a synchronizing data word for $\mathcal{A}$?	Does there exist some data word that is not accepted by $\mathcal{A}$?

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[1] Neven, Schwentick, Vianu: Finite state machines for strings over infinite alphabets, 2004.
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The Bounded Synchronizing Problem

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Results		
NRA	NEXPTIME-complete	NEXPTIME-complete
1-NRA	NEXPTIME-complete	NEXPTIME-complete

NEXPTIME-completeness of the Bounded Non-Universality Problem

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	Bounded Non-Universality Problem for Regular-Like Expressions
Instance:	Regular-like expression expr (built from $\{a, b\}$ and operations $\cdot, +, ^2$), $N \in \mathbf{N}$ (in binary)
Question:	Does there exist some string u with $\|u\| \leq N$ and $u \notin L(expr)$?

[1] Stockmeyer, Meyer: Word Problems requiring exponential time, 1973.

$\mathrm{expr} = (aa + a)^2, \quad N = 100$

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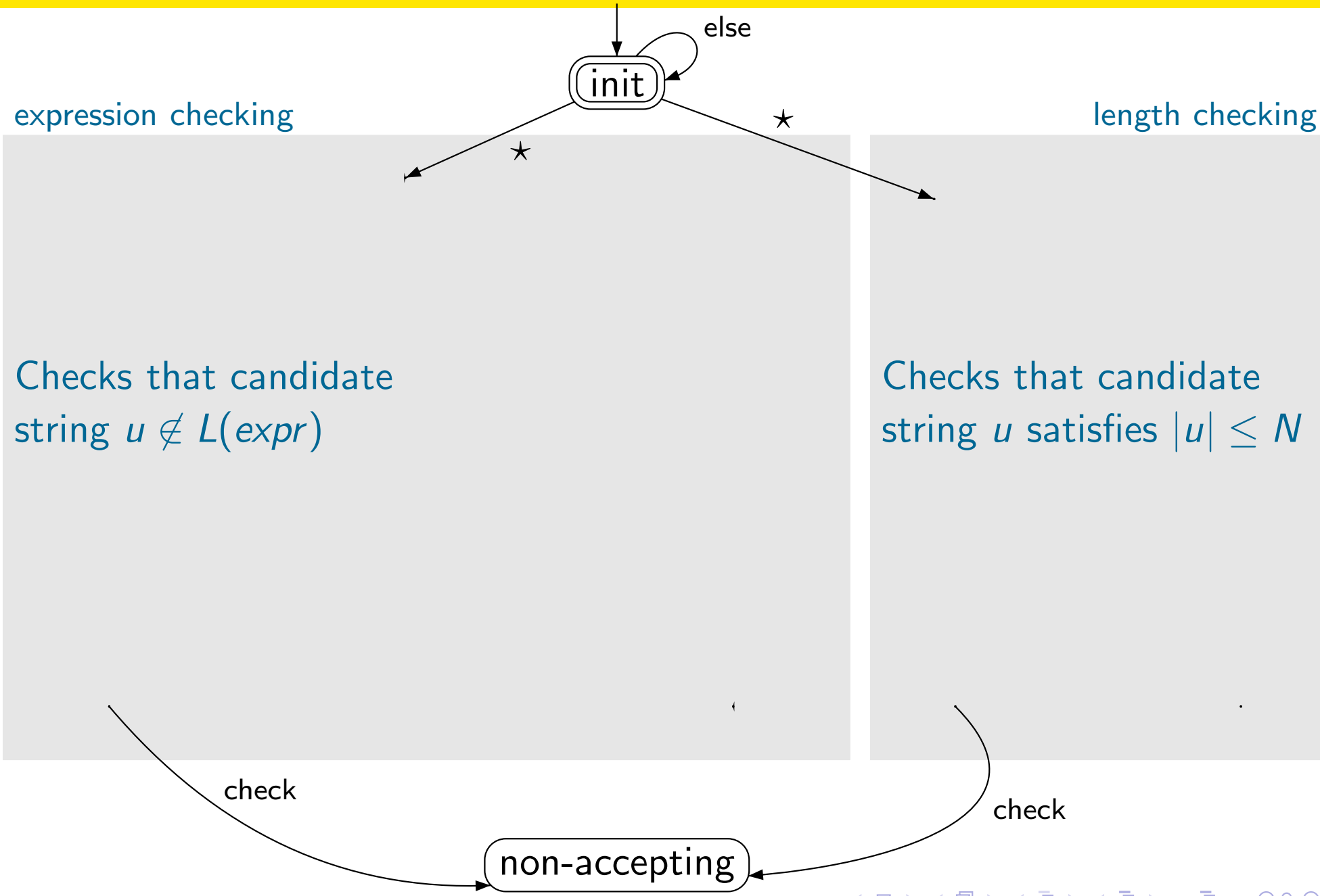

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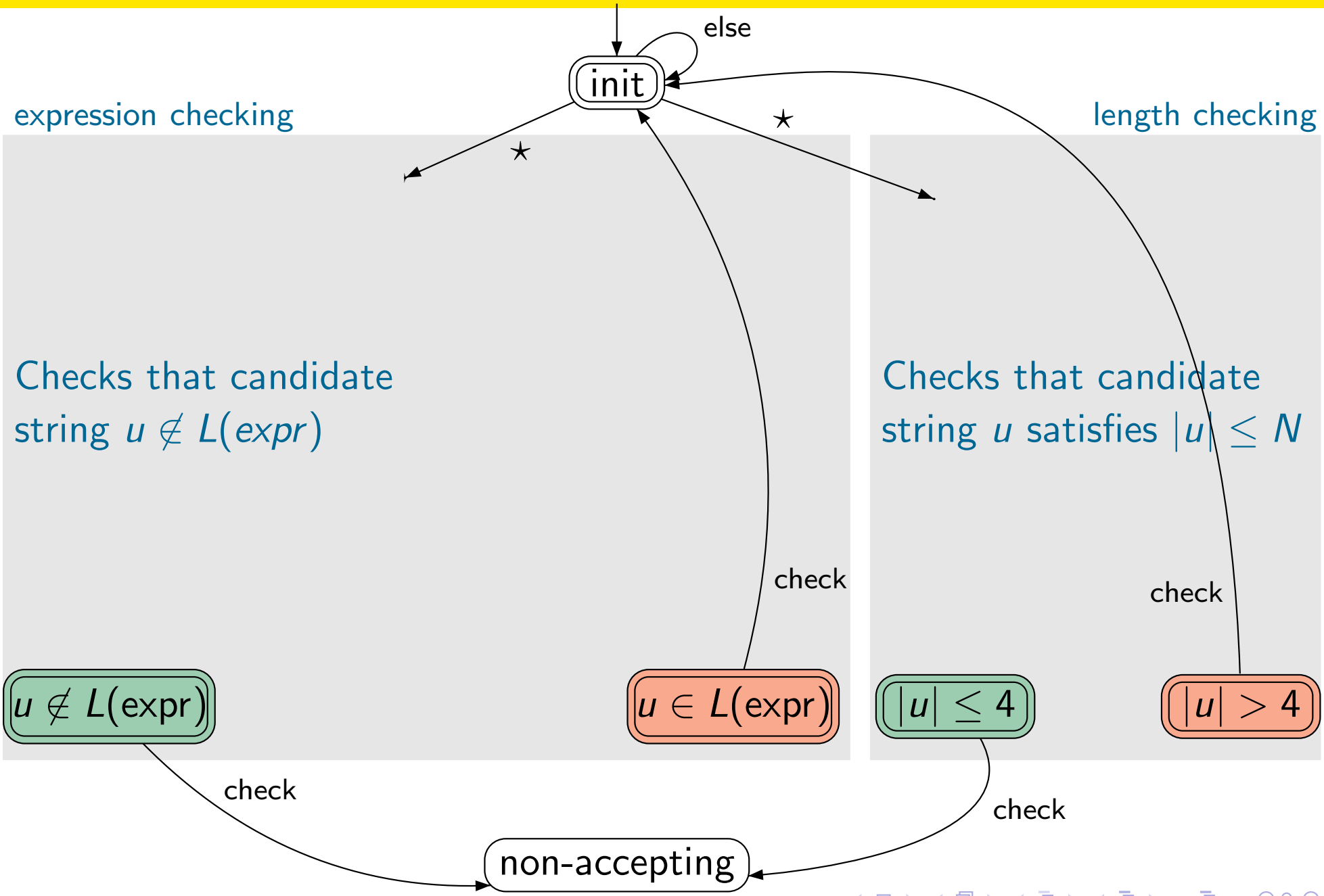

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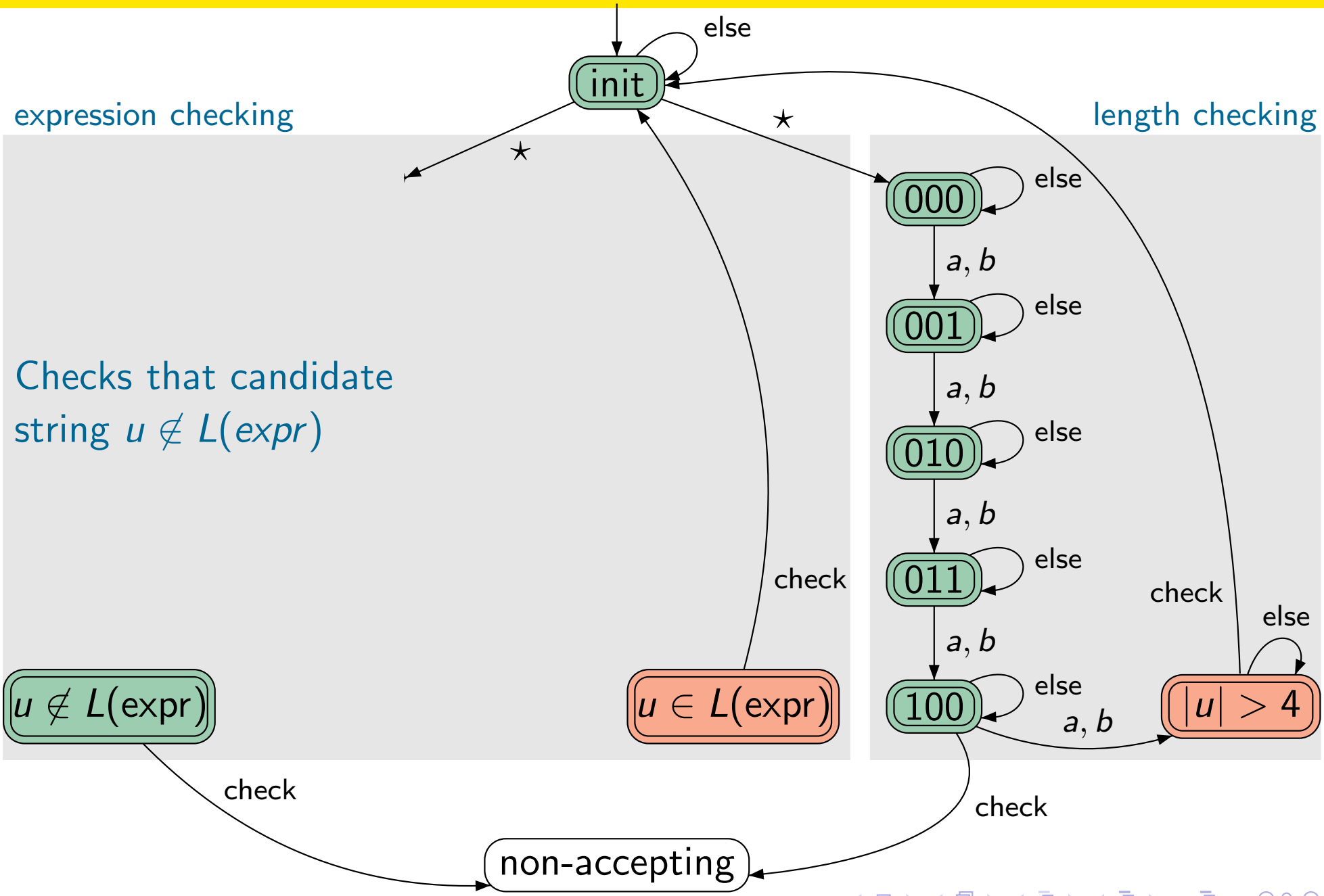

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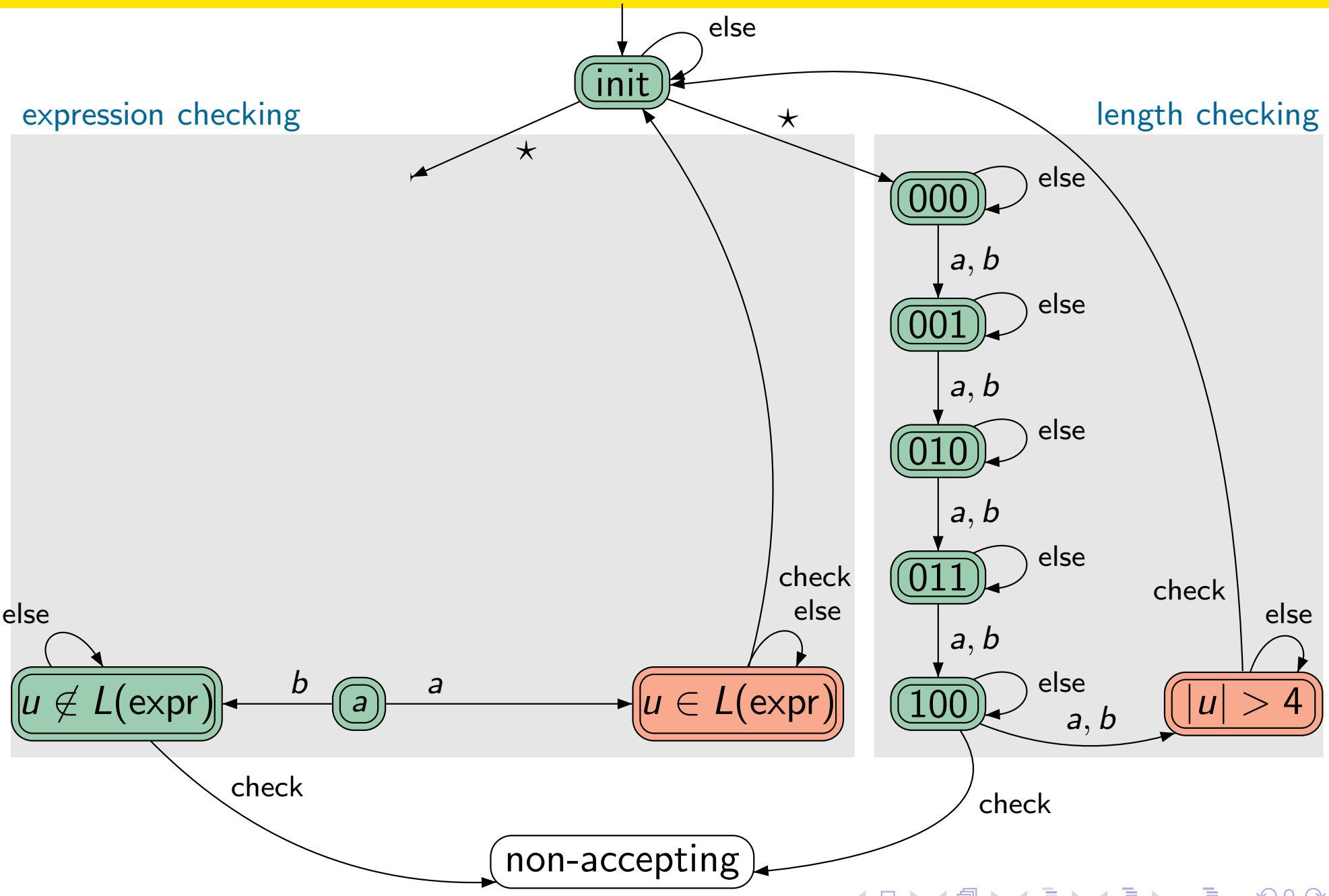

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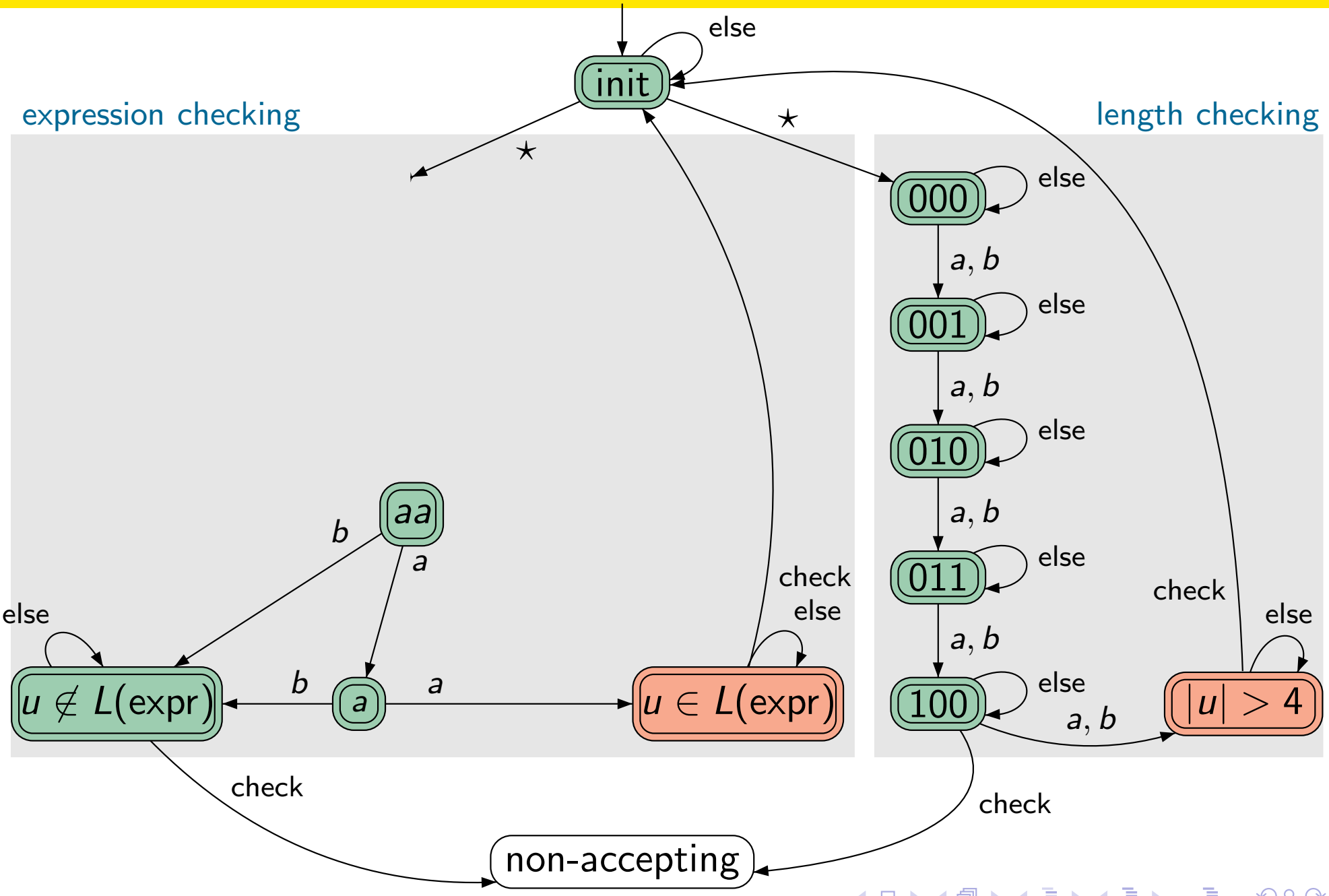

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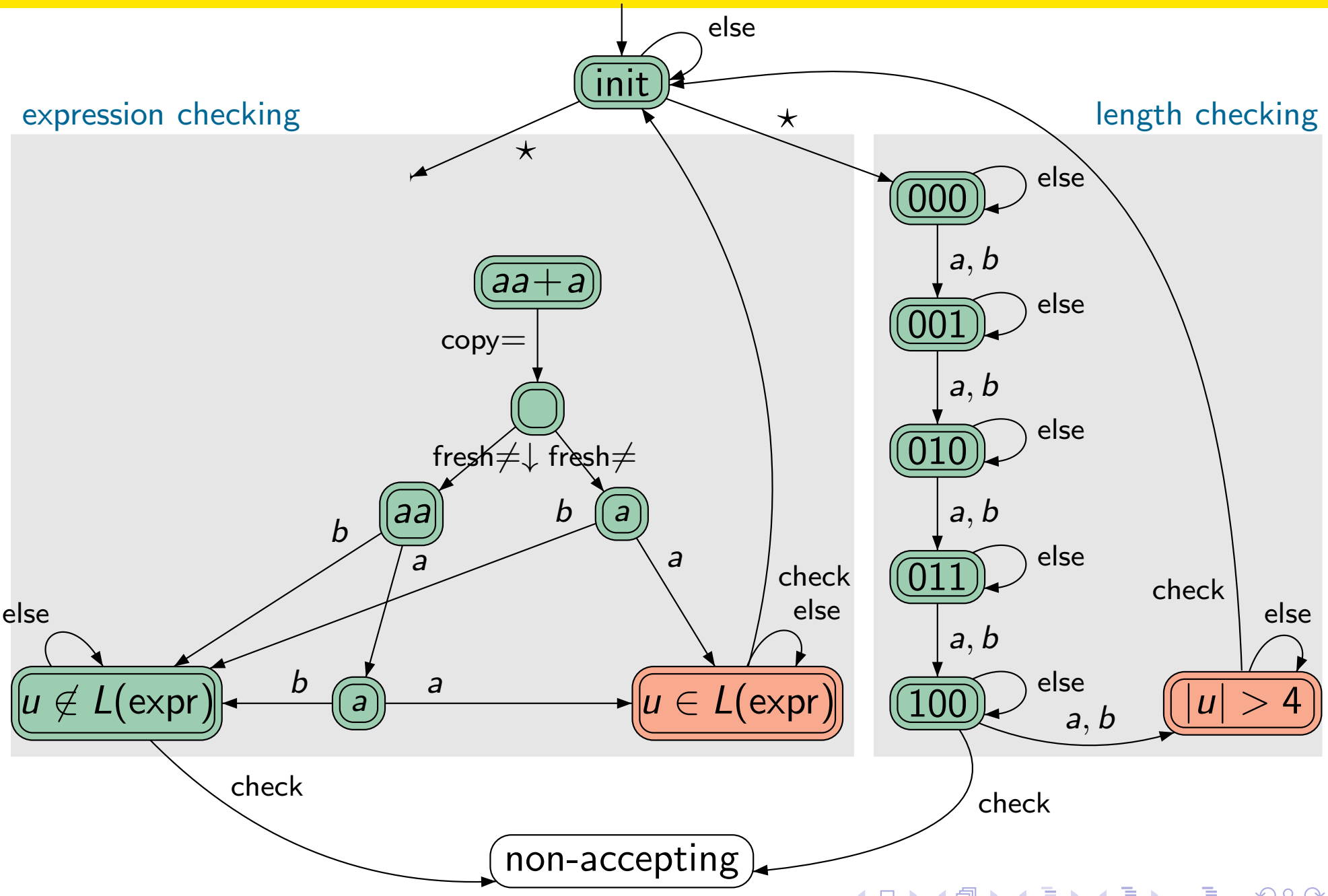

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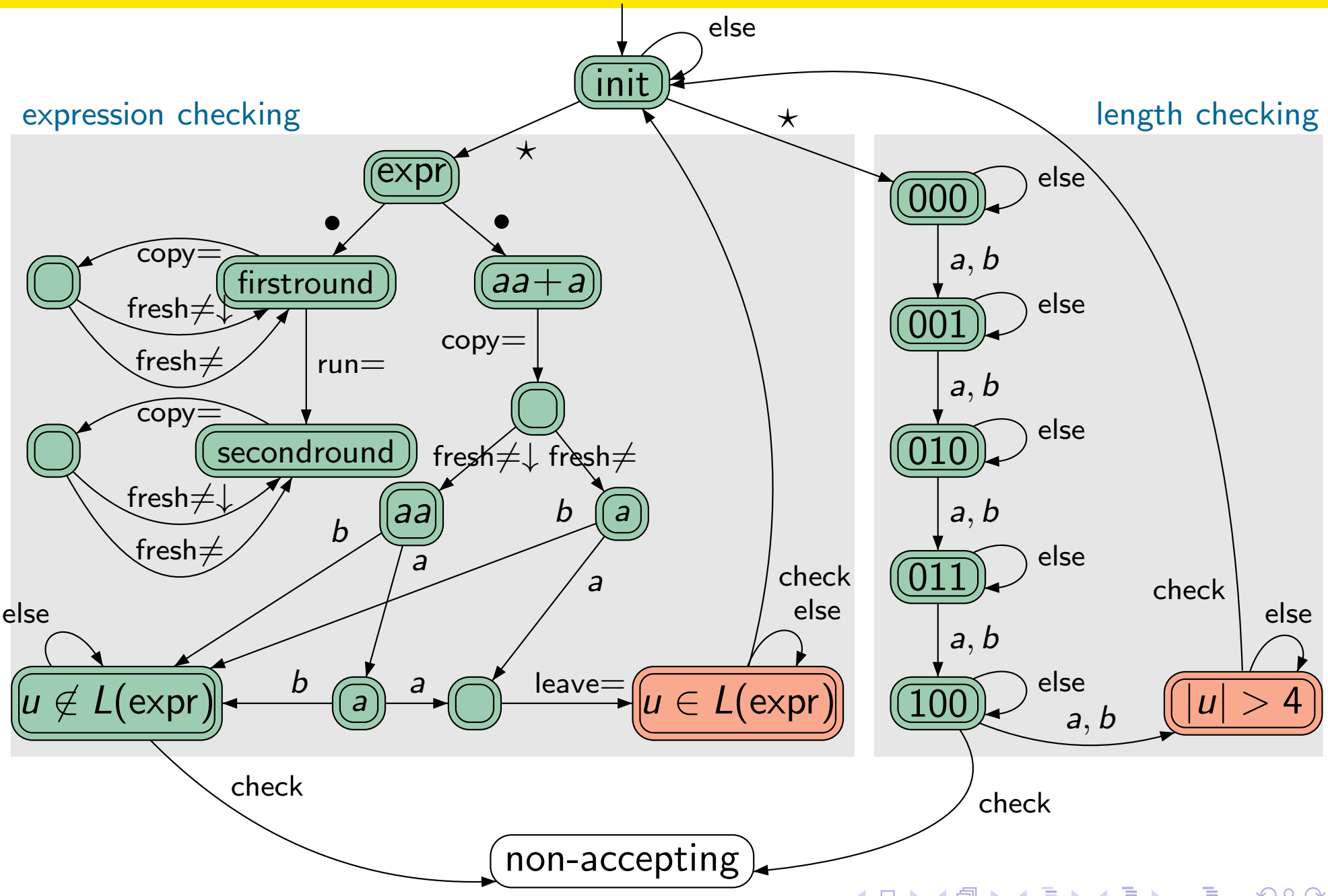

Summary and Open Problems

The synchronizing/non-universality problem is

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- NLOGSPACE-complete for 1-DRA
- undecidable for NRA
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Open: Precise complexity for the bounded synchronizing/non-universality problem for DRA.