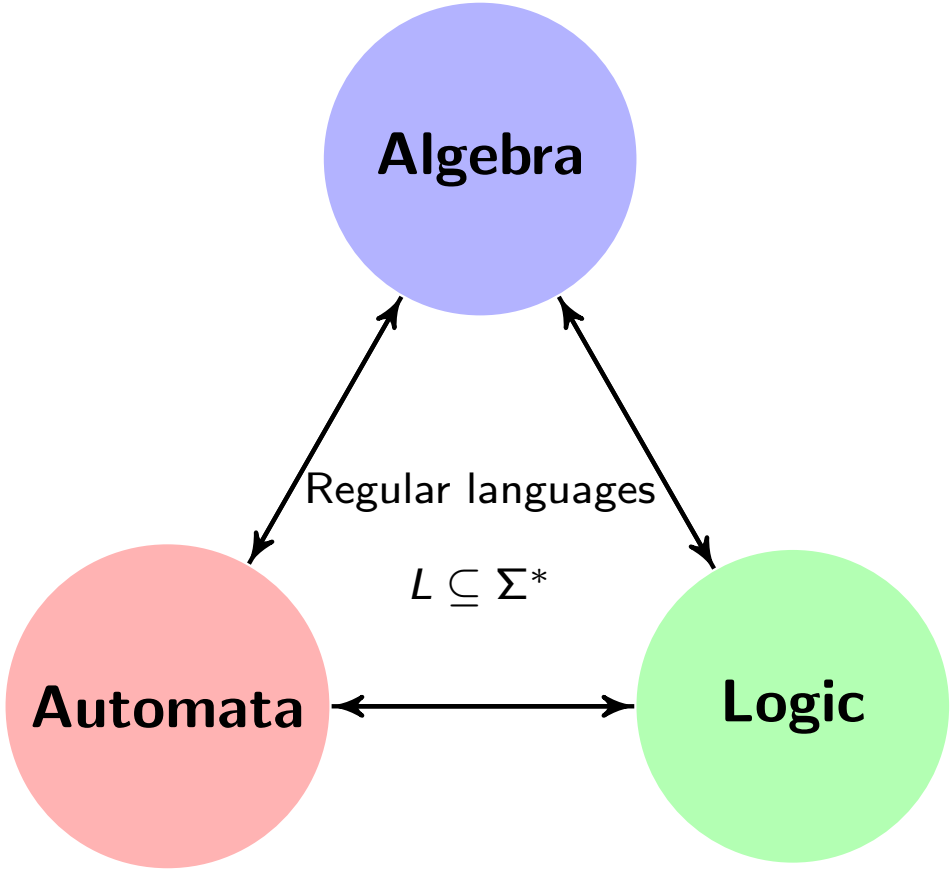
Automata, Logic and Algebra for (Finite) Word Transductions

Emmanuel Filiot

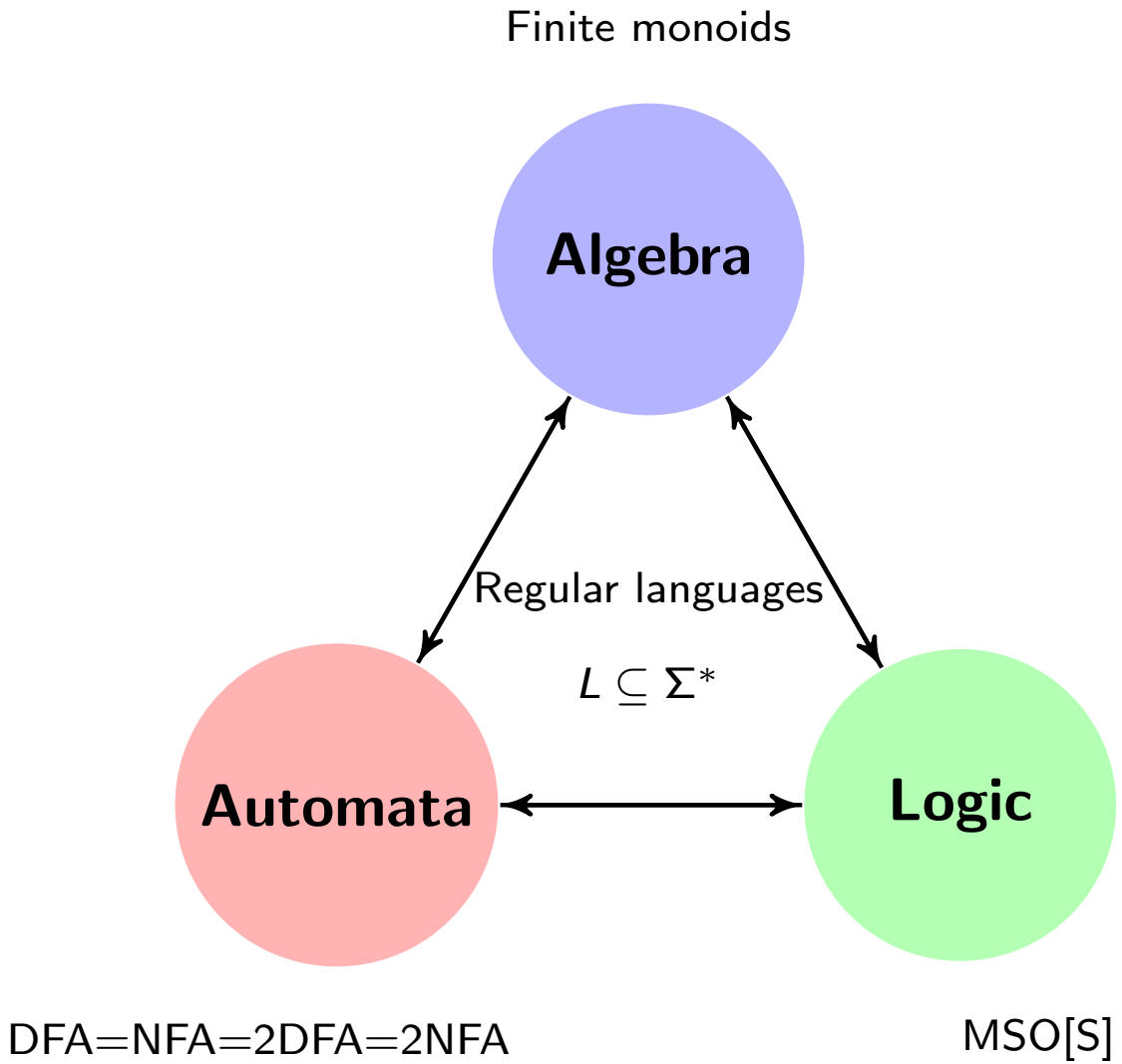
Université libre de Bruxelles & FNRS

Highlights of Logic, Automata and Games 2016

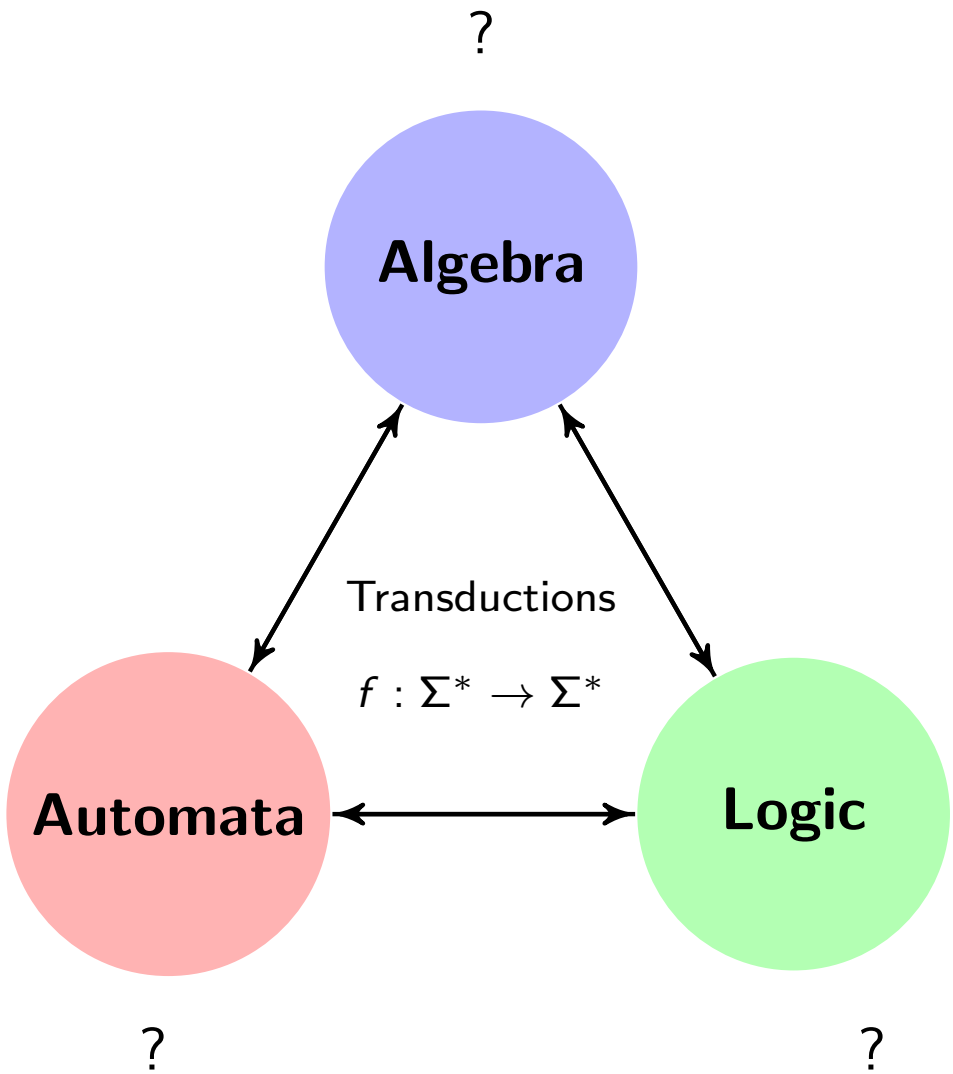
Trinity for Regular Languages



Trinity for Regular Languages

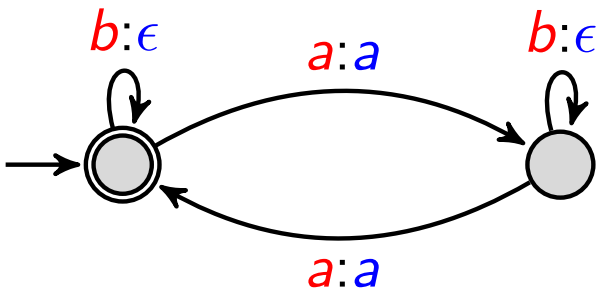


Objective of the talk



Automata for transductions: transducers

f_{del} :



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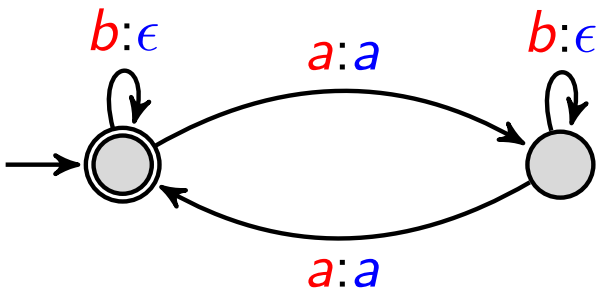

$$aabaa \mapsto aaaa$$

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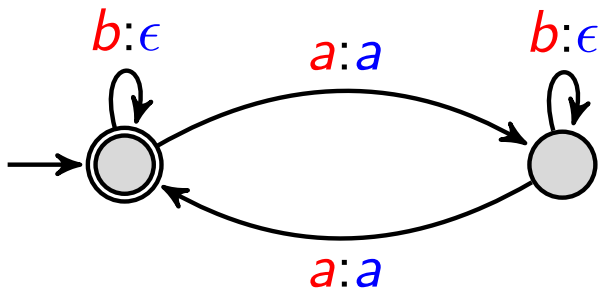

$$aabaa \mapsto aaaa$$

$$aaba \mapsto \text{undefined}$$

Automata for transductions: transducers

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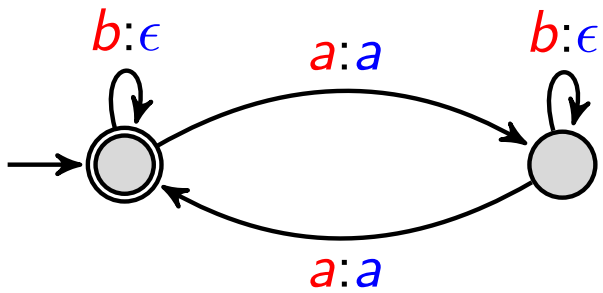


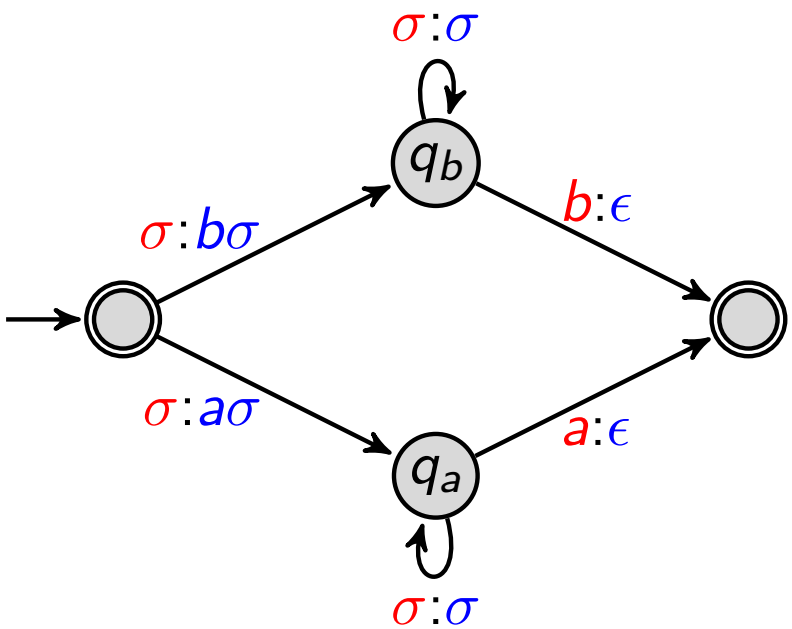
$$aabaa \mapsto aaaa$$

$$aaba \mapsto \text{undefined}$$

$$dom(f_{del}) = \text{'even number of } a\text{'}$$

Automata for transductions: transducers

f_{sw} :



for all $\sigma \in \Sigma$

Automata for transductions: transducers

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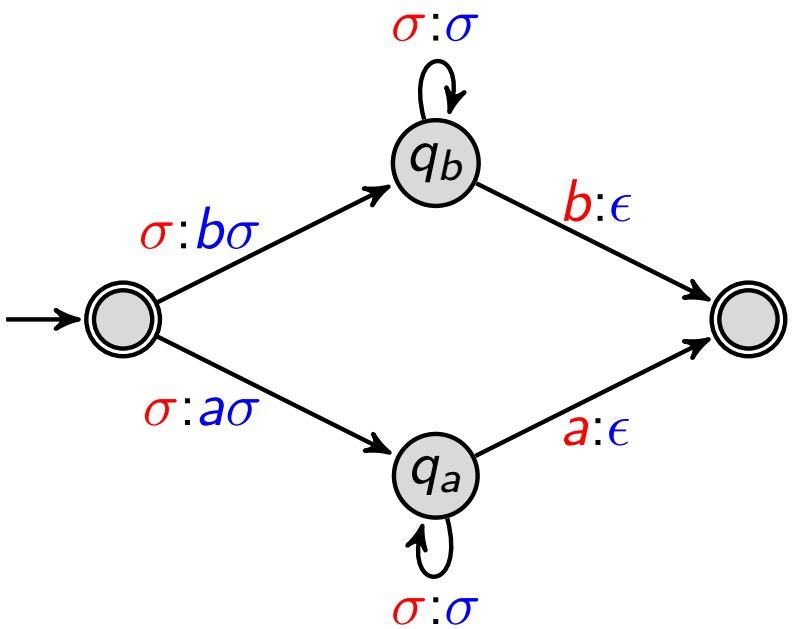

for all $\sigma \in \Sigma$

$$babaa \mapsto ababa$$

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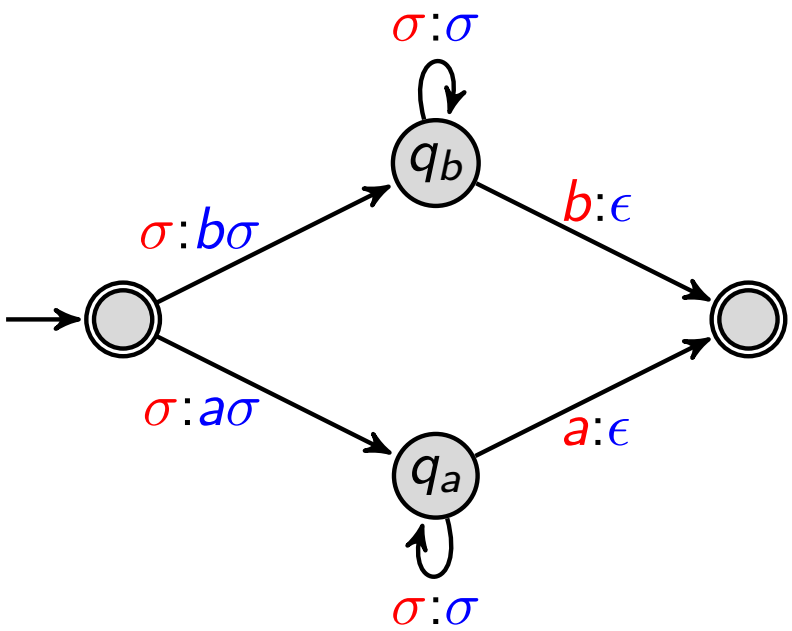


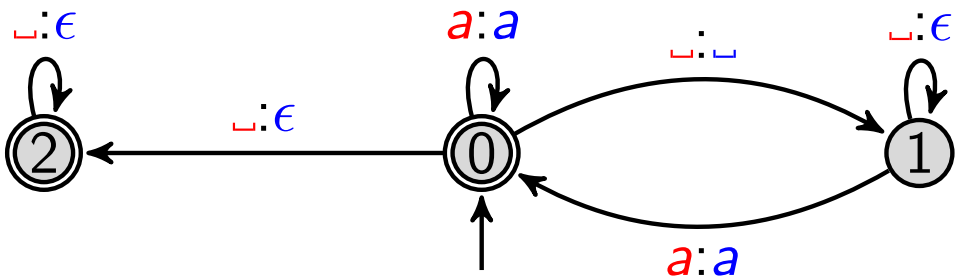
for all $\sigma \in \Sigma$

$$babaa \mapsto ababa$$

$$u\sigma \mapsto \sigma u \qquad |u| \geq 1$$

input-determinism (aka sequential) $<$ non-determinism

Automata for transductions: transducers



␣ = white space

Automata for transductions: transducers

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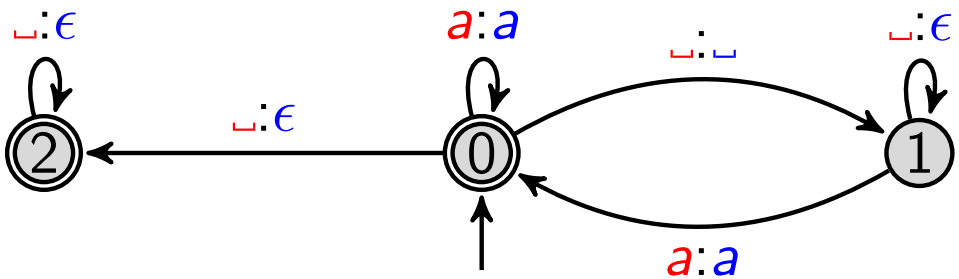

␣␣aa␣␣␣a␣␣ ↦ ␣aa␣a

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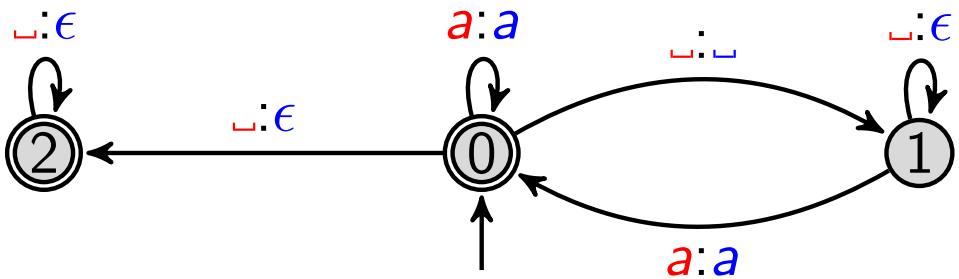

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Is non-determinism needed ?

Automata for transductions: transducers

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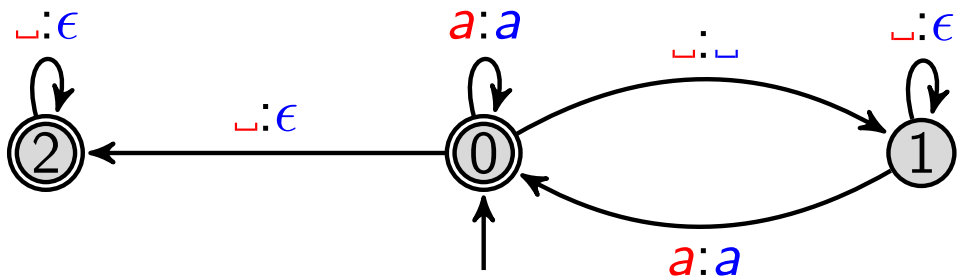
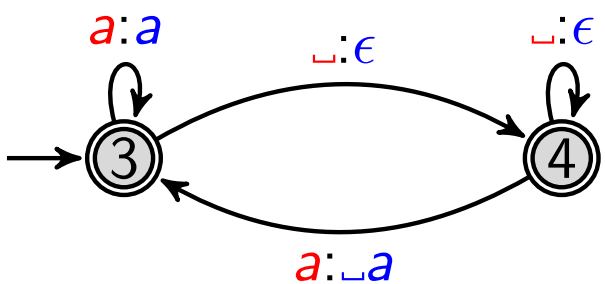
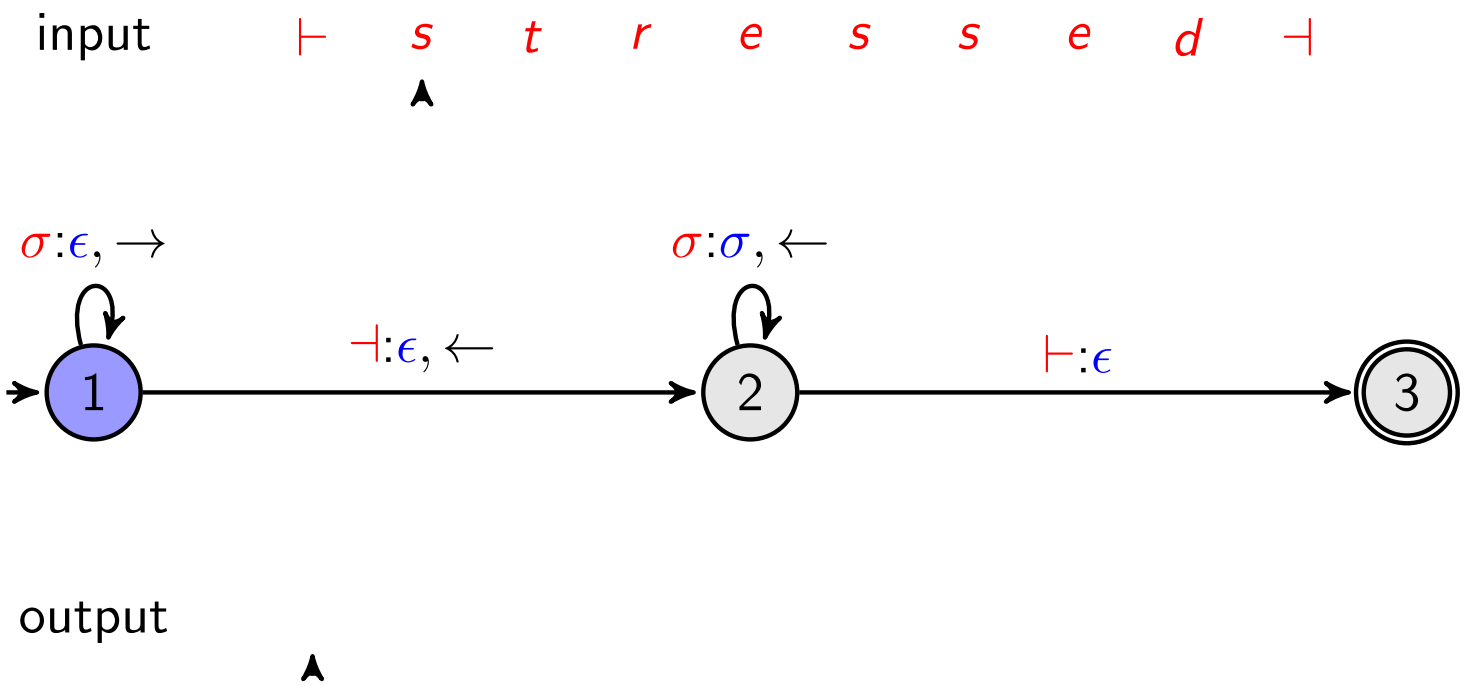


␣␣aa␣␣␣a␣␣ ↦ ␣aa␣a



Is non-determinism needed ? No.

Automata for transductions: two-way transducers



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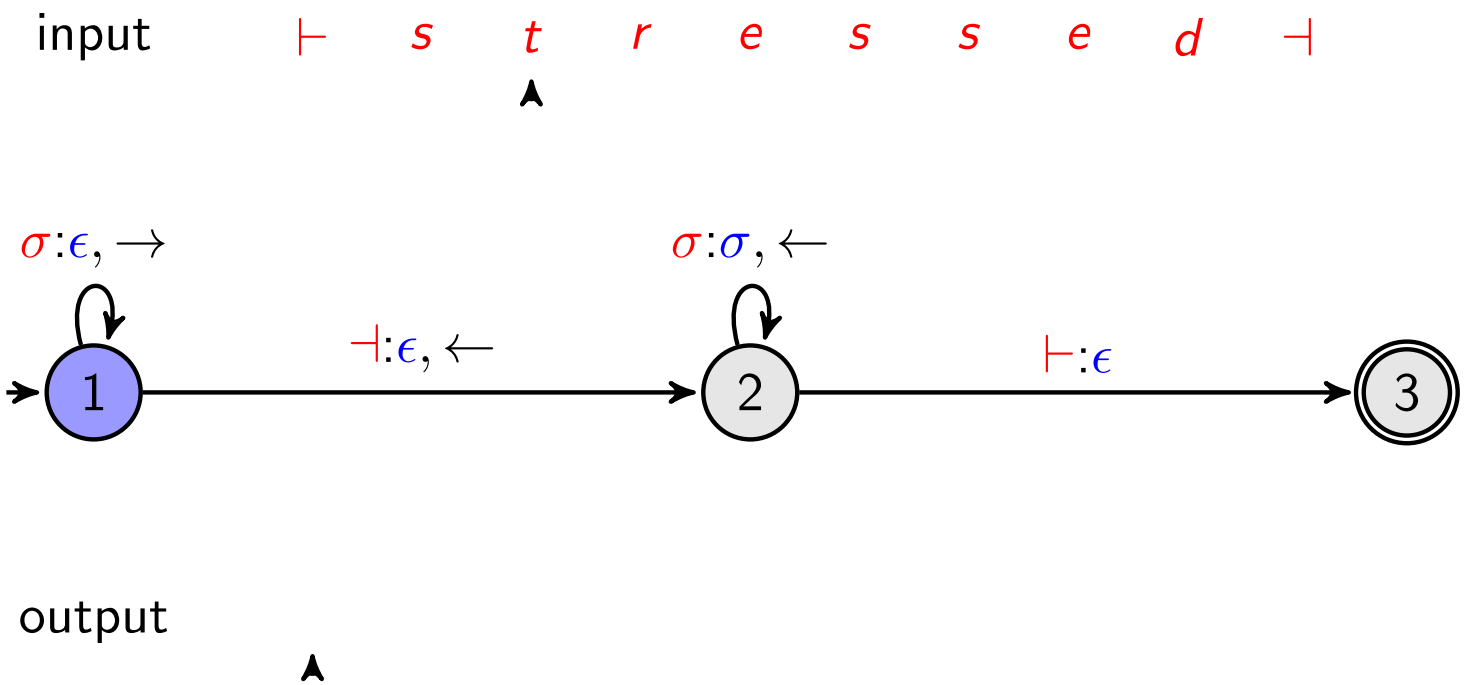

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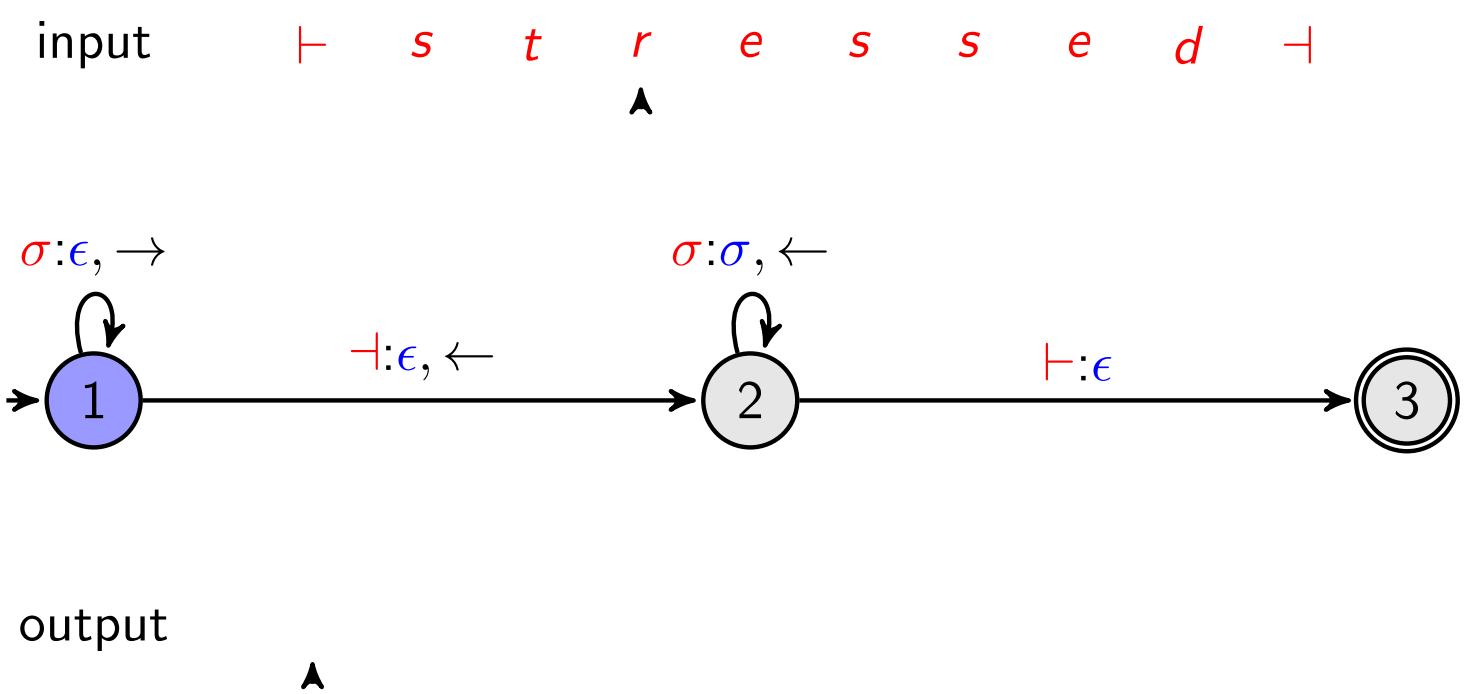

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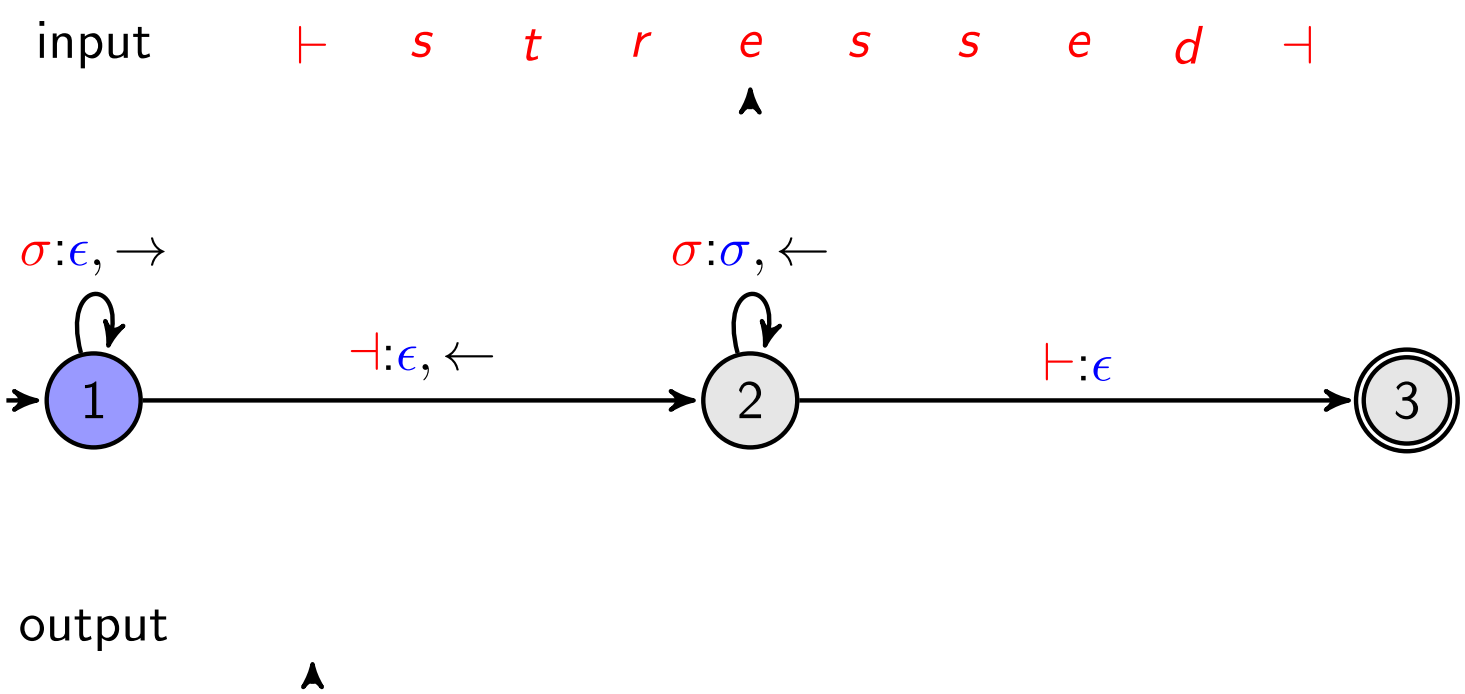

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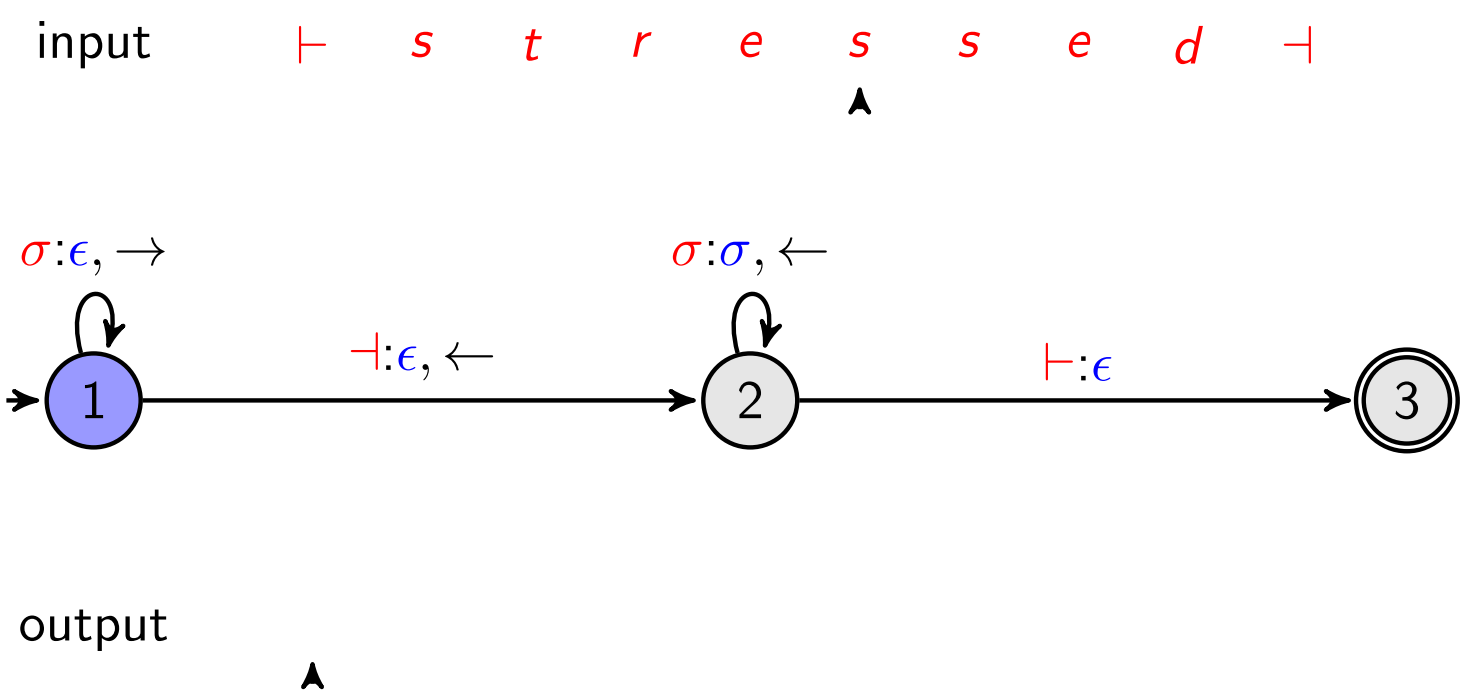

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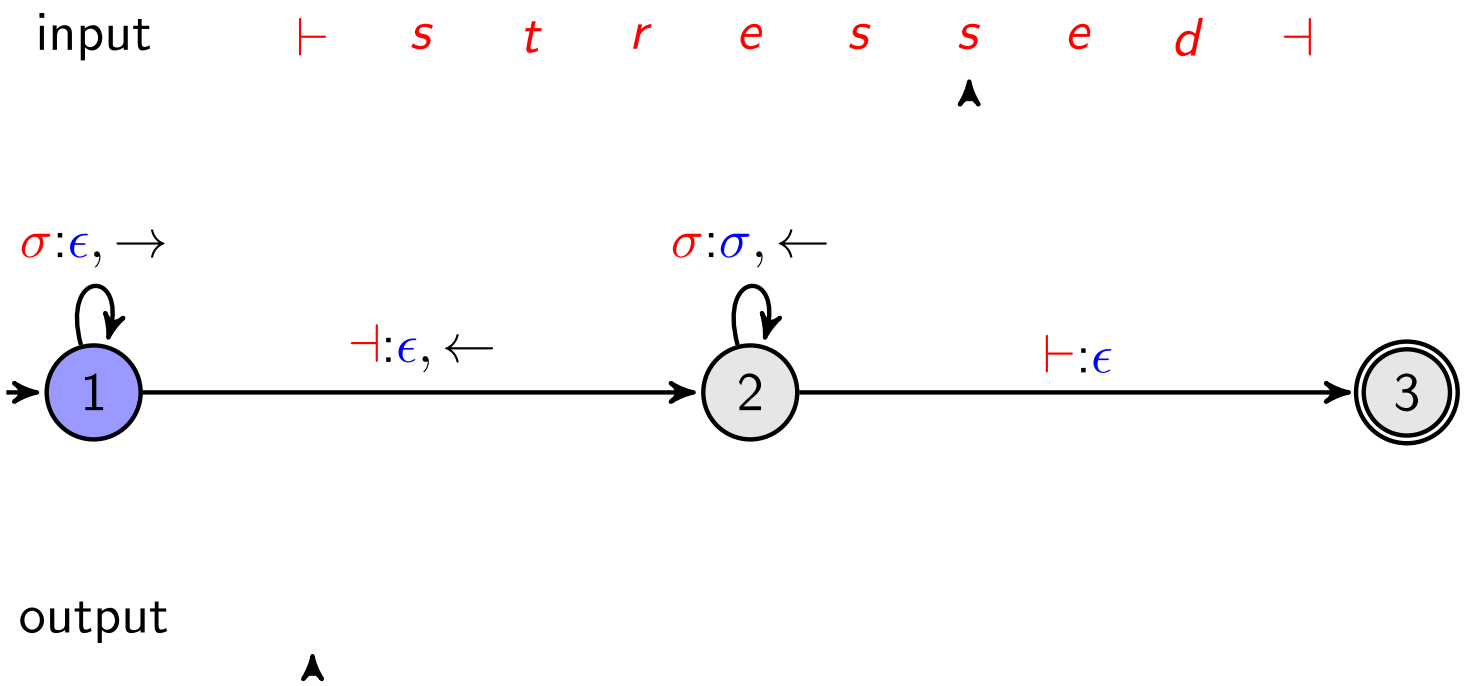

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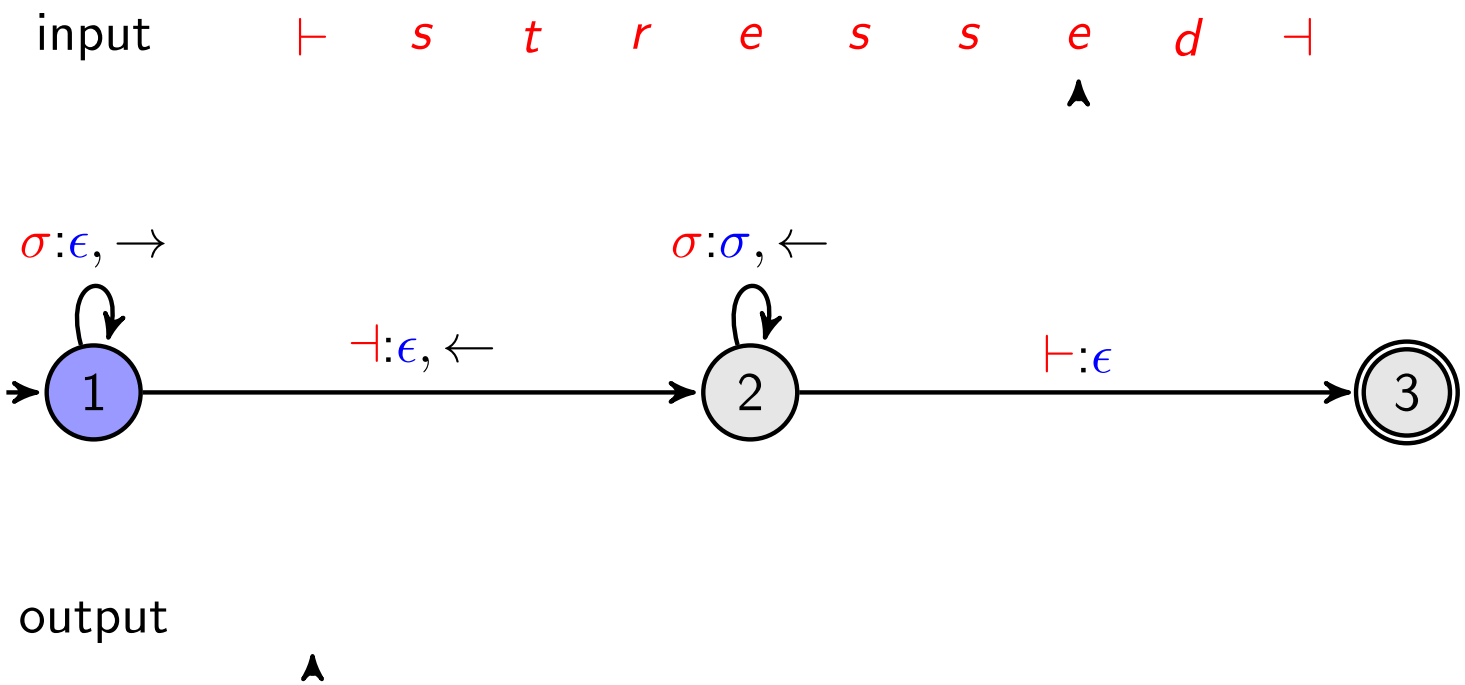

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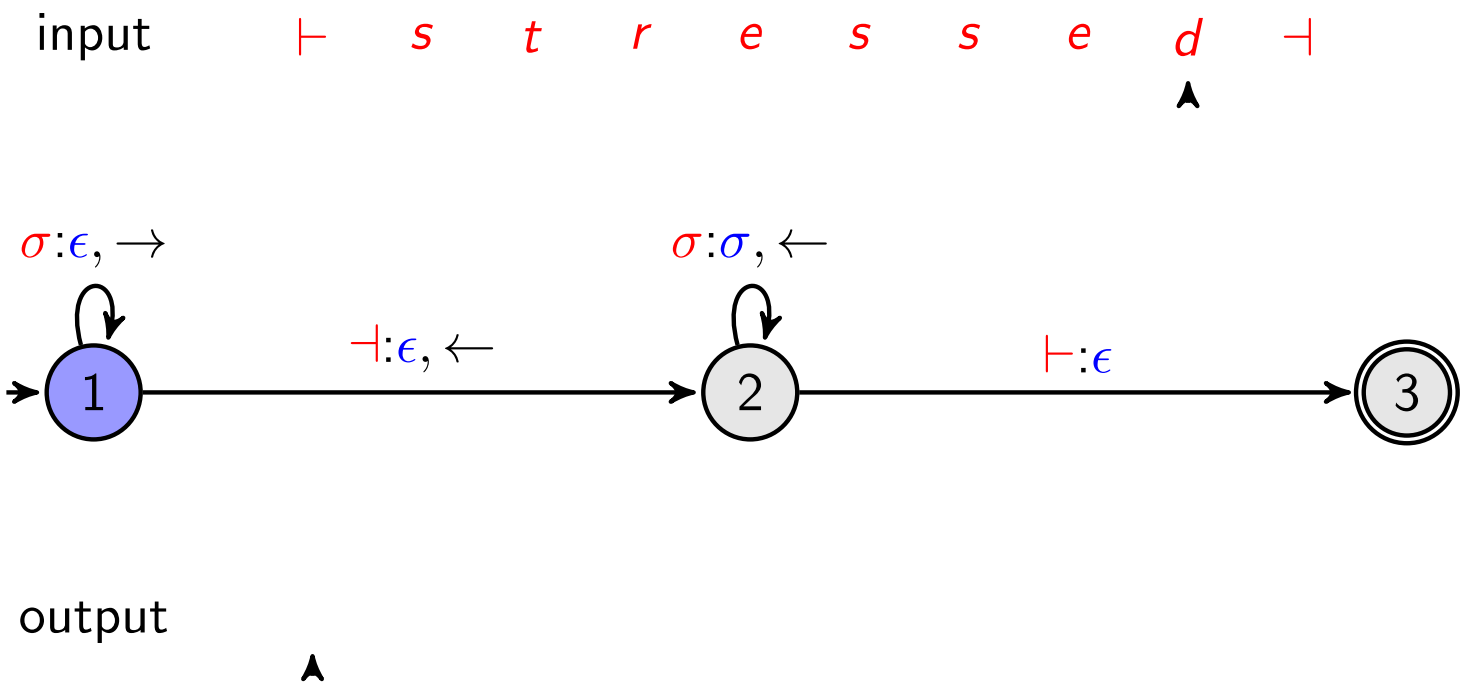

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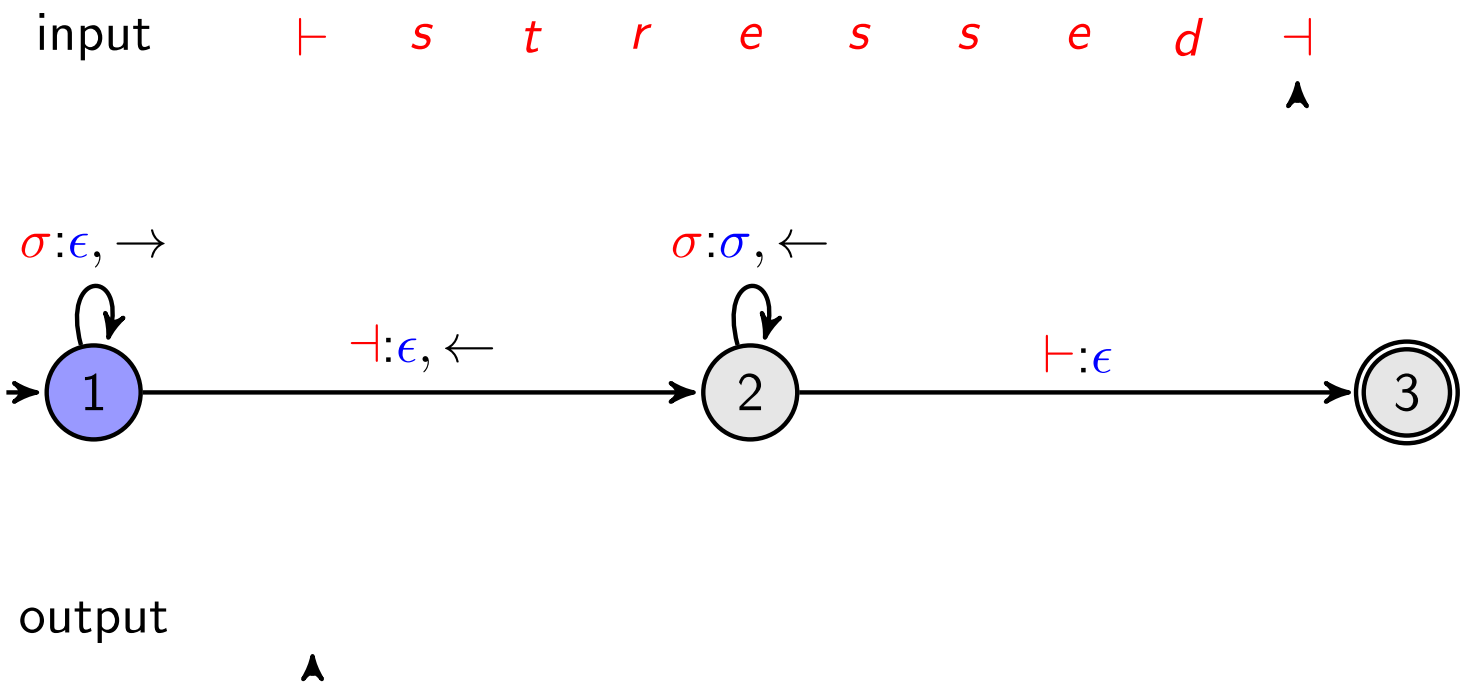

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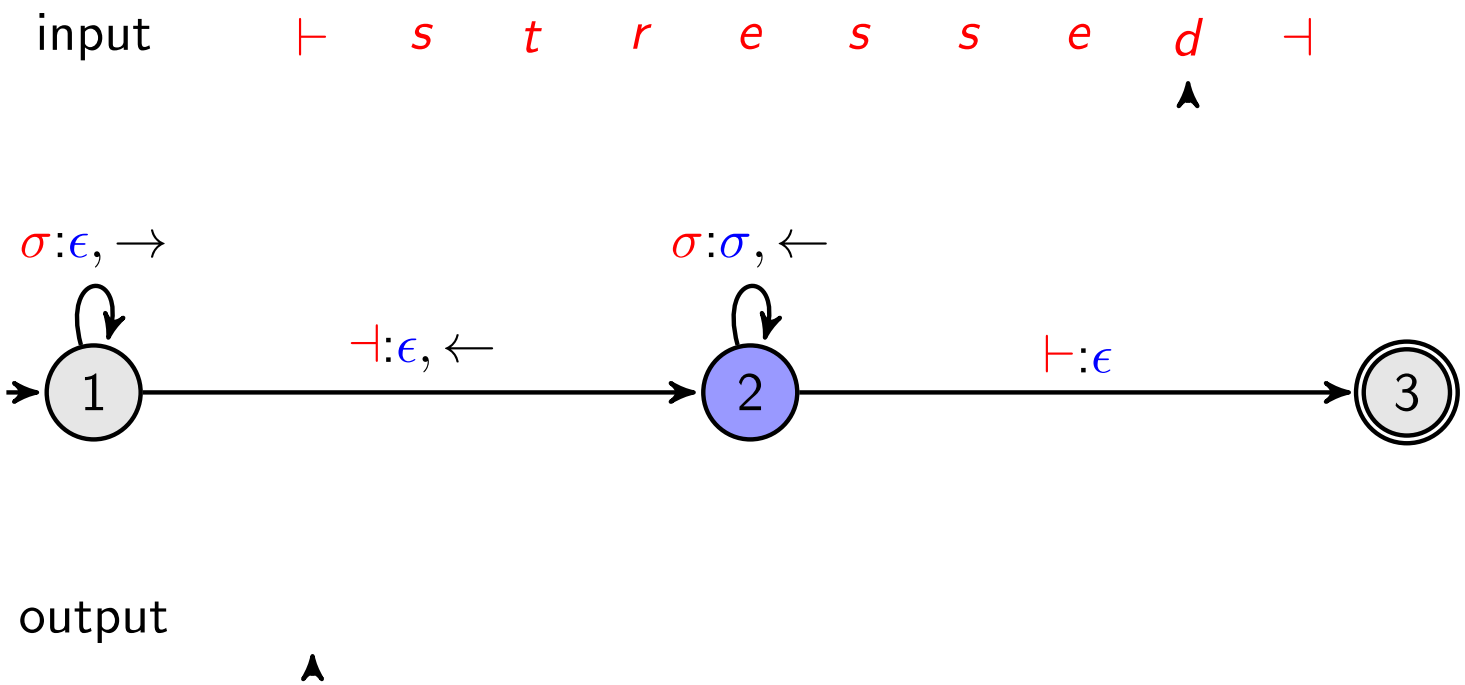

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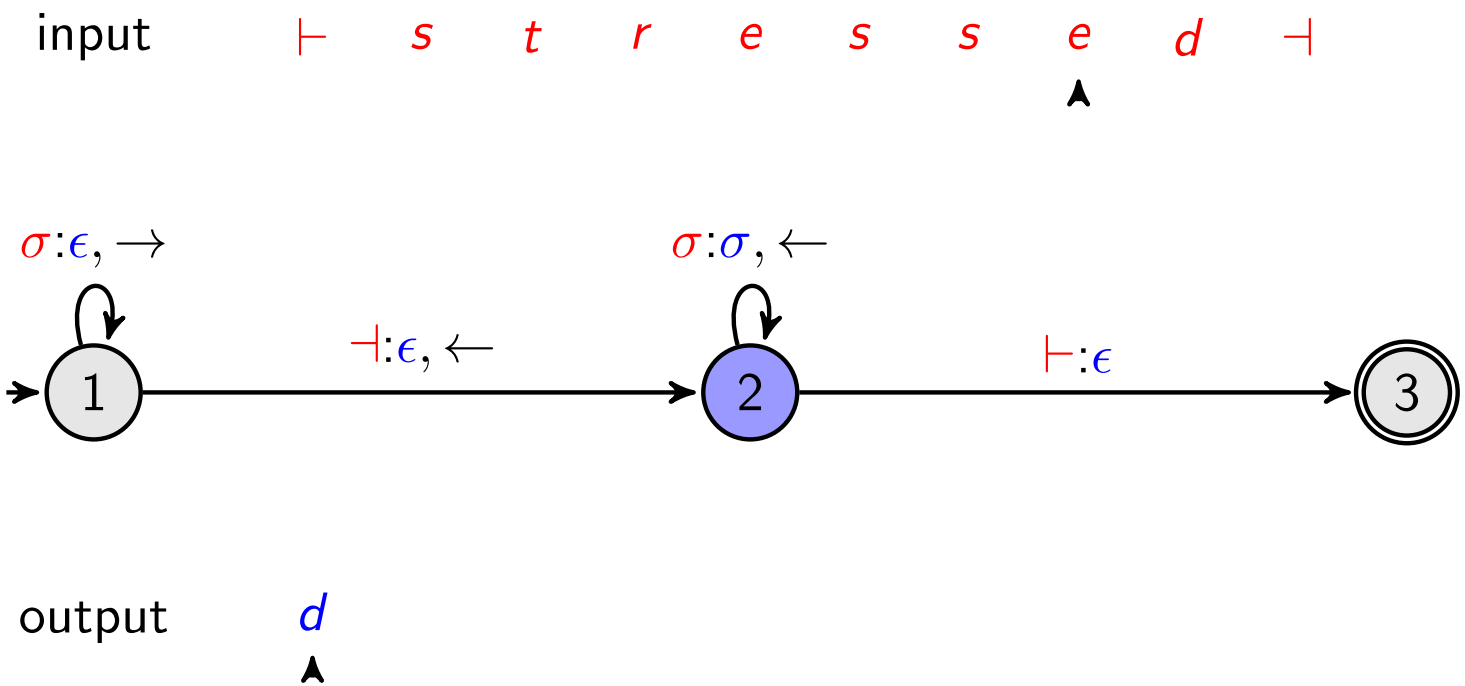
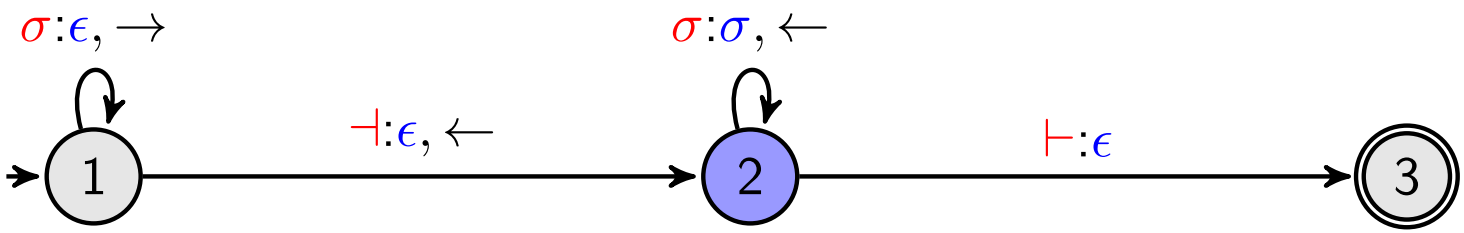


Automata for transductions: two-way transducers

input $\vdash$ s t r e s s e d $\dashv$



output d e

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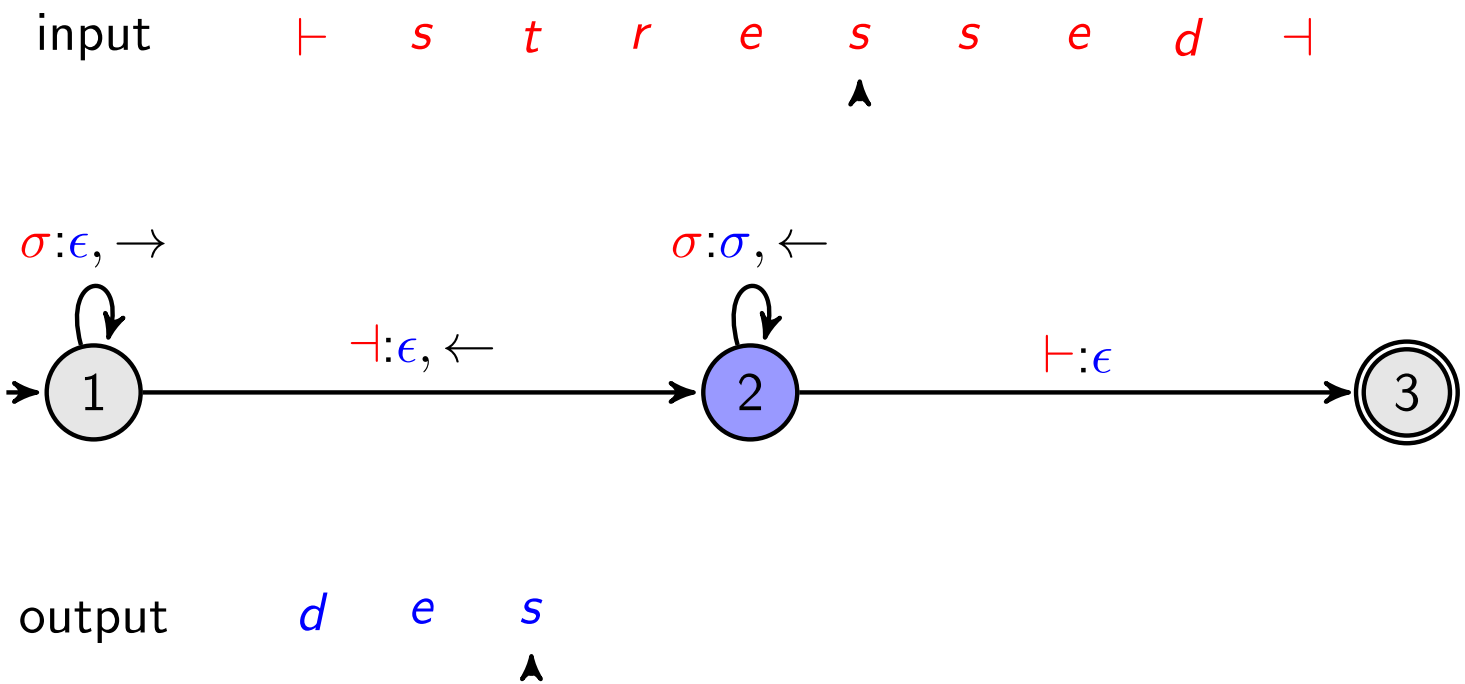

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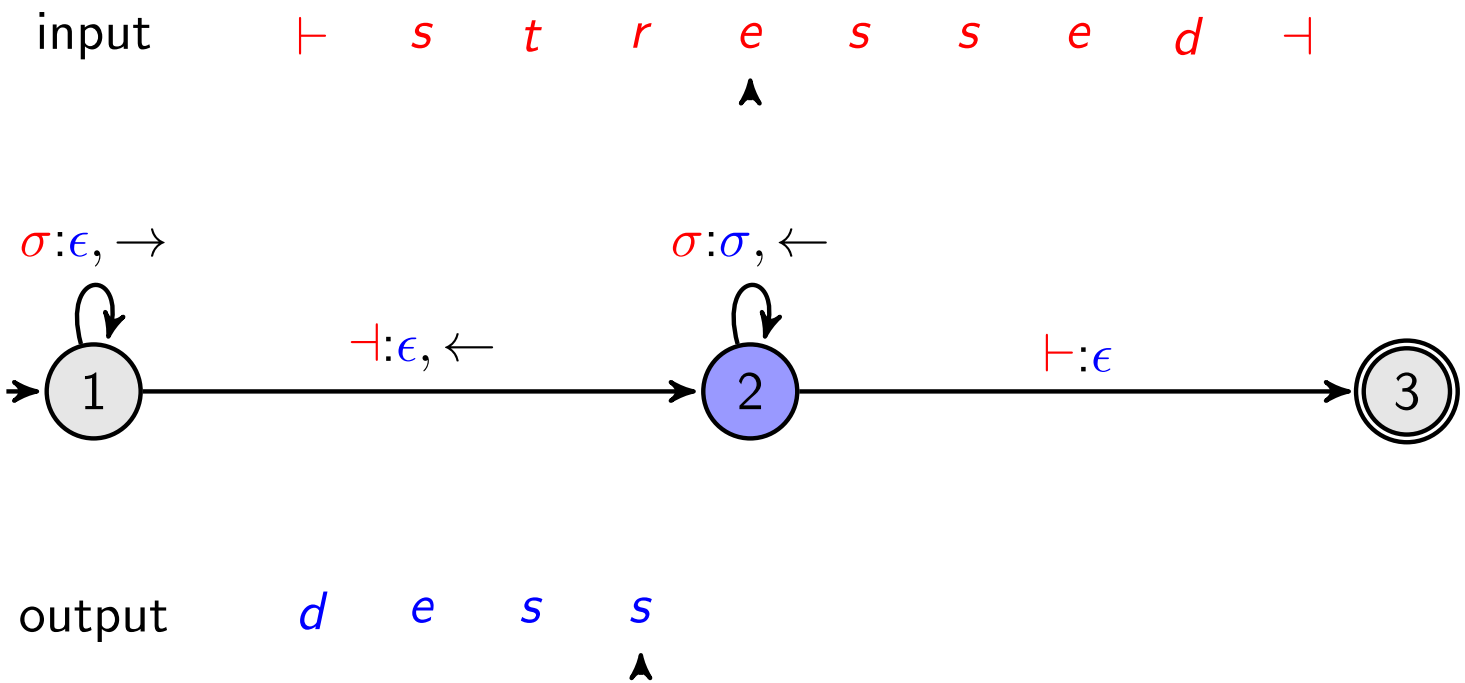

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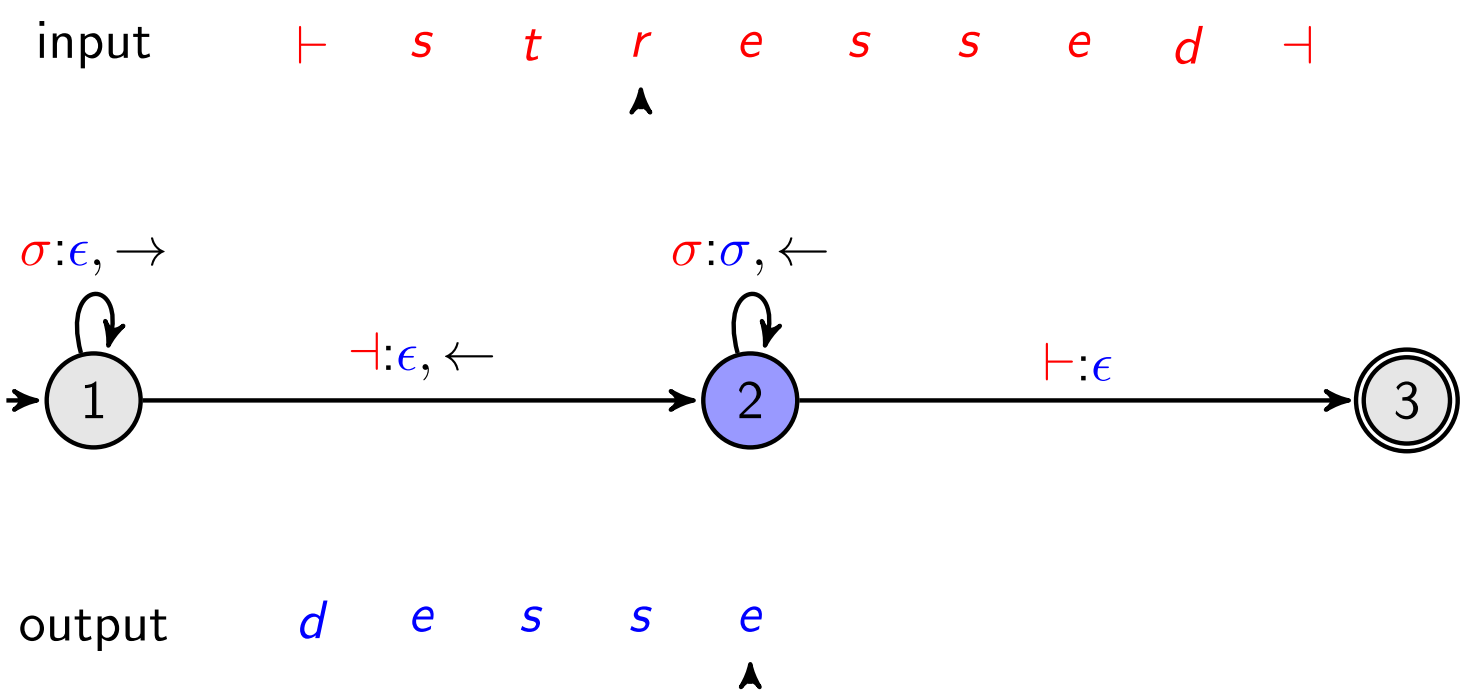

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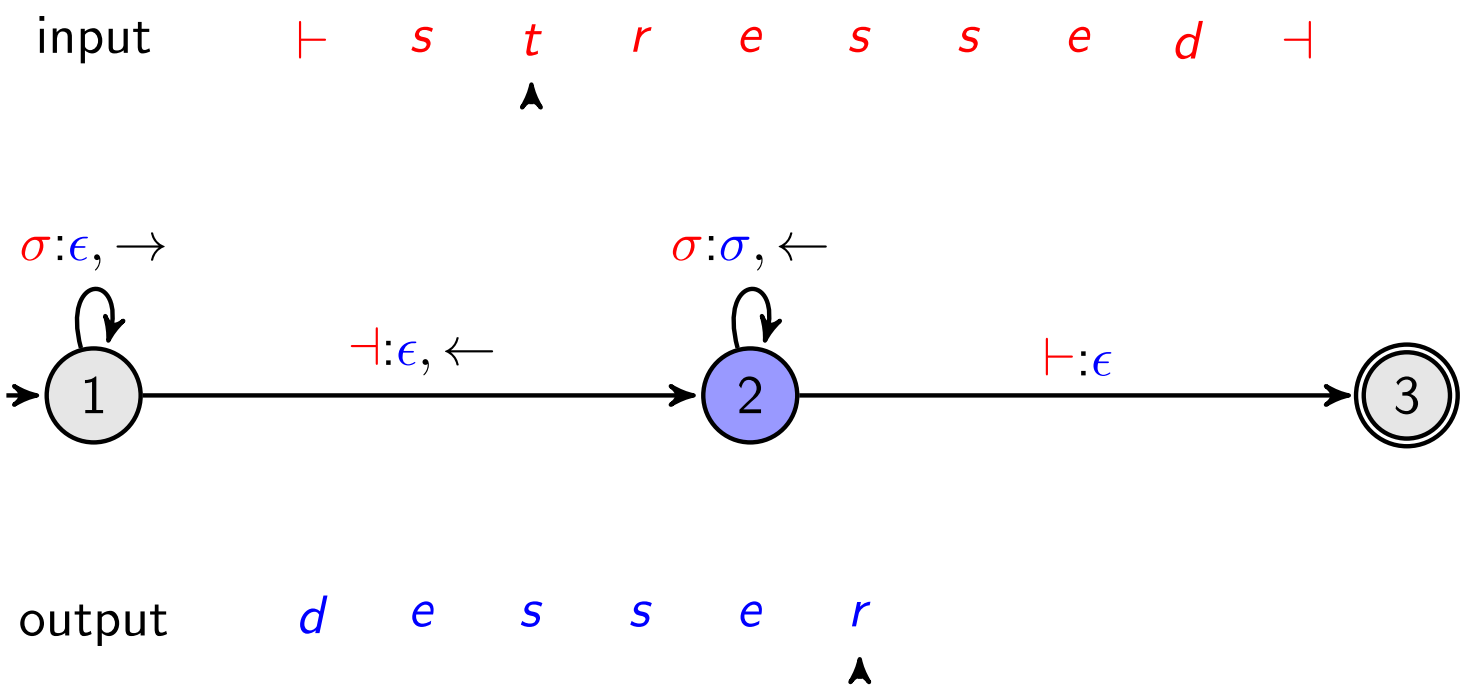

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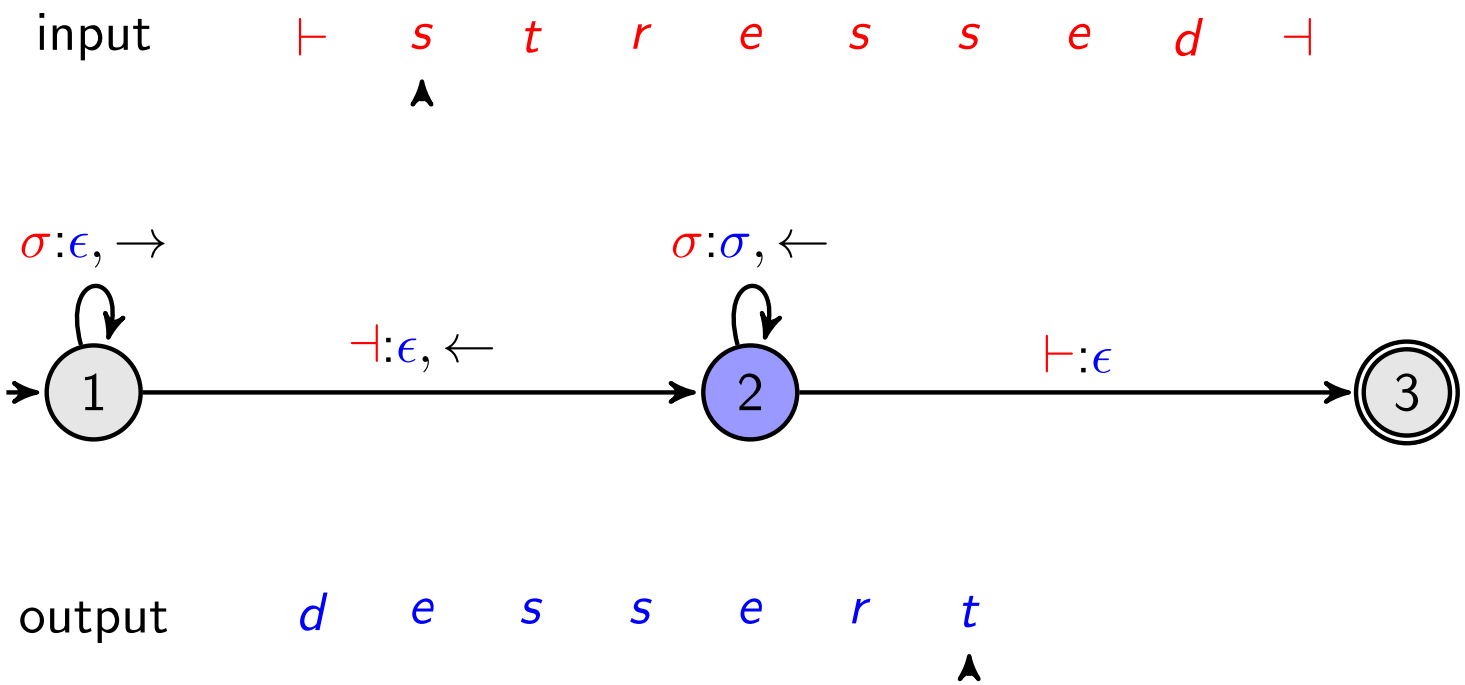

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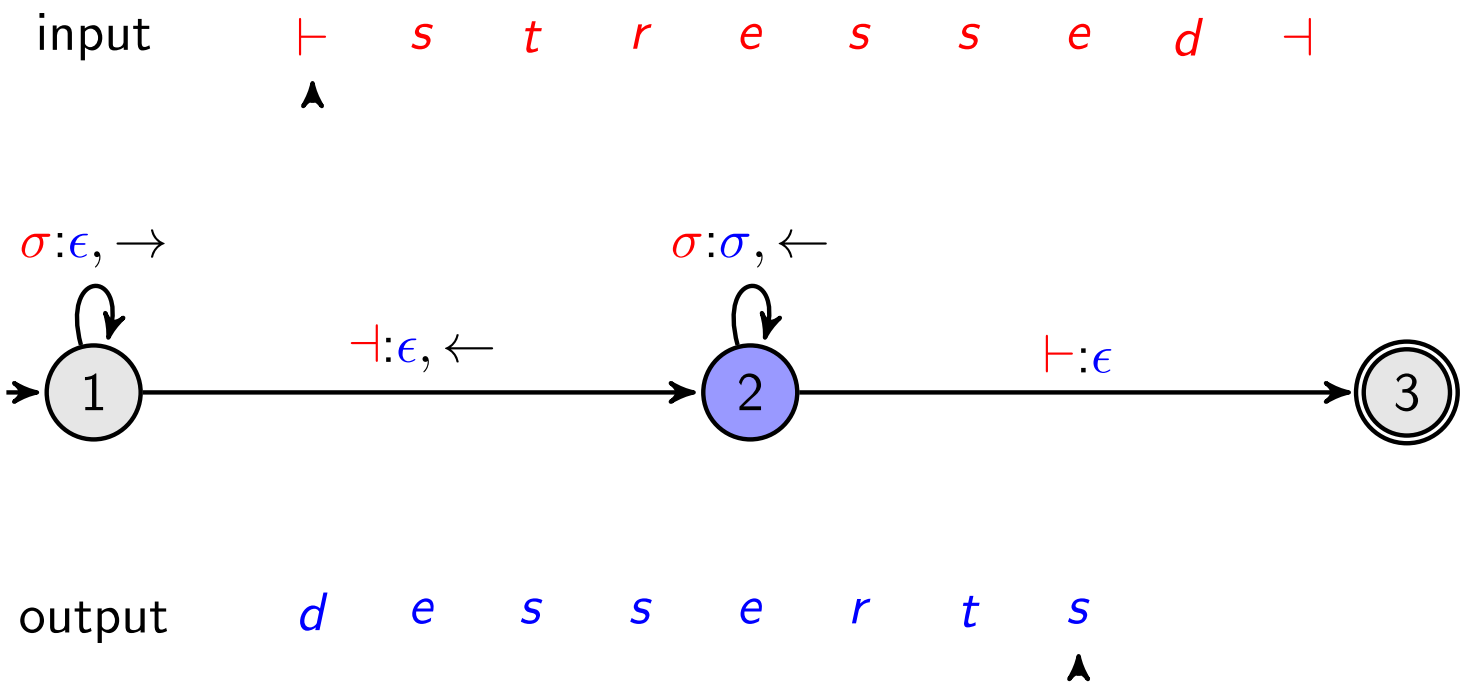

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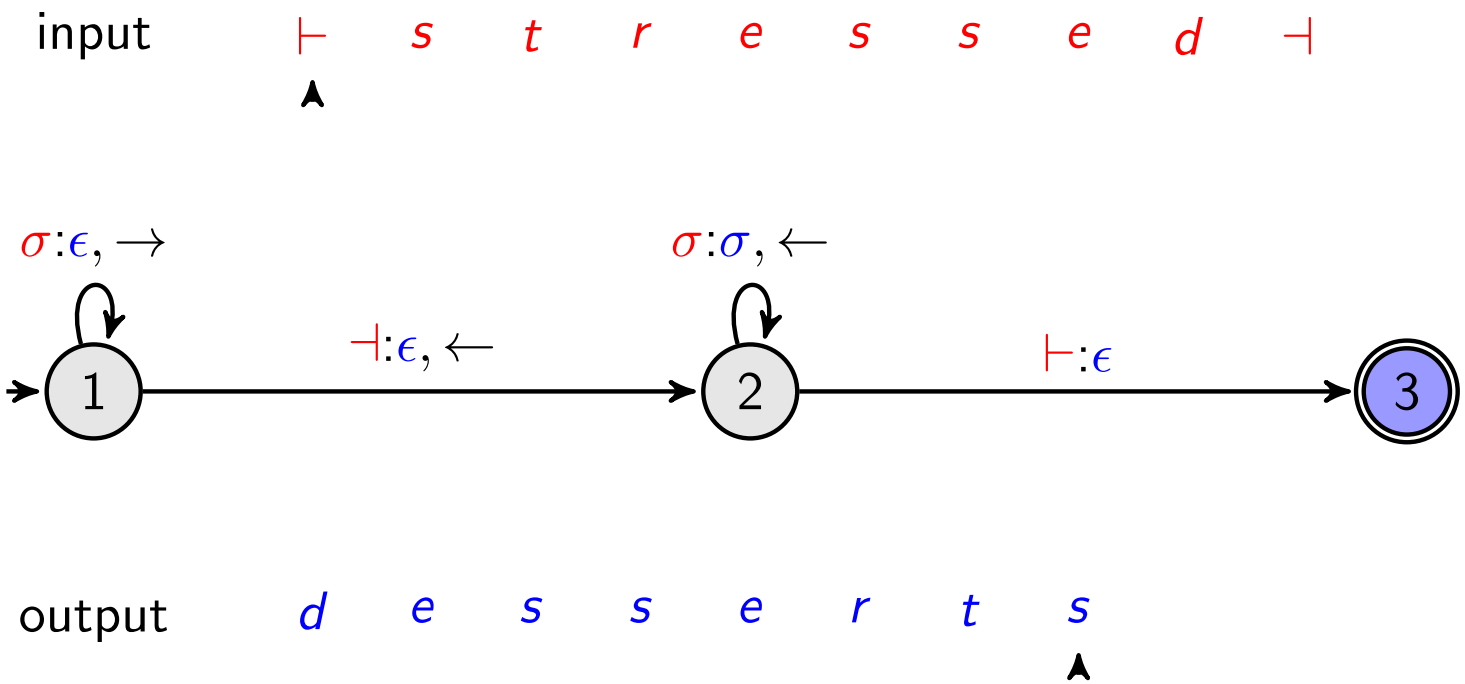

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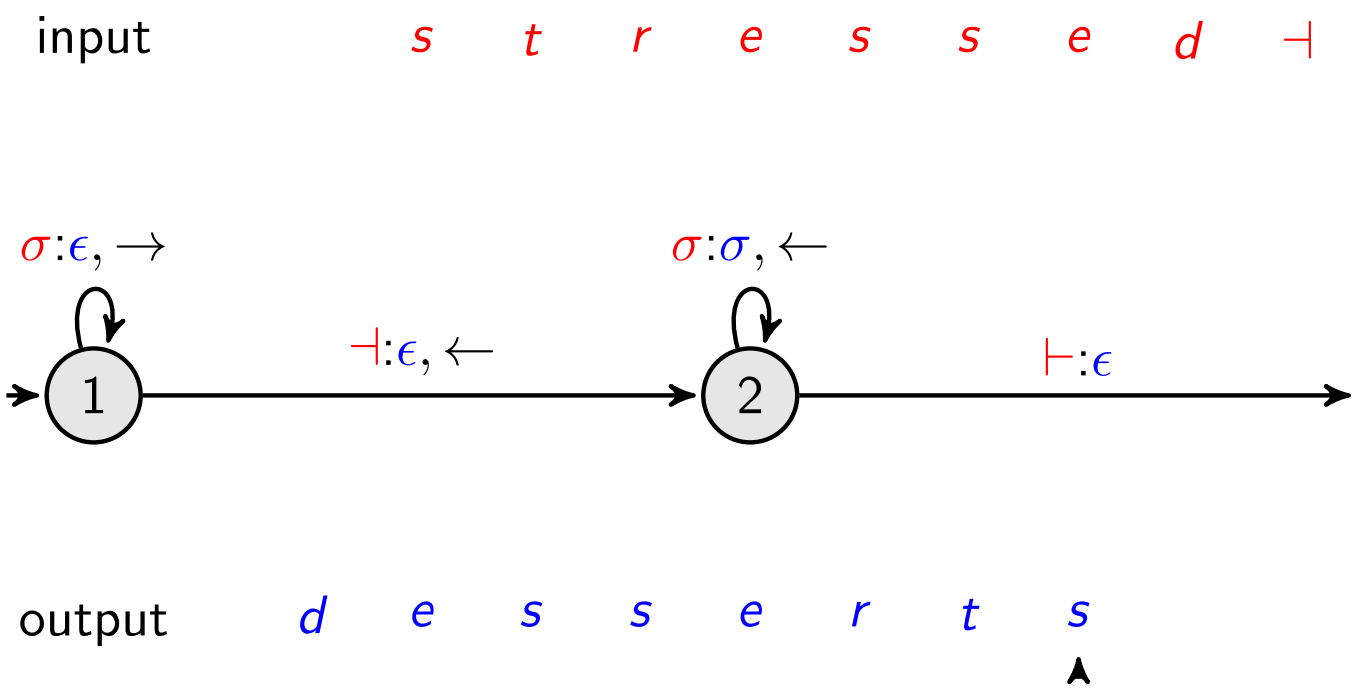
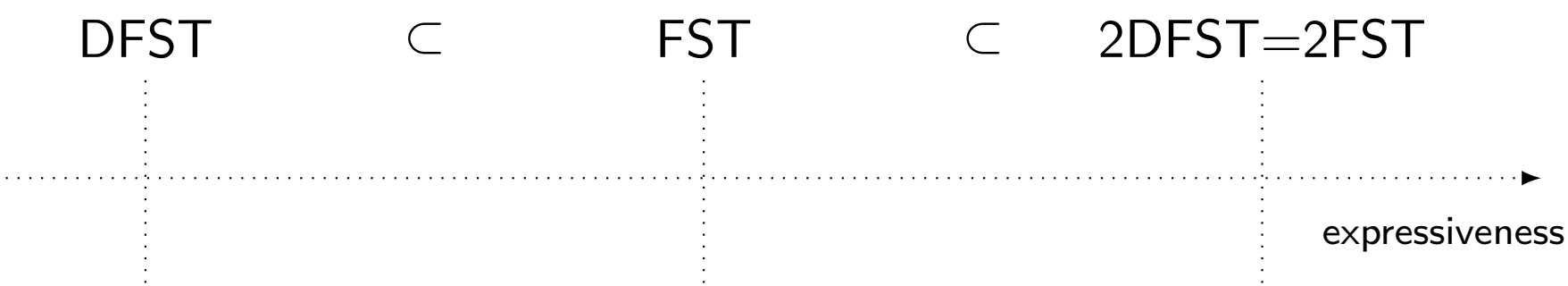


one-way $<$ two-way

- ☺ decidable equivalence problem (Culik, Karhumaki, 87).
- ☺ closed under composition $\circ$ (Chytil, Jakl, 77)

Landscape of Transducer Classes



sequential transductions

rational transductions

regular transductions

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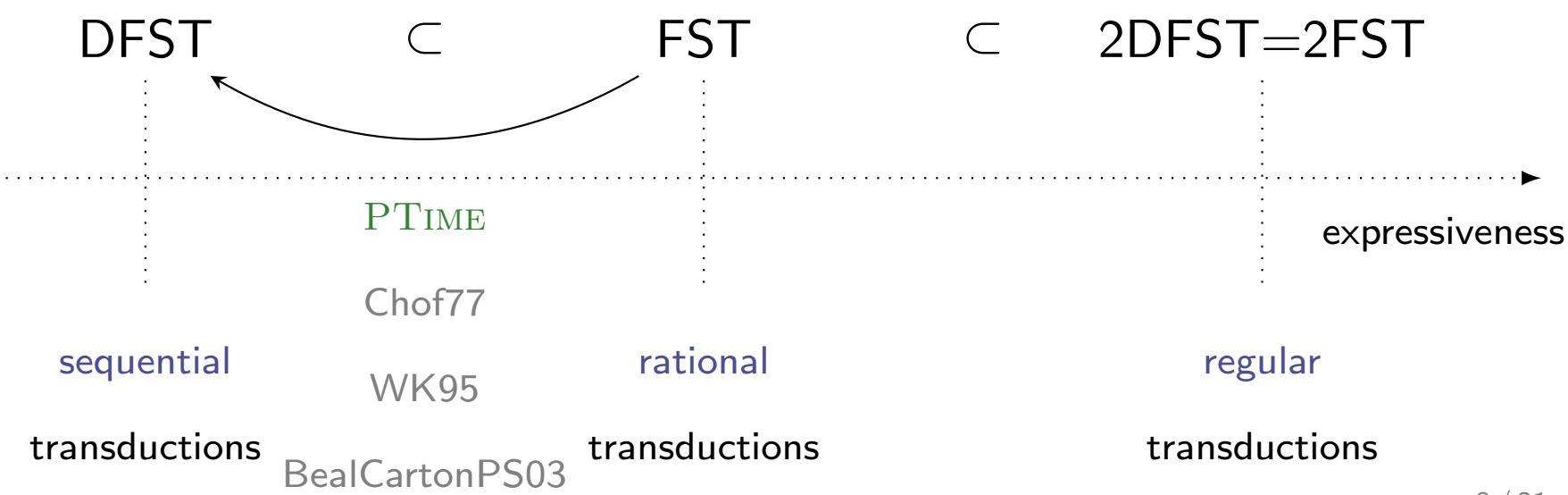

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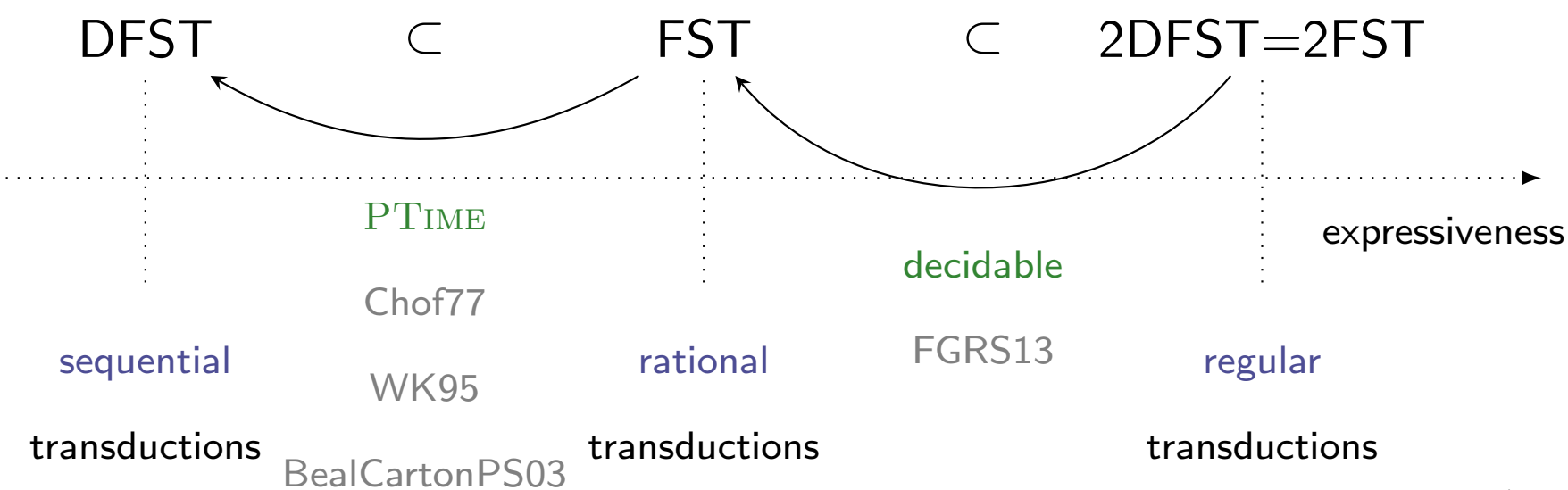

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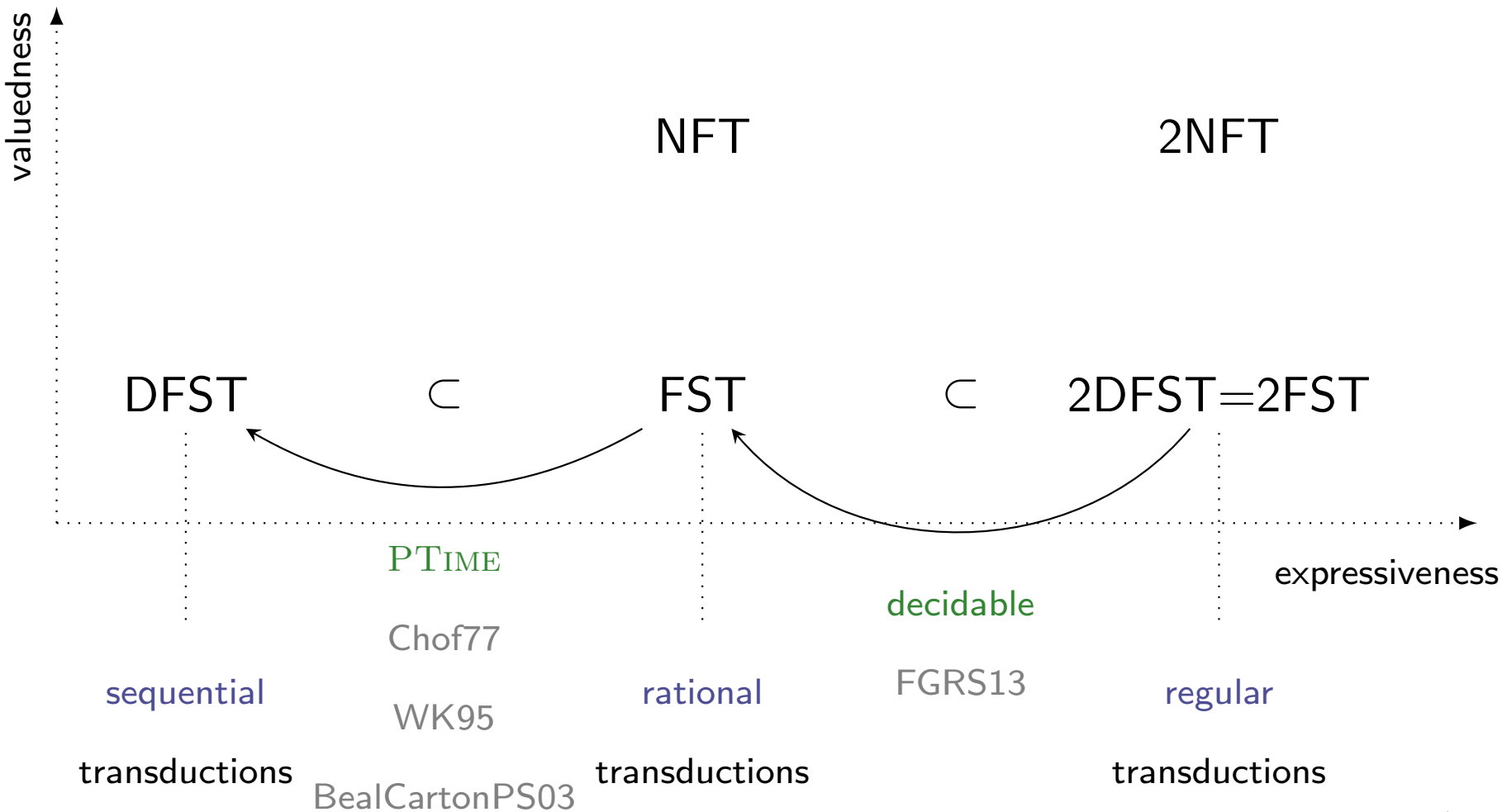

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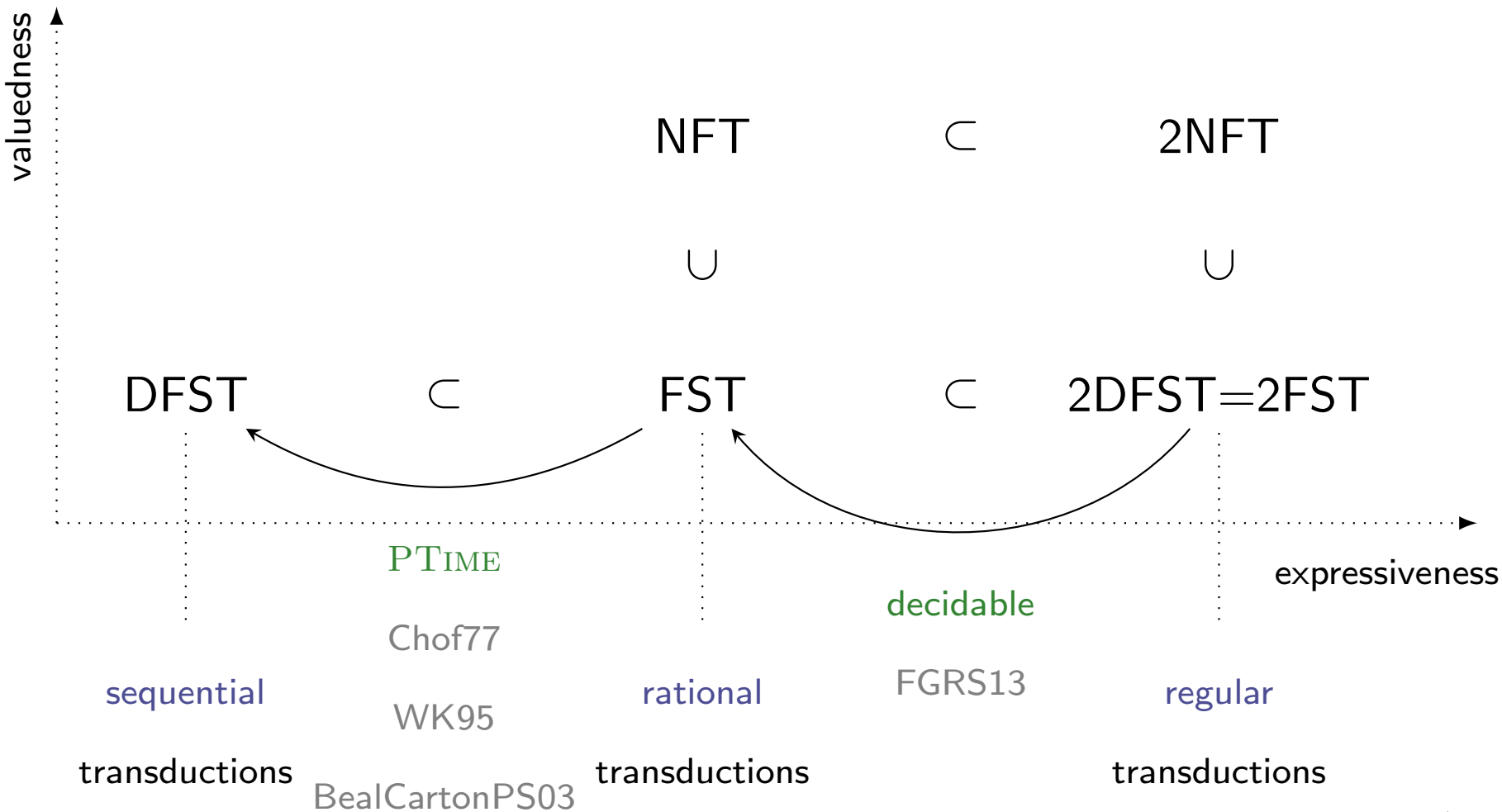

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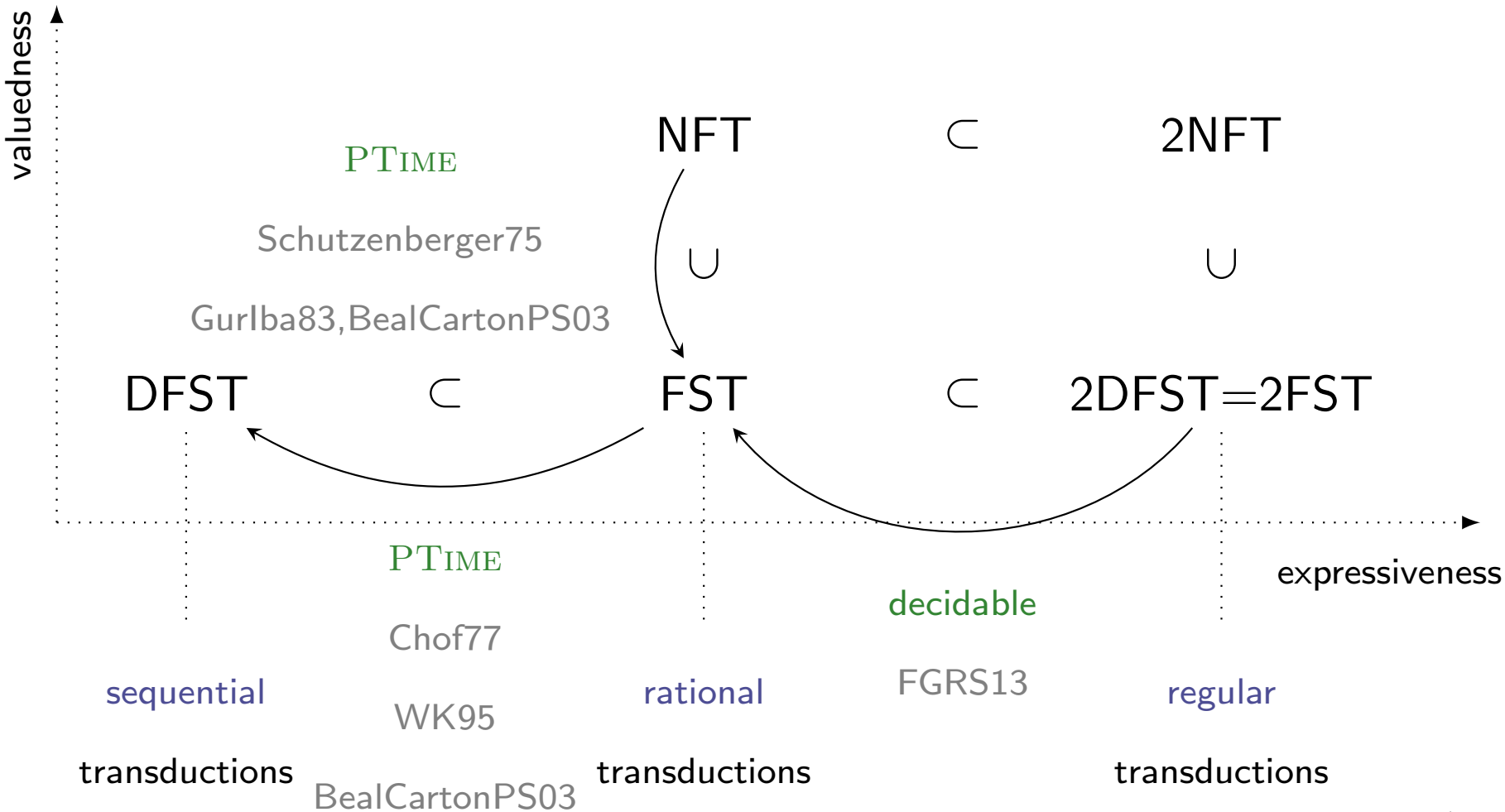

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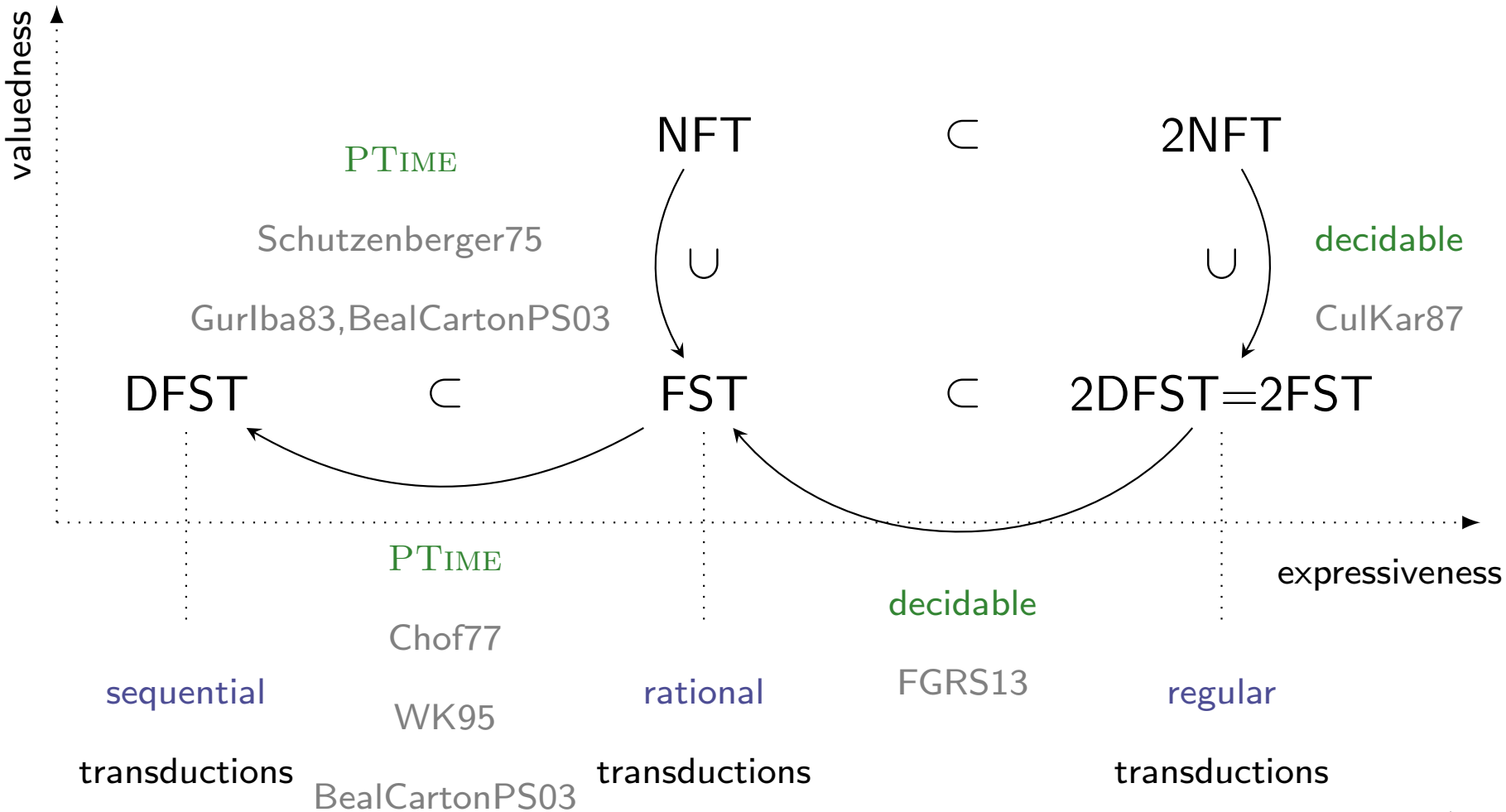

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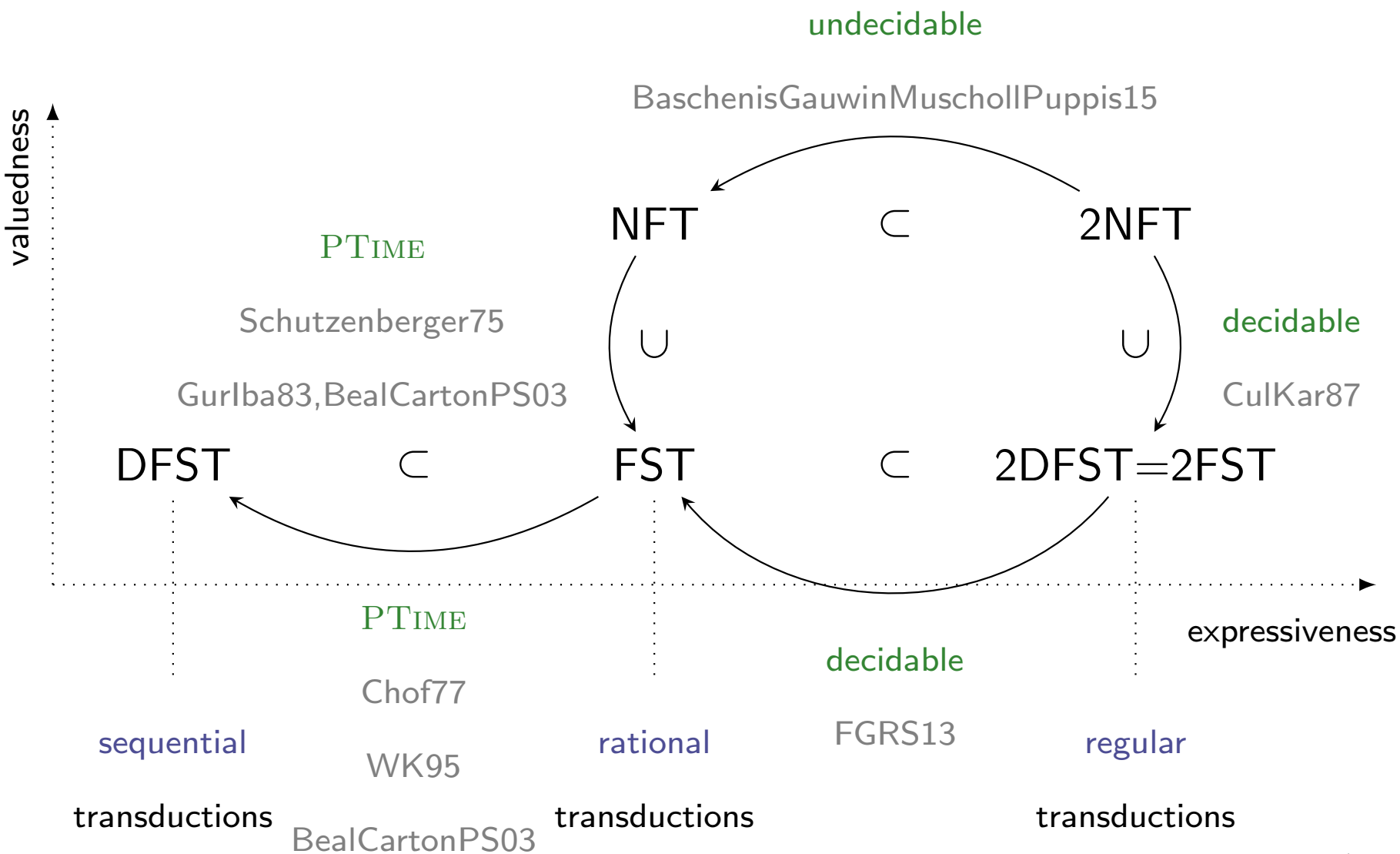

(Courcelle) MSO Transformations

"interpreting the output structure in the input structure"

- output predicates defined by MSO[S] formulas interpreted over the input structure

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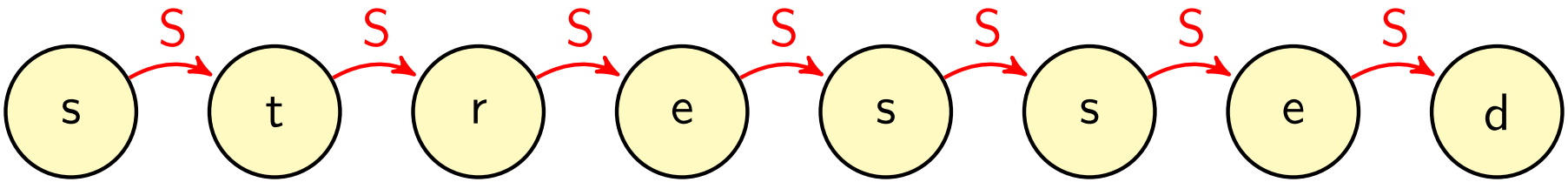

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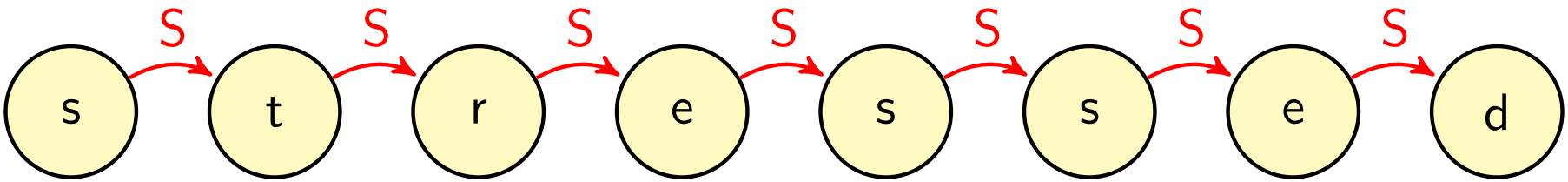

$$\phi_S(x,y) \equiv S(y,x)$$

$$\phi_\sigma(x) \equiv \sigma(x)$$

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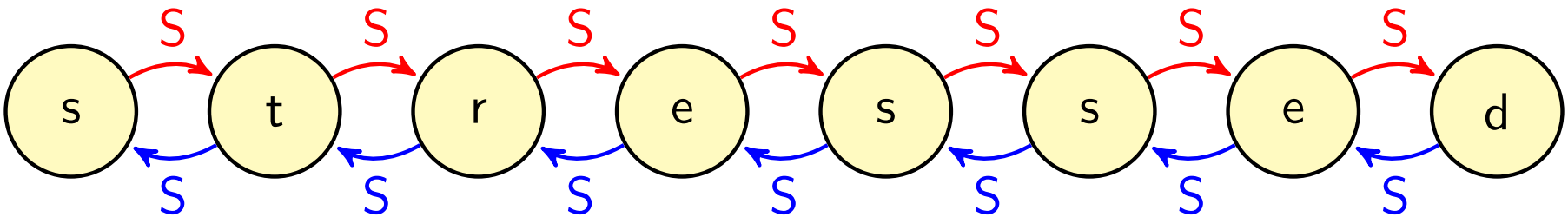

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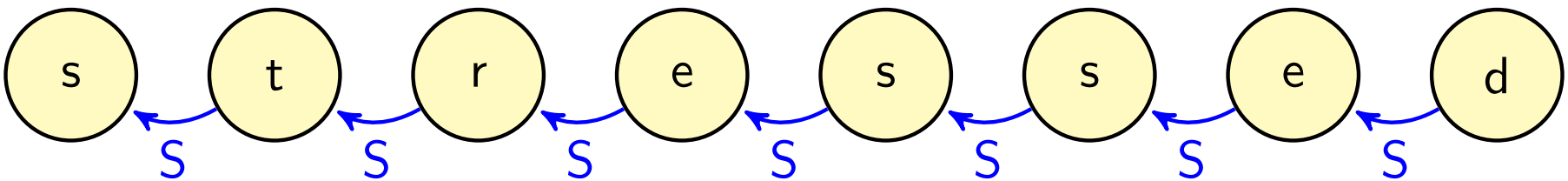

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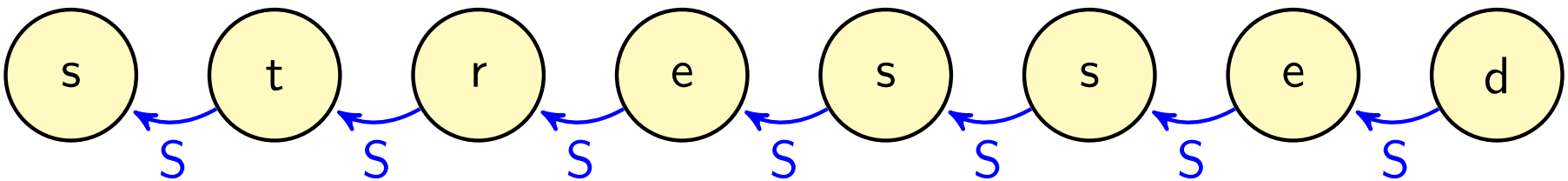

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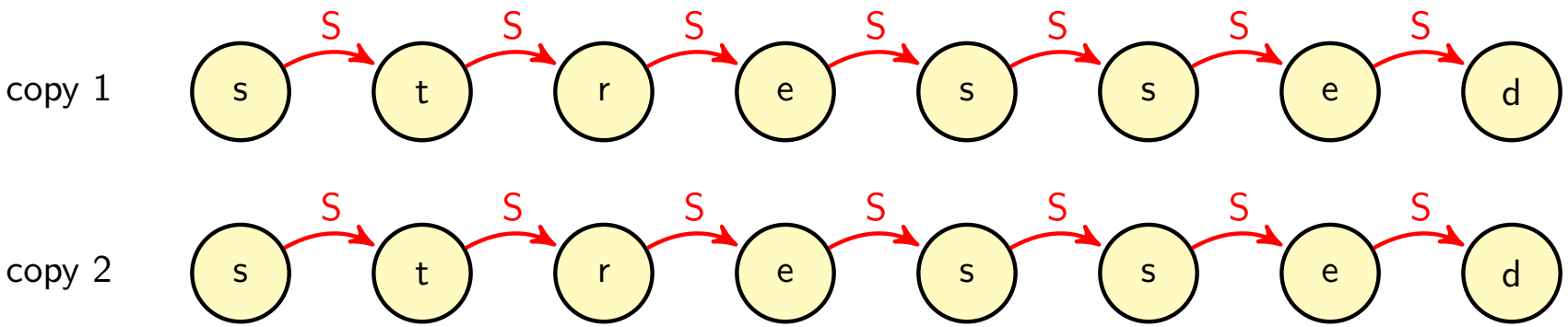

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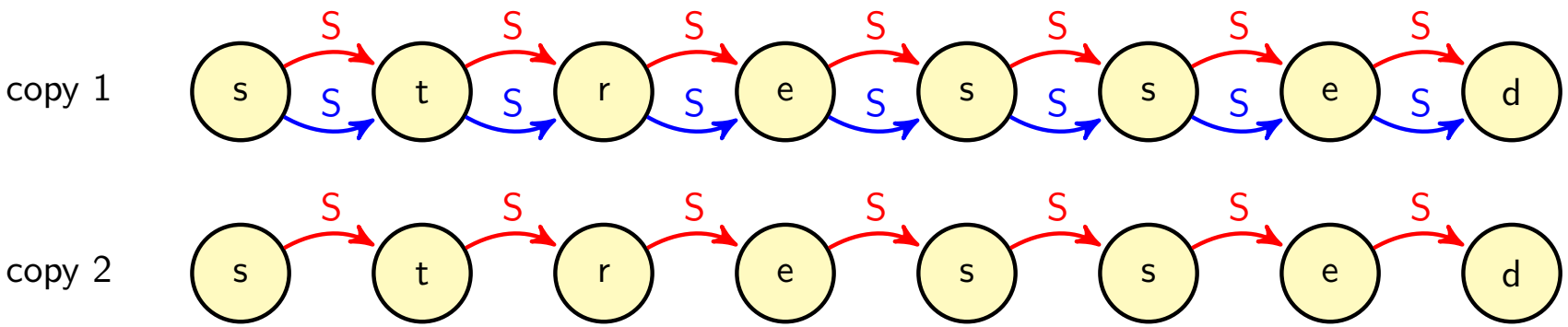

Formulas

copy 1: $\phi^1_S(x,y) \equiv S(x,y)$

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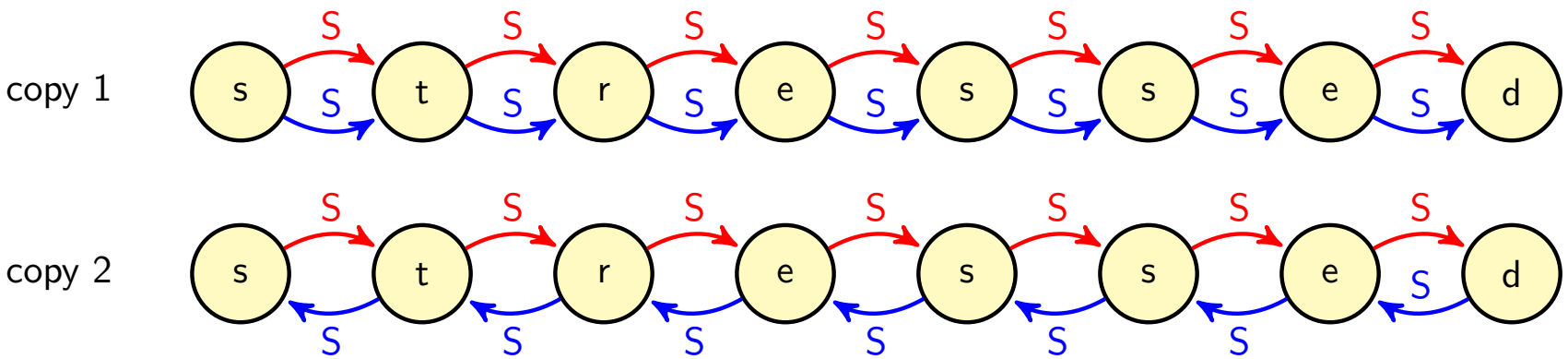

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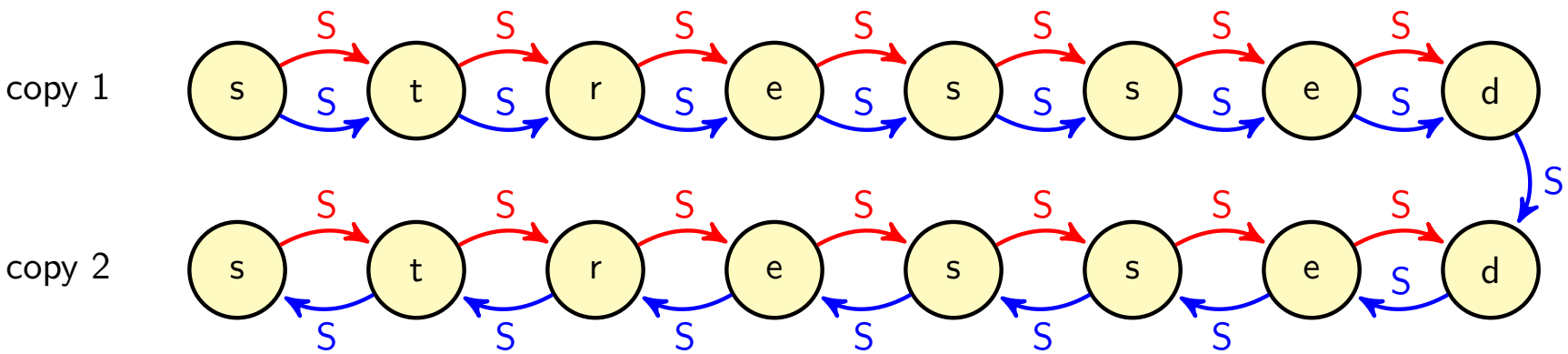

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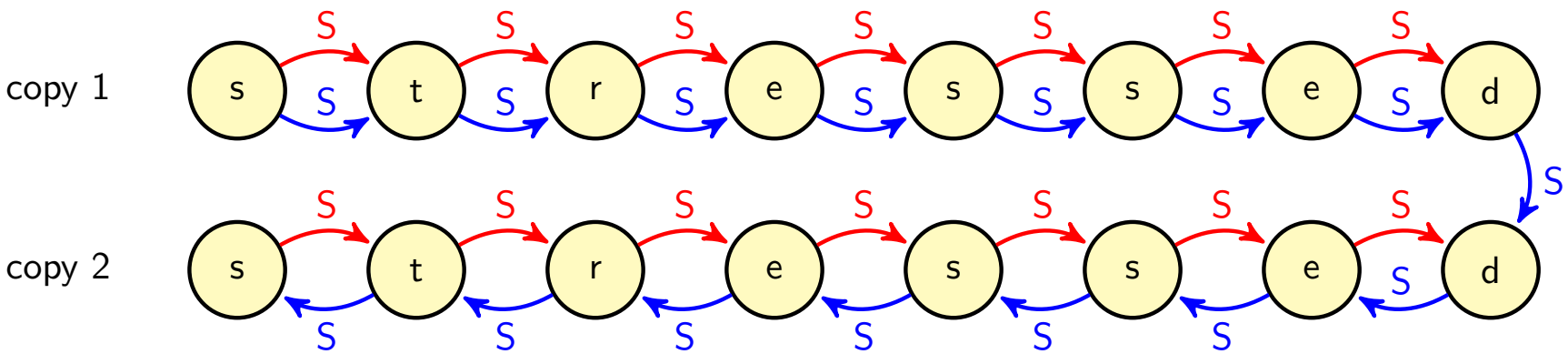

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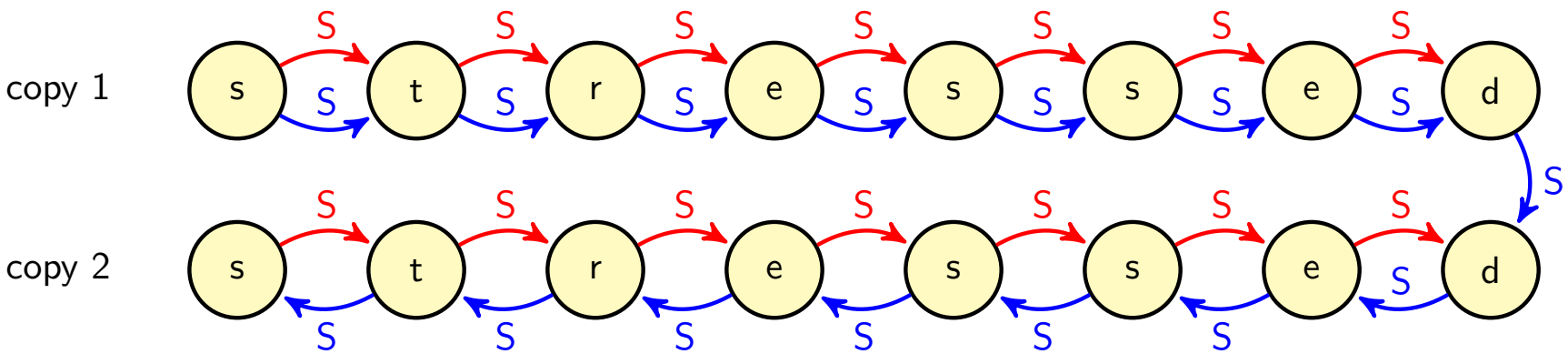

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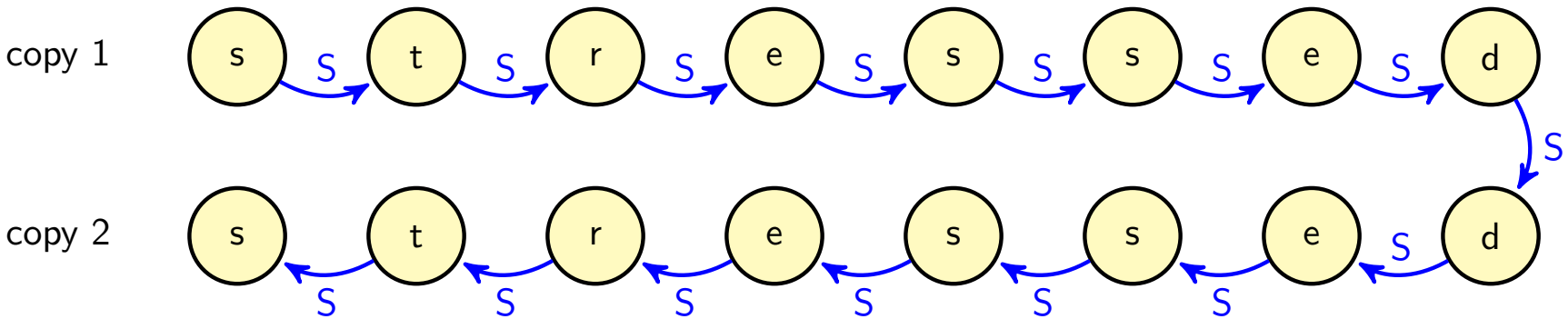

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Büchi Theorems for Word Transductions

Theorem (Engelfriet, Hoogeboom, 01)

Let $f : \Sigma^* \to \Sigma^*$. The following are equivalent:

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Theorem (Bojanczyk 14, F. 15)

Let $f : \Sigma^* \to \Sigma^*$. The following are equivalent:

1. *f is definable by a (one-way) transducer*
2. *f is definable in order-preserving MSO*

Order-preserving MSO: $\phi_S^{i,j}(x, y) \models x \preceq y$.

Myhill-Nerode congruence for $L \subseteq \Sigma^*$

- $u \sim_L v$ if: for all $w \in \Sigma^*$, $uw \in L$ iff $vw \in L$
- u and v have the same "effect" on continuations w
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Goal

Extend Myhill-Nerode's theorem to classes of transductions

Sequential transductions (Choffrut)

Refinement of the MN congruence.

Two ideas

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Theorem (Choffrut)

f is sequential iff $\sim_f$ has finite index

$\sim_f$ is a right congruence $\rightsquigarrow$ canonical and minimal transducer !

$$\text{Transitions: } [u] \xrightarrow{\sigma | F(u)^{-1} F(u\sigma)} [u\sigma]$$

Rational transductions are almost sequential

- $f_{sw} : u\sigma \mapsto \sigma u$ is not sequential
- but sequential modulo *look-ahead information* $\mathcal{I} = \{a, b, \epsilon\}$.

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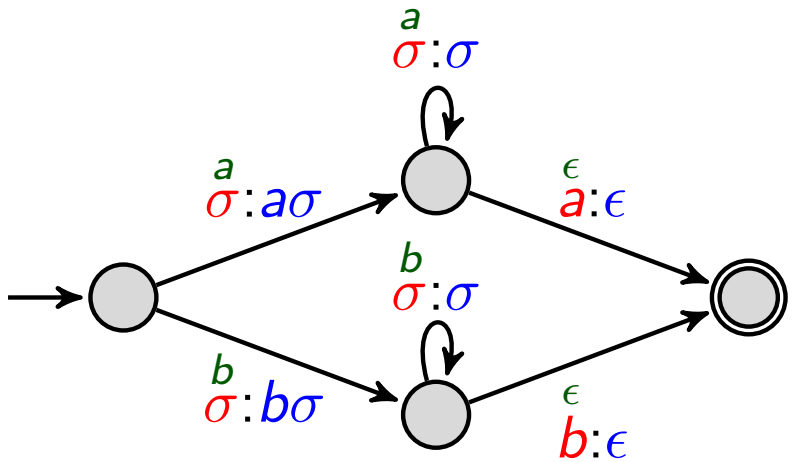

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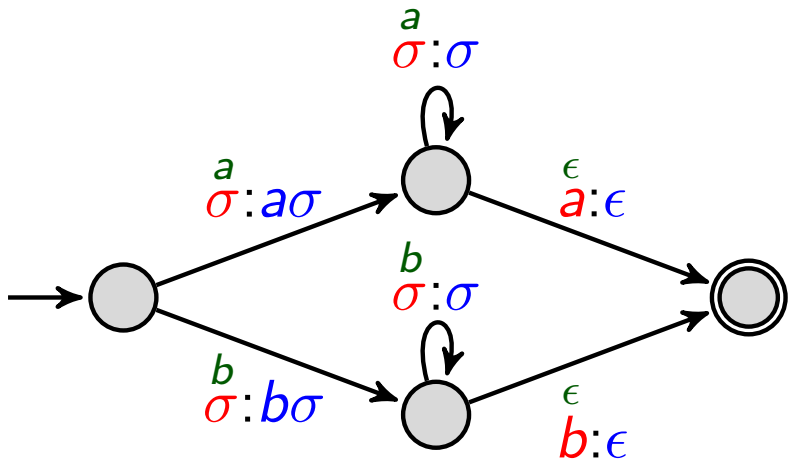

- look-ahead information: $\mathcal{L} : \Sigma^* \to \mathcal{I}$
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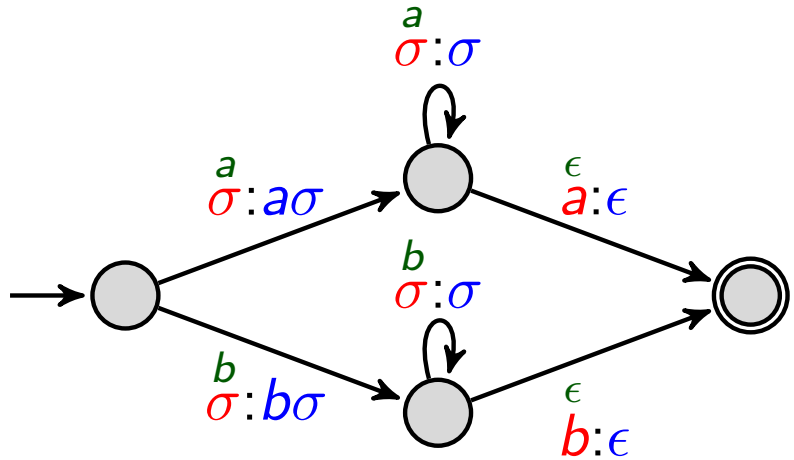

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Theorem (Elgot, Mezei, 65)

f is rational iff $f[\mathcal{L}]$ is sequential, for some finite look-ahead information $\mathcal{L}$ computable by a right sequential transducer.

Original statement: $RAT = SEQ \circ RightSEQ$.

A canonical look-ahead

Identify suffixes with a 'bounded' effect on prefixes.

Equivalent suffixes

- $u \equiv_f v$ if $\{d(f(wu), f(wv)) \mid wu, wv \in dom(f)\}$ is finite[a].

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[a] $d(\alpha, \beta) = |\alpha| + |\beta| - 2|lcp(\alpha, \beta)|$.

Ex: For f_{sw}, $\Sigma^*/_{\equiv} = \{\{ua \mid u \in \Sigma^*\}, \{ub \mid u \in \Sigma^*\}, \{\epsilon\}\}$.

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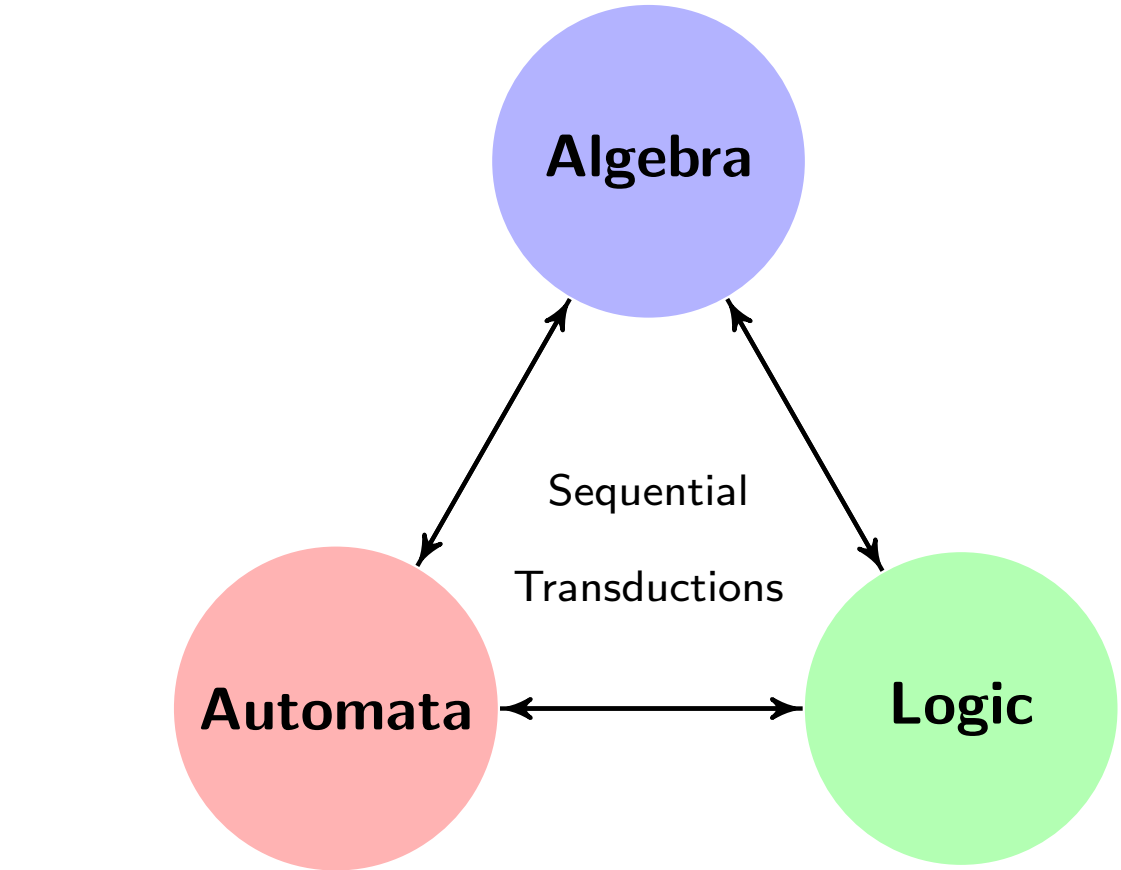
Theorem (Reutenauer, Schützenberger, 91)

The following are equivalent:

- *f is rational*
- *$\equiv_f$ has finite index and $f[\equiv_f]$ is sequential*
- *$\equiv_f$ and $\sim_{f[\equiv_f]}$ have finite index.*

Summary: sequential transductions

finiteness of $\sim_f$ (Choffrut)



input-deterministic
one-way transducers

prefix-independent MSO, ?

Summary: rational transductions

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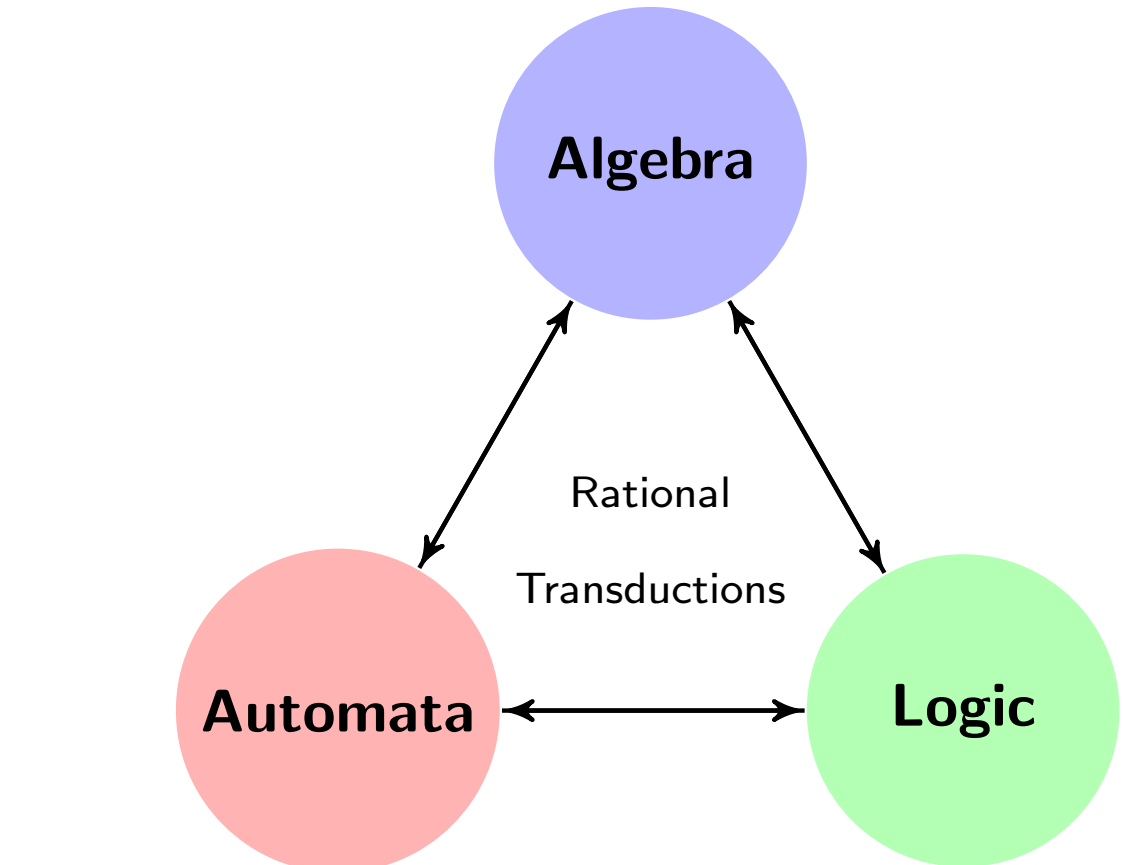

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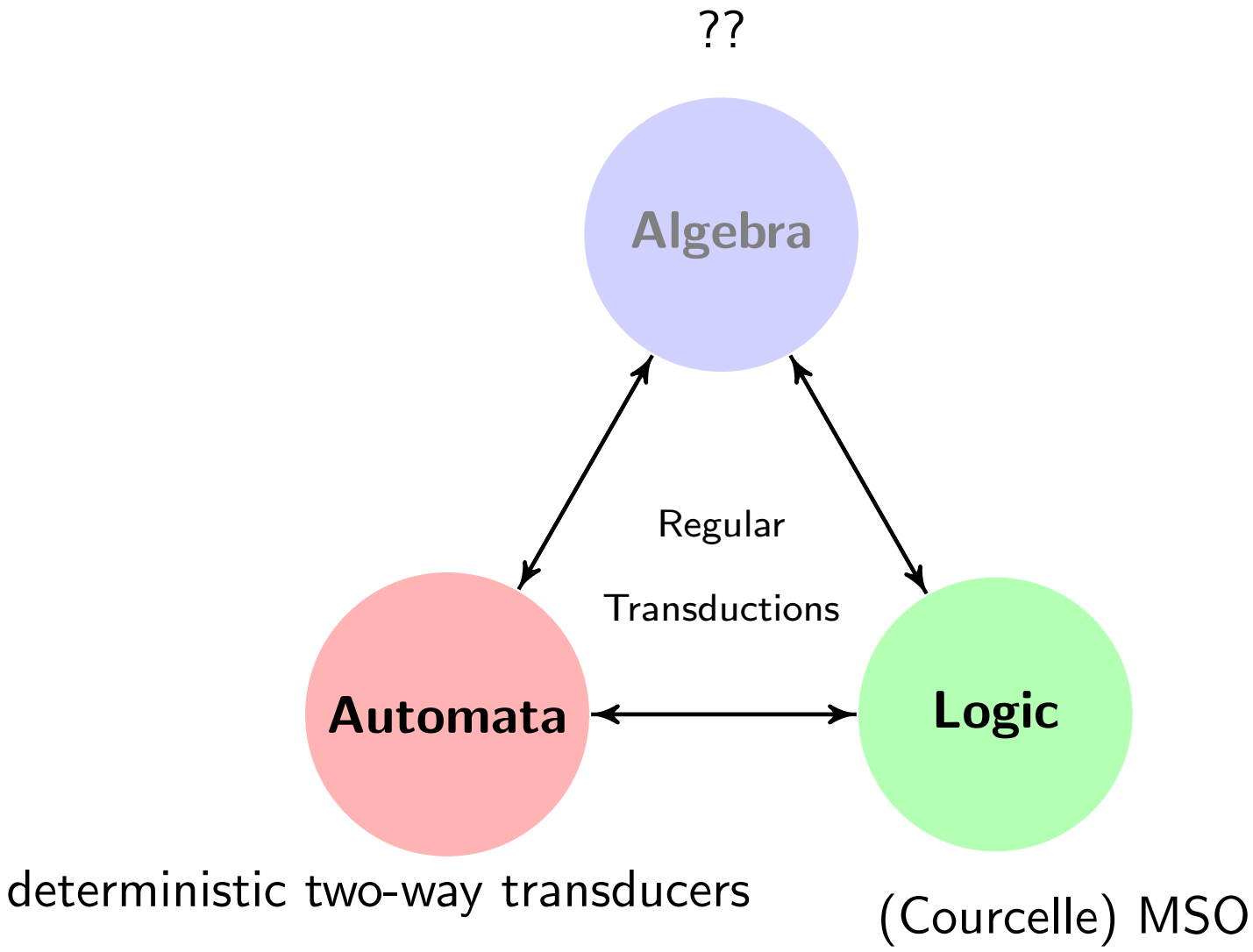
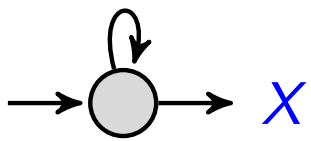


Transducers with Registers FST_{re} (Alur, Cerny, 10)

- deterministic one-way
- register updates:
  $X := YZ$, $X := u$

$\sigma \mid X := \sigma X$



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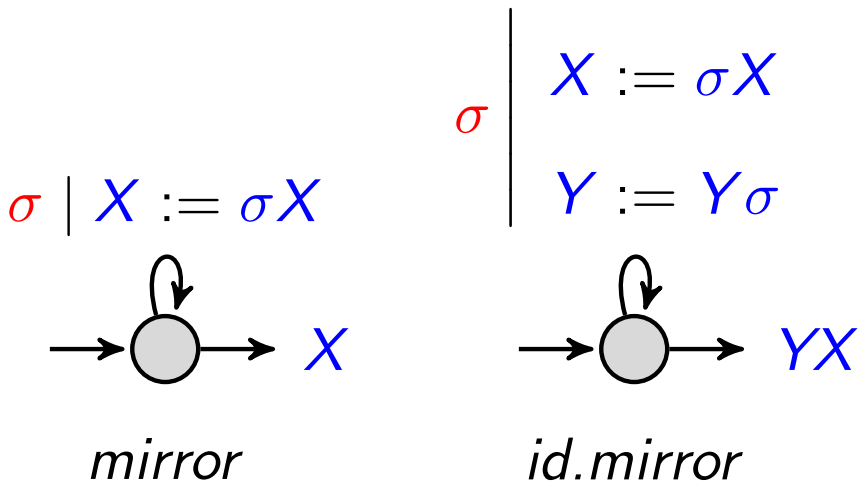

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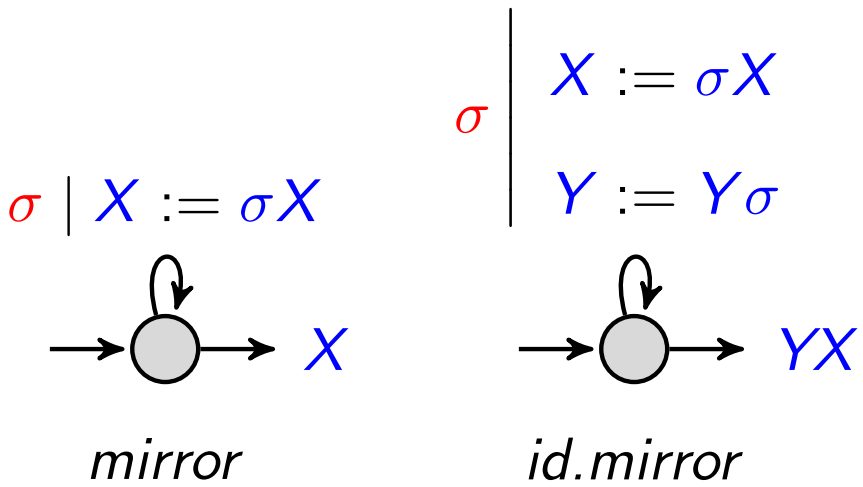

Results

- MSO-equivalent if copyless updates (Alur, Cerny, 10)
- decidable equivalence problem (F., Reynier) $\sim$ HDT0L

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Results

- machine-independent characterization of regular transductions with origin and first-order definability (Bojanczyk, 14)
- relaxation of origin semantics & decision problems (F., Jecker, Löding, Winter, 16)

Other works

- regular combinators (Alur, Freilich, Raghothaman, 14)
- variants of two-way transducers (Guillon, Choffrut, 14,15,16), (Carton, 12) (McKenzie, Schwentick, Thérien, Vollmer, 06)
- extensions with non-determinism

SIGLOG News 9th

Transducers, Logic and Algebra for Functions of Finite Words

Emmanuel Filiot, Université Libre de Bruxelles
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