Rational Verification
in
Iterated Electric Boolean Games

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r:

	q	$\neg q$
p	$0, 2, 1$	$0, 0, 1$
$\neg p$	$-1, 2, 1$	$-1, 0, 1$

$\neg r$:

	q	$\neg q$
p	$0, 2, -1$	$0, 0, -1$
$\neg p$	$-1, 2, -1$	$-1, 0, -1$

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$P1 : \{p, q\}$

$P2 : \{r\}$

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Initial endowment : $e_0 = (1, 1)$

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$\neg r$

$P1 : \{p, q\}, \Phi_1 \equiv \mathsf{F}\ (p \wedge q)$

$P2 : \{r\}, \Phi_2 \equiv \mathsf{G}\neg r$

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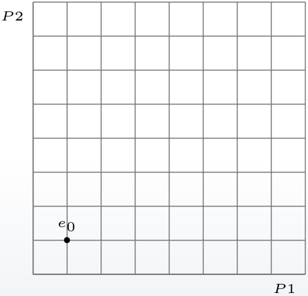

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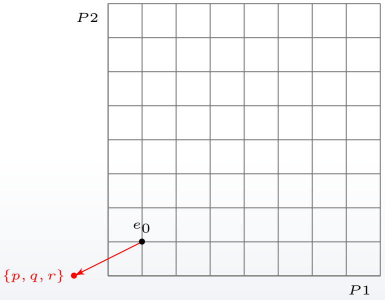

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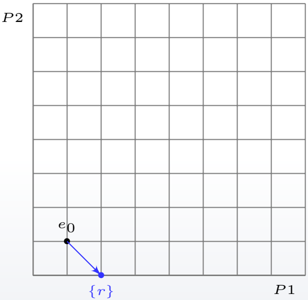

	q	$\neg q$
p	0, 2, 1	0, 0, 1
$\neg p$	−1, 2, 1	−1, 0, 1

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p	0, 2, −1	0, 0, −1
$\neg p$	−1, 2, −1	−1, 0, −1

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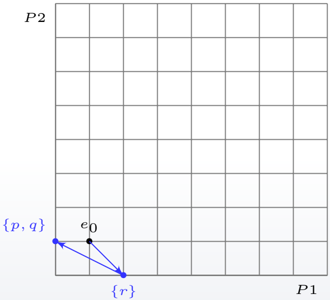

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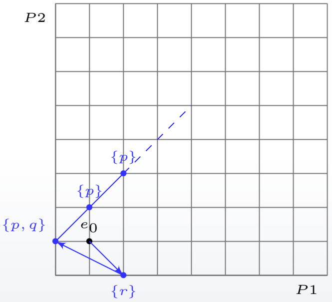

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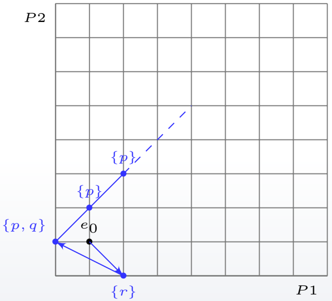
Initial endowment : $e_0 = (1, 1)$

$\mathsf{Payoff}(P1) = 1, \mathsf{Payoff}(P2) = 0$



	q	$\neg q$
p	$0,2,1$	$0,0,1$
$\neg p$	$-1,2,1$	$-1,0,1$

r

	q	$\neg q$
p	$0,2,-1$	$0,0,-1$
$\neg p$	$-1,2,-1$	$-1,0,-1$

$\neg r$

$P1 : \{p, q\}, \Phi_1 \equiv \mathsf{F}\ (p \wedge q)$

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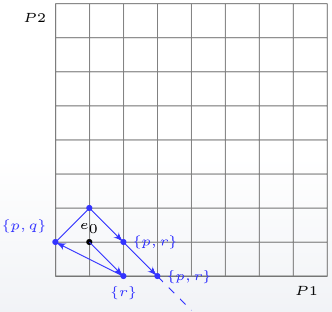
Initial endowment : $e_0 = (1, 1)$

$\mathsf{Payoff}(P1) = 0, \mathsf{Payoff}(P2) = 0$



Game semantics (Harrenstein, Turrini, Wooldridge 2015)

- Each player is given an *initial endowment*.
- Objectives are LTL formulae.
- The interaction is repeated for an infinite duration.
- Each valuation has a *cost*.

Strategy

A function
$$\sigma_i : (2^{AP})^* \to 2^{AP_i} \quad .$$

Payoff

$$\mathsf{Payoff}_i(\sigma) = \begin{cases} 1 & \text{if } \sigma \text{ is feasible, and } \langle \sigma \rangle \models \Phi_i \quad , \\ 0 & \text{otherwise.} \end{cases}$$

Definition (Nash equilibrium)

- N is the set of players.
- Σ_i is the set of possible strategies for player i.

$$\forall i \in N,\ \forall \tau_i \in \Sigma_i\ , \mathsf{Payoff}_i((\tau_i, \sigma_{-i})) \leq \mathsf{Payoff}_i(\sigma)\ .$$

Rational elimination

Is there a resource redistribution of e such that σ is not a Nash equilibrium?

Rational construction

Is there a resource redistribution of e such that σ is a Nash equilibrium?

~~Iterated~~ Electric Boolean Games

Theorem (Harrenstein, Turrini, Wooldridge 2015)

Rational elimination is NP*-complete.*
Rational construction is $\mathsf{co\text{-}NP}$*-hard and in* P^{NP}.

Rational Verification Problem

Definition

Given an EBG $\mathcal{B}^{c,e}$ and a finite memory profile σ, decide whether σ is a Nash equilibrium.

Rational Verification Problem

Definition

Given an EBG $\mathcal{B}^{c,e}$ and a finite memory profile σ, decide whether σ is a Nash equilibrium.

Theorem

Rational Verification Problem is PSPACE*-complete for iterated EBG.*

Decision Procedure in the non Electric Case

Let σ be a profile, i be a player, and ϕ_i its goal.

Lemma (Gutierrez, Harrenstein, Wooldridge 2015)

There exists an LTL formula $\psi(\sigma_{-i})$ such that:
i has a rational deviation iff $\psi(\sigma_{-i}) \wedge \phi_i$ is satisfiable.

Theorem

LTL satisfiability is PSPACE*-complete.*

Decision Procedure in the Electric Case

Let i be a player, and ϕ_i its goal.

- Construct $\mathcal{A}_i$ a Büchi automaton accepting the models of ϕ_i.
- Abstract the behaviour of the other players using a weighted graph G.
- Check whether there exists a word w accepted by $\mathcal{A}_i$ while being *consistent* with G.
- Make sure the new profile is *feasible*.

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Remark

The synchronisation of $\mathcal{A}_i$ with G yields an *energy Büchi* game.

Detecting a Rational Deviation

Lemma

Let σ be a finite memory profile, and i be a player such that $\mathsf{Payoff}_i(\sigma) = 0$ then, i has a rational deviation iff there exists a winning strategy in $G[\sigma_{-i}]$.

Where

- $G[\sigma_{-i}]$ is a one player energy Büchi game.

PSPACE Problems

Theorem

The RC problem and the RE problem are PSPACE*-complete.*

Proof.

Guess a resource redistribution and check whether the profile is an equilibrium. □

Conclusion

Summary

- The rational verification PSPACE-complete.
- Redistribution schema are PSPACE-complete.

Next step

- The synthesis problem.
- Other logics.
- Extend to the dynamic setting.