Average Controllability Measures for Solitaire Games

Marco Faella

University of Naples "Federico II", Italy

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Is Video Poker

a game of **skill**

or a game of **chance**?

- March 11th, 1980. A video poker machine is seized in Pennsylvania as an illegal gambling device
- Joseph Kadane (statistician at CMU) testifies for the defense:
  - Empirically compares a “dumb” strategy and a “smart” strategy for 128 rounds
  - Dumb strategy: always stand
  - Smart strategy: change the cards as to maximize expected payoff
  - Result: smart outperforms dumb by a factor of 4.68

[Kadane, Statistics and the Law, 2008]

What did the courts say?

- County Court: game of skill
- Superior Court: game of skill

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- County Court: game of skill
- Superior Court: game of skill
- Pennsylvania Supreme Court: game of **chance**

  "While appellee has demonstrated that some skill is involved in the playing of Electro-Sport, we believe that **the element of chance predominates** *and the outcome is largely determined by chance."*

Can we help?

1. Assign a real number between 0 and 1 to 1-player games
   - 0 means player has no control on outcome (pure chance)
   - 1 means player has full control on outcome (pure skill)

2. Set a threshold: if you play a game with value $< p$, you are gambling!

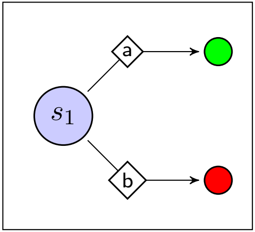
actually, an *order* and a *reference game* would suffice

Some axioms

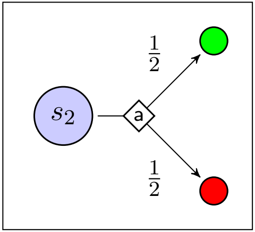
1. Completely random games have measure 0
2. Games with no randomness have measure 1
3. For all $p \in [0, 1]$, the *balanced game* B_p has measure p
4. ...

model: MDP with partial info

The Balanced Game B_p, with $p \in [0, 1]$

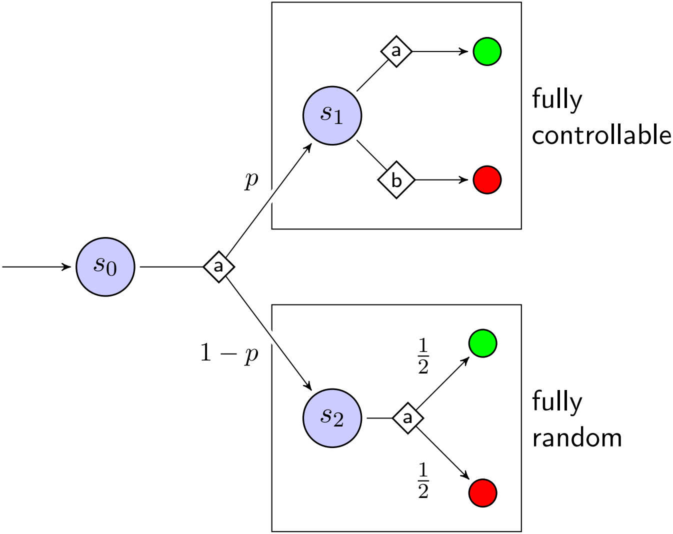


fully
controllable



fully
random

The Balanced Game B_p, with $p \in [0, 1]$



The measure of Borm et al.

expected value of
smart strategy

$$m(G) = \frac{E_{\text{smart}} - E_{\text{dumb}}}{E_{\text{cheat}} - E_{\text{dumb}}}$$

smart maximize expectation

cheat smart + knows random outcomes in advance

dumb plays uniformly at random every move

[Borm, Dreef and van Der Genugten, 2001, 2004]

The measure of Borm et al.

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$$m(G) = \frac{E_{\text{smart}} - E_{\text{dumb}}}{E_{\text{cheat}} - E_{\text{dumb}}}$$

$m(B_p) = 1$ for all $p \in [0, 1]$

Violates Axiom 3!

Analysis of variance

Attributing data variance to different sources

Two *effect-size* measures:

- η: percentage of standard deviation explained
- γ: percentage of absolute deviation explained

$$\gamma = \frac{\text{expected abs. dev. between strategies}}{\text{total abs. dev.}} = \frac{\sum_{i=1}^{n} |E_{\sigma_i} - \overline{E}|}{2n\left(\overline{E} - \overline{E}^2\right)}.$$

Good news

Theorem

Measures η and γ satisfy Axioms 1-3.

but violate certain stronger axioms...

Bad news

Theorem

Measures η and γ induce different orders between games.

There are two games such that $\eta(G_1) > \eta(G_2)$ but $\gamma(G_1) < \gamma(G_2)$

Experiments

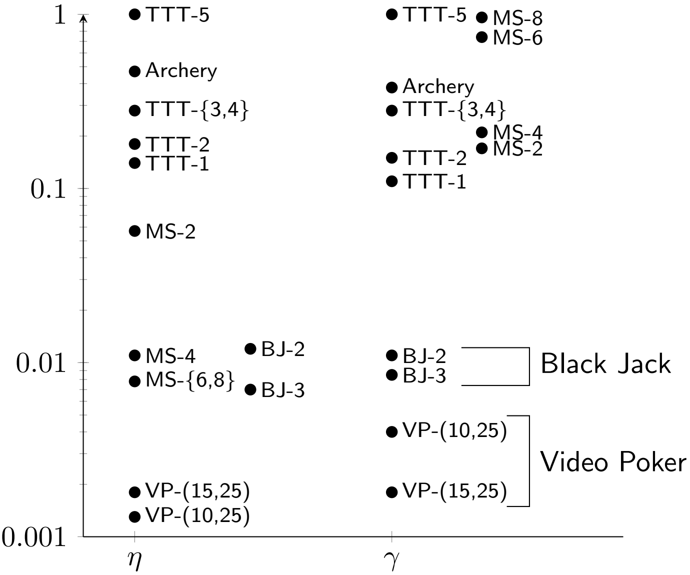
A tool simulating thousands of random plays

- Tic-Tac-Toe (five 1-player versions)
  (1) player 2 plays randomly
  ... ...
  (5) player 2 plays the first empty cell (deterministic)

- Black Jack
- Video Poker
- Minesweeper

- Archery (the sport)

Experiments



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m.faella@unina.it

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