Model and Objective Separation with Conditional Lower Bounds

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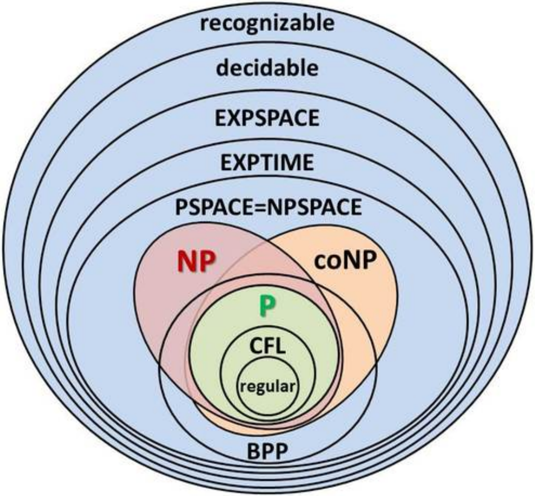
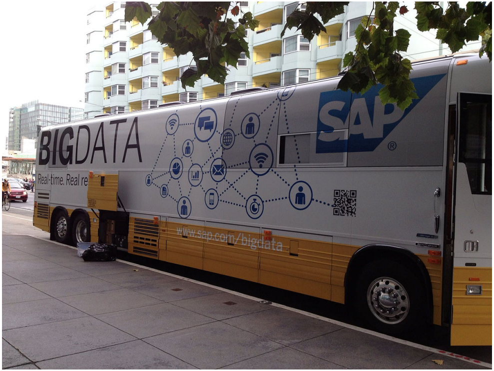
When we think of Computational Complexity . . .



But . . . Big Data!



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- Even quadratic time algorithms might be infeasible
- Super-linear lower bounds?
- So far in model checking: “polynomial time problems algorithmically equivalent”

This Paper

We study polynomial-time model checking problems in

- Graphs and
- Markov Decision Processes (MDPs),

with respect to several fundamental objectives,

- e.g., Rabin and Streett objectives.

Our contributions are two-fold:

- we present several improved algorithms, and
- the first conditional super-linear lower bounds.

Conjectures

Boolean Matrix Multiplication Conjecture (BMM)

There is no $O(n^{3-\varepsilon})$-time combinatorial algorithm for BMM for any $\varepsilon > 0$.

Strong Exponential Time Hypothesis (SETH)

For each $\varepsilon > 0$ there is a k such that k-CNF-SAT on n variables cannot be solved in $O(2^{(1-\varepsilon)n}\,\mathrm{poly}(n))$ time.

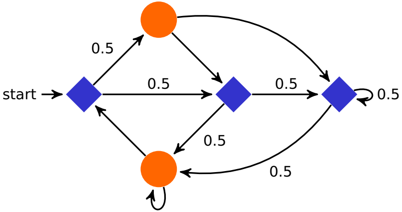
Models

Graphs

Finite directed graph $G = (V, E)$ with $n = |V|$ and $m = |E|$

Markov Decision Processes (MDP)

- Player 1 vertices V_1 ●
- Random vertices V_R ◆ (graph: $V_R = \emptyset$)
- Probability distribution δ: $V_R \to \mathcal{D}(V)$
  - here: uniform over outgoing edges



Formal Specification

Objective: desired subset ϕ of infinite paths Ω

Let T be a subset of vertices.

Büchi(T) paths that visit T infinitely often

coBüchi(T) paths that visit T only finitely often
$= \Omega \setminus$ Büchi(T)

Let $(L_1, U_1), \ldots (L_k, U_k)$ be k pairs of vertex sets.

Streett($\{(L_i, U_i)\}_{1 \le i \le k}$) for all i: If Büchi($L_i$) then Büchi($U_i$)

Rabin($\{(L_i, U_i)\}_{1 \le i \le k}$) for one i: Büchi($L_i$) and coBüchi($U_i$)
$= \Omega \setminus$ Streett($\{(L_i, U_i)\}_{1 \le i \le k}$)

Algorithmic Questions

Graphs: Is there an infinite path that satisfies the objective?

MDPs: Is there a strategy for player 1 to ensure that the objective is satisfied with probability 1?

(Almost-Sure) Winning Set: set of start vertices for which there exists winning path/strategy (with probability 1)

Example Model Separation (MDPs vs Graphs)

see paper for more results

Disjunction of k coBüchi Singleton Objectives

- upper bound graph: $O(m)$
- lower bound MDP: $\Omega(m^{2-o(1)})$, $\Omega((km)^{1-o(1)})$

n vertices, m edges

singleton: all target sets have cardinality 1

Example Objective Separation: Streett vs. Rabin

- graph games: coNP-c and NP-c
- memoryless strategies for Rabin but not Streett

Graphs:

	upper bound	lower bound	
Streett	$n^2 + nk \log n$	$k \cdot n^{2-o(1)}$	Rabin

MDPs:

	upper bound	lower bound	
Streett	$n^2 + nk \log n$	$k \cdot n^{2-o(1)}$	Rabin
	$(m^{1.5} + nk) \log n$	$m^{2-o(1)}$	

n vertices, m edges, k target sets

Rabin is algorithmically harder than Streett!

(assuming widely believed conjectures)

Open Problems

- linear time or conditional lower bound for Streett?
- some bounds are not tight
- remove “combinatorial” assumption or apply fast matrix multiplication?
- quantitative objectives like mean-payoff
- graph games
  - “Conditionally Optimal Algorithms for Generalized Büchi Games” MFCS’16
  - Understanding the complexity of parity games?