A **logic** for **transductions** and its decision through **data words**

Ongoing work by:

Luc Dartois, Emmanuel Filiot and **Nathan Lhote**

HIGHLIGHTS
of Logic, Games and Automata

BRUSSELS, 6–9 SEPTEMBER 2016

LaBRI

université de BORDEAUX

ULB UNIVERSITÉ LIBRE DE BRUXELLES

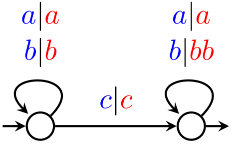
Transductions

Transductions: subsets of $\Sigma^* \times \Sigma^*$

Transductions

Transductions: subsets of $\Sigma^* \times \Sigma^*$

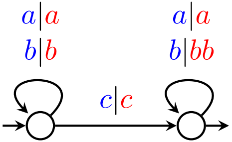
Example



Transductions

Transductions: subsets of $\Sigma^* \times \Sigma^*$

Example



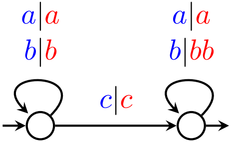
$abcba$

$abcbba$

Transductions

Transductions: subsets of $\Sigma^* \times \Sigma^*$

Example



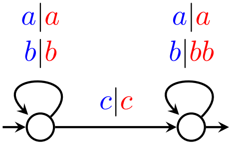
$a\ b\ c\ b\ \ a$

$a\ b\ c\ bb\ a$

Transductions

Transductions: subsets of $\Sigma^* \times \Sigma^*$

Example



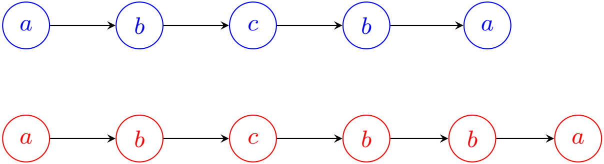
$a\ b\ c\ b\ \ a$

$a\ b\ c\ bb\ a$

origin information

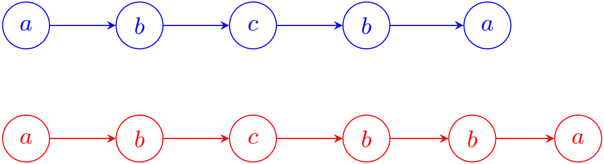
Logical transductions

MSO on labelled graphs



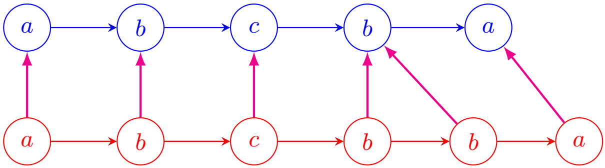
Logical transductions

MSO on labelled graphs



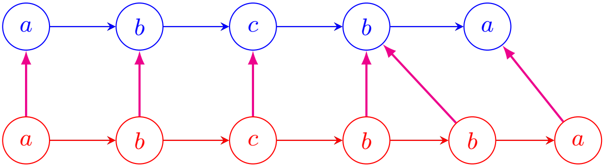
Logical transductions

MSO on labelled graphs



Logical transductions

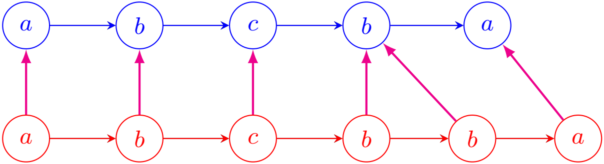
MSO on labelled graphs



$\phi_{\mathsf{word}} \wedge \phi_{\mathsf{word}} \wedge \phi_{\mathsf{function}}$

Logical transductions

MSO on labelled graphs

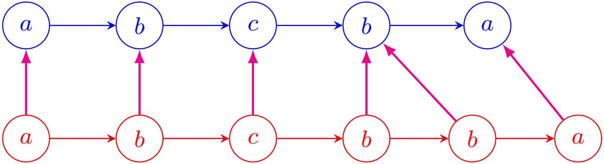


$\phi_{\mathsf{word}} \wedge \phi_{\mathsf{word}} \wedge \phi_{\mathsf{function}}$

$\mathsf{MSO}[<, <, O]$

Logical transductions

MSO on labelled graphs



$\phi_{\text{word}} \wedge \phi_{\text{word}} \wedge \phi_{\text{function}}$

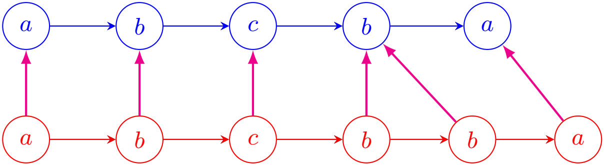
$\mathsf{MSO}[<, <, O]$

Example

- $\phi_{\text{shuffle}} = \begin{array}{l} \forall x \; \exists y \; O(y,x) \bigwedge_{\sigma\in\Sigma} (\sigma(x) \leftrightarrow \sigma(y)) \\ \bigwedge \forall x \; \forall y_1, y_2 \; O(y_1,x) \wedge O(y_2,x) \rightarrow y_1 = y_2 \end{array}$

Logical transductions

MSO on labelled graphs



$\phi_{\mathsf{word}} \wedge \phi_{\mathsf{word}} \wedge \phi_{\mathsf{function}}$

$\mathsf{MSO}[<, <, O]$

Example

- $\phi_{\mathsf{shuffle}} = \begin{array}{l} \forall x \ \exists y \ O(y,x) \bigwedge_{\sigma \in \Sigma} (\sigma(x) \leftrightarrow \sigma(y)) \\ \bigwedge \forall x \ \forall y_1, y_2 \ O(y_1, x) \wedge O(y_2, x) \rightarrow y_1 = y_2 \end{array}$
- $$\phi_{\mathsf{mirror}} = \phi_{\mathsf{shuffle}}$$
  $$\bigwedge \forall x_1, x_2 \ \forall y_1, y_2 \ O(y_1, x_1) \wedge O(y_2, x_2) \wedge (x_1 < x_2) \rightarrow (y_1 > y_2)$$

Logical transductions

Problem:

Undecidable satisfiability

Logical transductions

Problem:

Undecidable satisfiability

Approach:

Weaker fragment but "expressive enough" (MSOT)

Logical transductions

Problem:

Undecidable satisfiability

Approach:

Weaker fragment but "expressive enough" (MSOT)

Our logic:

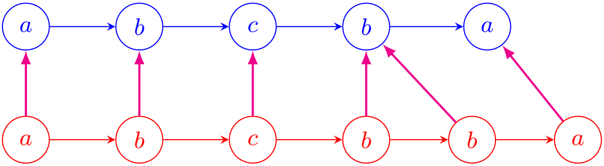
$$\mathsf{FO}^2[Reg, <, O]$$

Data words and transductions

Data word: $(\Gamma \times \mathbb{N})^*$

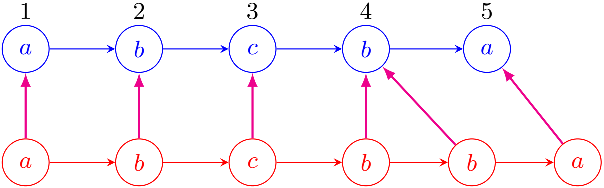
Data words and transductions

Data word: $(\Gamma \times \mathbb{N})^*$



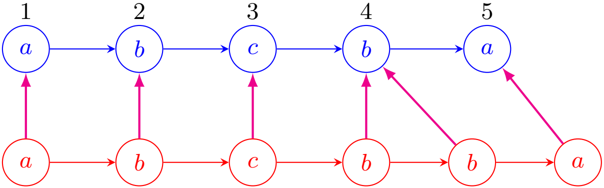
Data words and transductions

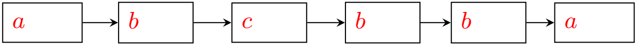
Data word: $(\Gamma \times \mathbb{N})^*$



Data words and transductions

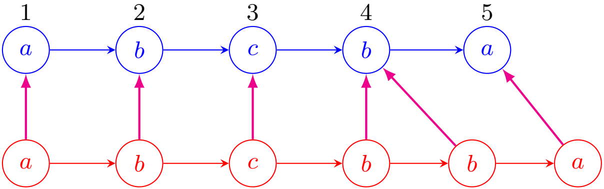
Data word: $(\Gamma \times \mathbb{N})^*$

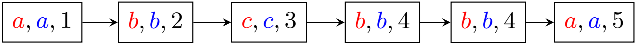




Data words and transductions

Data word: $(\Gamma \times \mathbb{N})^*$





[Schwentick & Zeume 2012]

$\mathsf{FO}^2[<, \prec]$ (and even $\mathsf{FO}^2[<, \prec, S]$) is decidable.

[Schwentick & Zeume 2012]
$\mathsf{FO}^2[<, \prec]$ (and even $\mathsf{FO}^2[<, \prec, S]$) is decidable.

Our results:

- $\mathsf{FO}^2[<, Reg_{\prec}]$ is decidable.

[Schwentick & Zeume 2012]

$\mathsf{FO}^2[<, \prec]$ (and even $\mathsf{FO}^2[<, \prec, S]$) is decidable.

Our results:

- $\mathsf{FO}^2[<, Reg_{\prec}]$ is decidable.
- satisfiability of $\mathsf{FO}^2[Reg, <, O]$ *reduces* to $\mathsf{FO}^2[<, Reg_{\prec}]$.

[Schwentick & Zeume 2012]

$\mathsf{FO}^2[<, \prec]$ (and even $\mathsf{FO}^2[<, \prec, S]$) is decidable.

Our results:

- $\mathsf{FO}^2[<, Reg_{\prec}]$ is decidable.
- satisfiability of $\mathsf{FO}^2[Reg, <, O]$ *reduces* to $\mathsf{FO}^2[<, Reg_{\prec}]$.
- Any MSOT is expressible in $\mathsf{FO}^2[Reg, <, O]$.

Thanks !