The Complexity of Rational Synthesis[1]

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[1] Joint work with Emmanuel Filiot, Jean-François Raskin (ULB) and Raffaella Gentilini (Perugia, Italy)

Motivation

- Classical reactive system synthesis:
  - One system and one antagonist environment
  - Synthesize a system to ensure the specification

$$\boxed{\text{Environment Model}} \parallel ? \models \boxed{\text{Specification}}$$

  - Synthesis $\approx$ two-player zero-sum game

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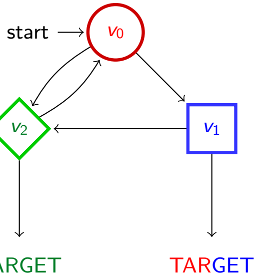
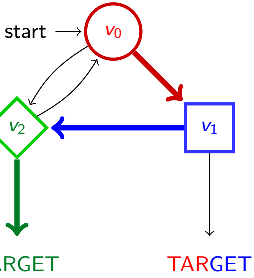
  - Synthesis ≈ two-player zero-sum game

- Rational synthesis:
  - Multi-component environment
  - Non-antagonist objectives
  - Rational synthesis ≈ multiplayer turn-based game

- $\mathcal{G} = \langle \mathbb{P}, V, (V_i)_{i\in\Omega}, E, v_0, (\mathcal{O}_i)_{i\in\mathbb{P}} \rangle$ where $\mathbb{P} = \{0, 1, \ldots, k\}$ and $\mathcal{O}_i \subseteq V^\omega$



- Strategy of Player i : $\sigma_i : V^* V_i \rightarrow V$
- Strategy profile $\bar{\sigma} = (\sigma_i)_{i \in \mathbb{P}}$,
- $pay(\bar{\sigma}) \in \{0,1\}^n$ s.t. $pay_i(\bar{\sigma}) = 1$ iff $out(\bar{\sigma}) \in \mathcal{O}_i$



Strategies and Nash Equilibria

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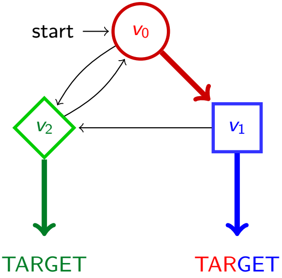

Nash Equilibrium

Definition (Nash Equilibrium (Nash51))

$\bar{\sigma}$ is Nash Equilibrium iff no incentive to deviate

$$pay_i(\bar{\sigma}_{-i}, \tau_i) \leq pay_i(\bar{\sigma}) \ \forall i \in \mathbb{P} \text{ and } \tau_i \text{ strategy of Player } i$$

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- $\bar{\sigma}$ is 0-fixed Nash Equilibrium iff

$$pay_i(\bar{\sigma}_{-i}, \tau_i) \leq pay_i(\bar{\sigma}) \ \forall i \in \mathbb{P} \setminus \{0\} \text{ and } \tau_i \text{ strategy of Player } i$$

Rational Synthesis

$$\boxed{\begin{array}{c}\text{Rational}\\ \text{Environment}\end{array}} \parallel ? \models \boxed{\text{Specification}}$$

- Rationality = behavior according to a Nash equilibrium.

[1] D. Fisman, O. Kupferman, and Y. Lustig. Rational synthesis. CoRR, abs/0907.3019, 2009.
[2] O. Kupferman, G. Perelli, and M. Y. Vardi. Synthesis with rational environments. EUMAS 2014

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Definition (Rational Synthesis Problems)

cooperative(CRSP):[2] $\exists \sigma_0 \ \exists \bar{\sigma}$ s.t. $\langle \sigma_0, \bar{\sigma} \rangle \in NE_{\sigma_0} \ \wedge \ pay_0(\sigma_0, \bar{\sigma}) = 1$?

non-cooperative(NCRSP):[3] $\exists \sigma_0 \ \forall \bar{\sigma}$: $\langle \sigma_0, \bar{\sigma} \rangle \in NE_{\sigma_0} \rightarrow pay_0(\sigma_0, \bar{\sigma}) = 1$?

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- Reduce to Model Checking of Strategy Logic formulas
- 2EXPTIME-complete
- Goal: fine understanding and complexities for particular objectives

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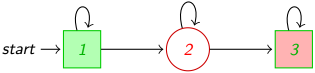
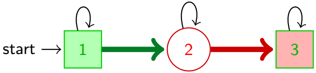
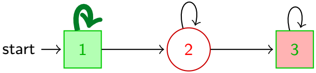
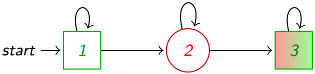
Rational Synthesis - Example1



Reachability objectives: $R_{\bigcirc} = \{3\}$, $R_{\square} = \{1\}$

Cooperative	**non-Cooperative**
	 no solution

Rational Synthesis - Example2



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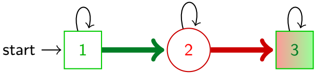

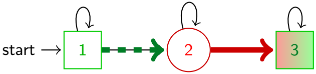

Cooperative Rational Synthesis

cooperative:[2] $\exists \sigma_0 \ \ \exists \bar{\sigma} \ \ s.t. \ \langle \sigma_0, \bar{\sigma} \rangle \in NE_{\sigma_0} \ \wedge \ pay_0(\bar{\sigma}) = 1 ?$

- Nash equilibrium characterization on path properties (inspired by Ummels[3])
- Find $\pi \in V^\omega$ s.t.
  - If $\mathcal{O}_i = Reach(R_i)$ for some $R_i \subseteq V$ (for tail objectives is similar) :

$$\pi \models \Diamond R_0 \wedge \bigwedge_{i=1}^{k} (\Box \neg R_i \rightarrow \Box \neg Win_i)$$

  - If $\mathcal{O}_i = \mathrm{Safe}(S_i)$ for some $S_i \subseteq V$:

$$\pi \models \Box S_0 \wedge \bigwedge_{i=1}^{k} ((\neg Win_i \ \mathcal{U} \ \neg S_i) \vee \Box S_i)$$

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[3] M. Ummels. The complexity of Nash Equilibria in infinite multiplayer games. FOSSACS 2008

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If it exists such π, it exists $\pi = x(y)^\omega$ with $|xy|$ polynomial in $\mathcal{G}$

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Non-Cooperative Rational Synthesis Problem

non-cooperative: $\exists\sigma_0 \quad \forall\bar{\sigma} \quad : \quad \langle\sigma_0,\bar{\sigma}\rangle \in NE_{\sigma_0} \rightarrow pay_0(\bar{\sigma}) = 1$?

- First attempt: two player zero-sum game with objective

$$\pi \vDash \Big(\bigwedge_{i=1}^{k}(\Box\neg R_i \rightarrow \Box\neg Win_i)\Big) \rightarrow \Diamond R_0$$

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$$\pi \models \Big(\bigwedge_{i=1}^{k} (\Box \neg R_i \rightarrow \Box \neg Win_i) \Big) \rightarrow \Diamond R_0$$

Incorrect!

Fix σ_0. Only 0-fixed NE w.r.t. σ_0 should be considered !!!

Automata-based solution for NCRSP

- Encode strategies of Player 0 as trees
- Define a nondeterministic tree automata to accept solutions

$$\mathcal{L}(\mathcal{T}) = \{t_{\sigma_0} \mid \sigma_0 \text{ is solution to NCRSP}\}$$

  - for each branch π of t_{σ_0} compatible to σ_0, check that:
    - $\pi \in \mathcal{O}_0$ or
    - π not the outcome of a 0-fixed NE

      $\updownarrow$

      guess at least one player that wants to deviate from π
      and check he has a winning strategy under σ_0
  - exponential size in $|\mathbb{P}|$
  - monotonicity properties along paths of accepting runs
- Test emptiness on-the-fly using two-player games

Results and Future Work

	Cooperative		Non-Cooperative	
	Unfixed # Players	Fixed # Players	Unfixed # Players	Fixed # Players
Safety	NP-c	PTIME-c	PSPACE-c	PTIME-c
Reachability	NP-c	PTIME-c	PSPACE-c	PTIME-c
Büchi	PTIME-c[3]	PTIME-c[3]	PSPACE-c	PTIME-c
co-Büchi	NP-c[3]	PTIME-c	PSPACE-c	PTIME-c
Parity	NP-c[3]	$UP \cap co - UP$, parity-h	EXPTIME, PSPACE-h	PSPACE, NP-h, coNP-h
Streett	NP-c[3]	NP[3], NP-hard	EXPTIME,PSPACE-h	PSPACE-c
Rabin	P^{NP}, NP-h, coNP-h	P^{NP}, coNP-h	EXPTIME, PSPACE-h	PSPACE-c
Muller	PSPACE-c	PSPACE-c	EXPTIME, PSPACE-h	PSPACE-c
LTL	2EXPTIME-C[2]	2EXPTIME-C[2]	2EXPTIME-C[2]	2EXPTIME-C[2]

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- Future work:
  - Imperfect information
  - Other notions of rationality

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