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lfcs Laboratory for Foundations of Computer Science

The non-deterministic fragment of modal μ

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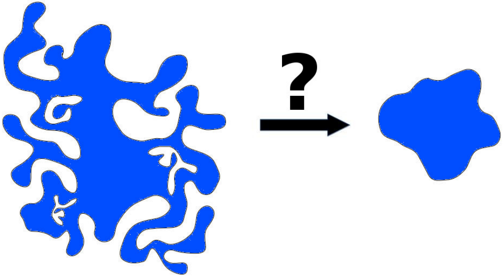
The problem: Given a L_μ formula, is there an equivalent simpler formula?



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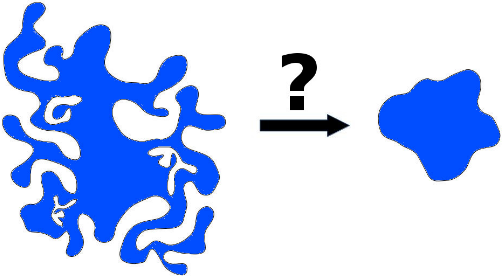
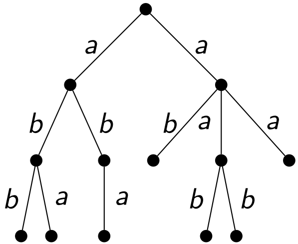


Deciding the alternation hierarchy / Mostowski-Rabin index

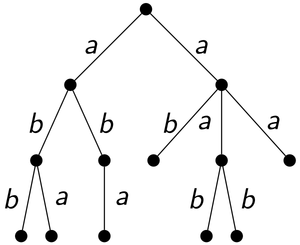
Describing properties of labelled transition systems



- Formulas

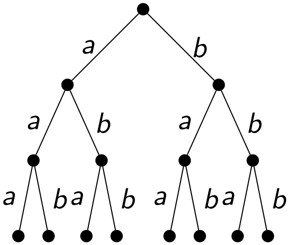
Describing properties of labelled transition systems

Unordered



- Formulas

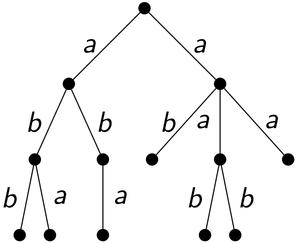
Ordered



- Automata

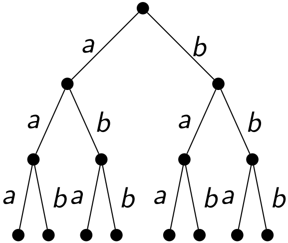
Describing properties of labelled transition systems

Unordered



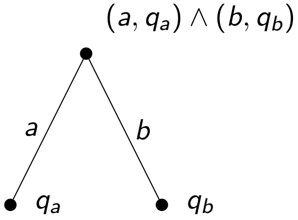
- Formulas
- Syntactic complexity
- Alternation hierarchy

Ordered



- Automata
- Acceptance condition
- Rabin-Mostowski hierarchy

Non-deterministic automata as L_μ formula



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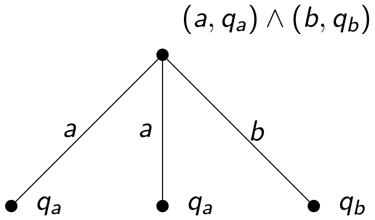
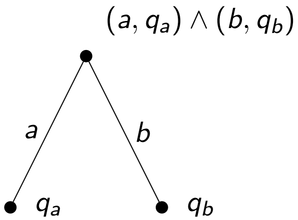
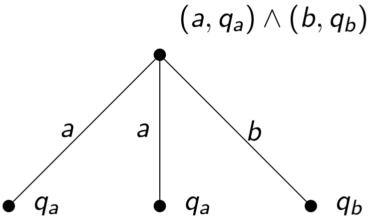
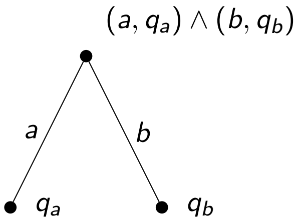




Non-deterministic automata as L_μ formula





- Subfragment of **disjunctive normal form**
- Extra restrictions on modalities
- May need more alternations/higher index than equivalent disjunctive formula

Non-deterministic L_μ

Key lemma: For non-deterministic formulas, semantic equivalence implies syntactic similarity.

$A \vee B \iff A' \vee B'$

Non-deterministic L_μ

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$$([a]A_0 \wedge [b]B_0) \vee ([a]A_1 \wedge [b]B_1)$$
$$\Longrightarrow$$
$$([a]A'_0 \wedge [b]B'_0) \vee ([a]A'_1 \wedge [b]B'_1)$$

Non-deterministic L_μ

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Then $[a]A_0 \wedge [b]B_0 \implies [a]A_i \wedge [b]B_i$ for some $i \in \{0, 1\}$

Non-deterministic L_μ

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If $[a]A_0 \wedge [b]B_0$ does not imply $[a]A_i \wedge [b]B_i$ then either $A_0 \wedge \neg A'_i$ or $B_0 \wedge \neg B'_i$ is satisfiable.

Non-deterministic L_μ

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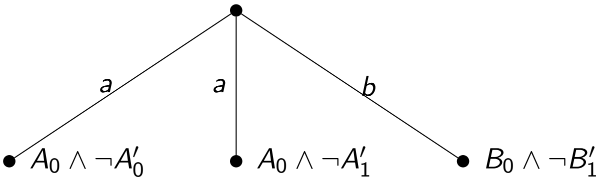

Non-deterministic L_μ

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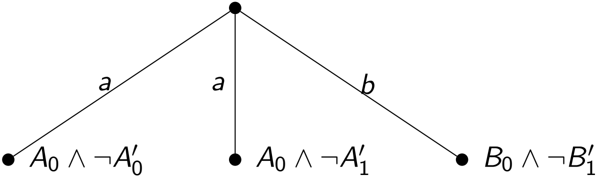

Then $[a]A_0 \wedge [b]B_0 \implies [a]A_i \wedge [b]B_i$ for some $i \in \{0, 1\}$

Non-deterministic L_μ

- Then **semantic** equivalence implies **syntactic** similarity
- Implications for the index problem

Conclusion

- The index-problems for L_μ and parity automata are different
- For example: non-deterministic $L_\mu \neq$ non-deterministic automata