Window parity games: an alternative approach toward parity games with time bounds

Véronique Bruyère[1] Quentin Hautem[1] Mickael Randour[2]

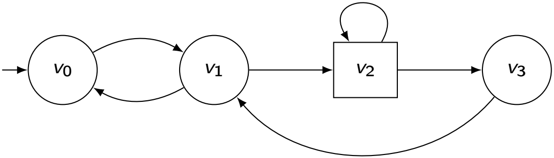
[1]University of Mons, [2]Université libre de Bruxelles

September 8th, 2016. Highlights 2016

Games on graphs

- Two-player game:

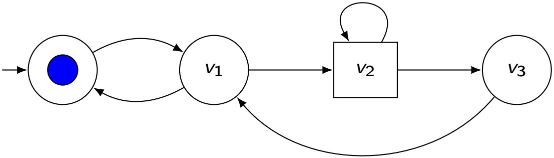
  system/player 1 (○) vs. the environment/player 2 (□)



Games on graphs

- Two-player game:

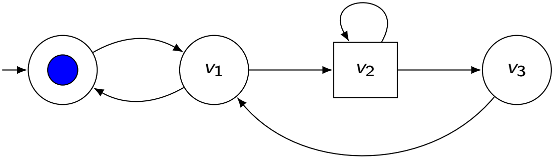
system/player 1 (○) vs. the environment/player 2 (□)



Games on graphs

- Two-player game:

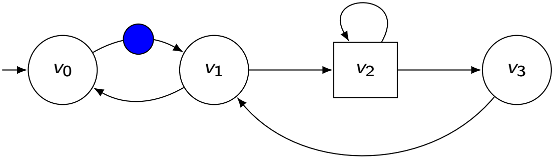
  system/player 1 (○) vs. the environment/player 2 (□)



Games on graphs

- Two-player game:

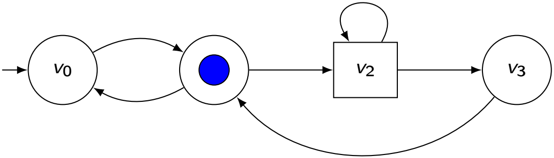
system/player 1 (○) vs. the environment/player 2 (□)



Games on graphs

- Two-player game:

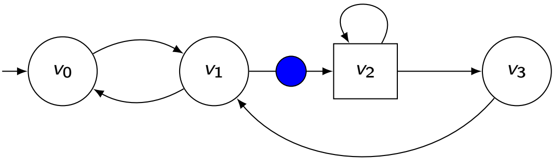
  system/player 1 (○) vs. the environment/player 2 (□)



Games on graphs

- Two-player game:

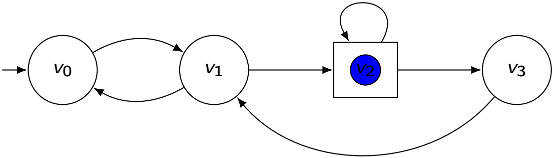
system/player 1 (○) vs. the environment/player 2 (□)



Games on graphs

- Two-player game:

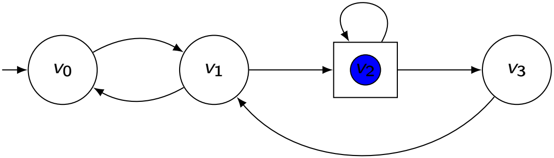
  system/player 1 ($\bigcirc$) vs. the environment/player 2 ($\square$)



Games on graphs

- Two-player game:

  system/player 1 (○) vs. the environment/player 2 (□)



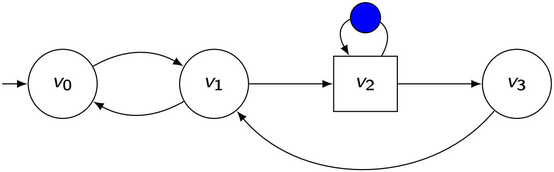
- Strategies: function that maps histories to vertex.
- Objective of player 1: set of plays.

→ Ex: v_2 has to be visited infinitely often.

Games on graphs

- Two-player game:

  system/player 1 (○) vs. the environment/player 2 (□)



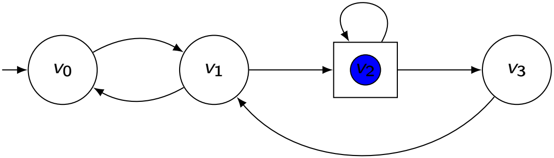
- Strategies: function that maps histories to vertex.
- Objective of player 1: set of plays.

→ Ex: v_2 has to be visited infinitely often.

Games on graphs

- Two-player game:

  system/player 1 (○) vs. the environment/player 2 (□)



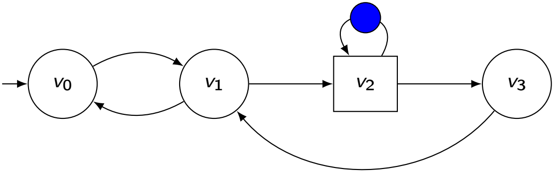
- Strategies: function that maps histories to vertex.
- Objective of player 1: set of plays.

→ Ex: v_2 has to be visited infinitely often.

Games on graphs

- Two-player game:

  system/player 1 (○) vs. the environment/player 2 (□)



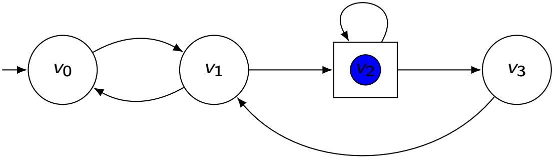
- Strategies: function that maps histories to vertex.
- Objective of player 1: set of plays.

→ Ex: v_2 has to be visited infinitely often.

Games on graphs

- Two-player game:

system/player 1 (○) vs. the environment/player 2 (□)



- Strategies: function that maps histories to vertex.
- Objective of player 1: set of plays.

→ Ex: v_2 has to be visited infinitely often.

Questions

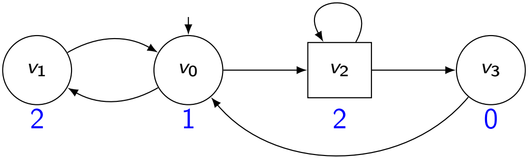
Given a game structure G, an objective Ω and an initial vertex v_0,

- Does one player have a winning strategy from the initial vertex ?
- If yes, can we decide which one ?
- What is the complexity class of the decision problem ?
- How much memory is needed for winning strategies ?

Objectives

- Let $p : V \to \{0, \ldots, k\}$ be a priority function.
  Parity objective : minimum priority seen infinitely often is even.

Player 1 has a memoryless winning strategy to ensure the Parity objective.



Known results [Jur98]

- The decision problem is in $UP \cap coUP$.
- Memoryless strategies are sufficient for both players.

Known results [Jur98]

- The decision problem is in $UP \cap coUP$.
- Memoryless strategies are sufficient for both players.

Open question : Is there a polynomial time algorithm to solve these games ?

Known results [Jur98]

- The decision problem is in $UP \cap coUP$.
- Memoryless strategies are sufficient for both players.

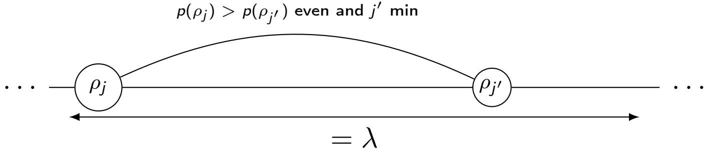
Open question : Is there a polynomial time algorithm to solve these games ?

Parity objective deals with limit behavior.

$\rightsquigarrow$ No explicit bound.

- Parity-response (PR(λ, p)):

Idea: every odd priority has to be followed by a smaller even priority in $\lambda - 1$ steps.

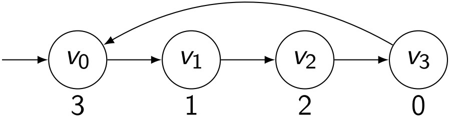


Objectives based on Parity-Response

- Fixed (Fix) objective : bound λ is given as a parameter.
- Bounded (Bnd) objective : looking for the existence of such a bound.
- Under approximations of parity objective

Example

Player 1 is winning for $\text{FixPR}(3, p)$



Results

	complexity	$\mathcal{P}_1$ mem.	$\mathcal{P}_1$ mem.
[1] Fixed PR	PSPACE-c.	exponential	$\leq$ exponential
[2] Bounded PR	P-easy.	memoryless	infinite

[1] [WZ16] : A. Weinert and M. Zimmermann. Easy to win, hard to master: Optimal strategies in parity games with costs.

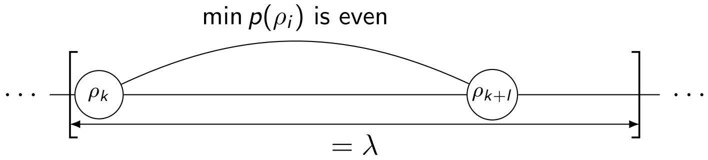
[2] [CHH09]: K. Chatterjee, T. Henzinger, and F. Horn. Finitary winning in omega-regular games.

Window Parity

Same idea as done for Window Mean-Payoff objective [CDRR15]!

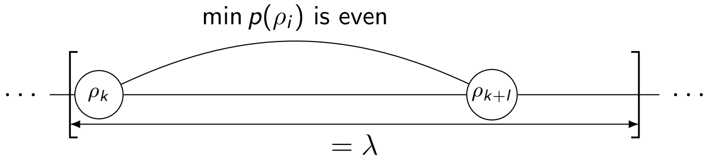
Window Parity

Same idea as done for Window Mean-Payoff objective [CDRR15]!



Window Parity

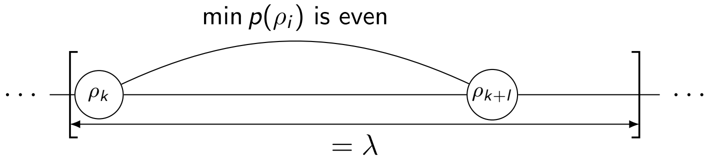
Same idea as done for Window Mean-Payoff objective [CDRR15]!



Idea: min of priorities has to be even before the end of the window.

Window Parity

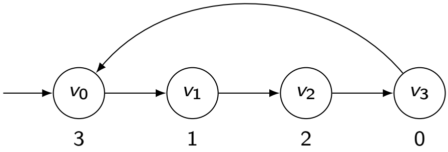
Same idea as done for Window Mean-Payoff objective [CDRR15]!



Idea: min of priorities has to be even before the end of the window.

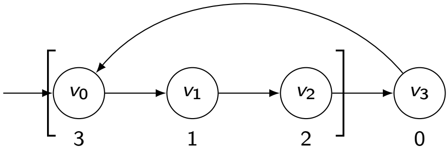
Again, we consider Fixed and Bounded objectives .

Example



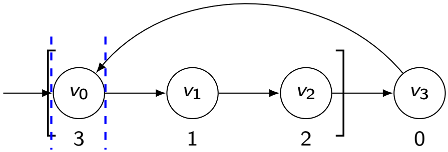
- $\rho \notin \text{FixWP}(\lambda = 3, p)$.
- $\rho \in \text{FixWP}(\lambda = 4, p)$.
- $\text{FixWP}(\lambda, p)$ and $\text{BndWP}(p) \Rightarrow \text{Parity}(p)$

Example



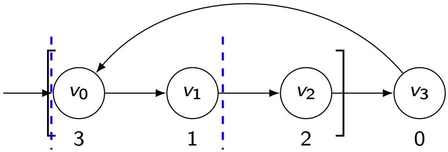
- $\rho \notin \text{FixWP}(\lambda = 3, p)$.
- $\rho \in \text{FixWP}(\lambda = 4, p)$.
- $\text{FixWP}(\lambda, p)$ and $\text{BndWP}(p) \Rightarrow \text{Parity}(p)$

Example



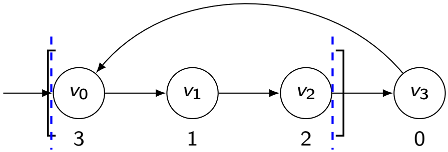
- $\rho \notin \text{FixWP}(\lambda = 3, p)$.
- $\rho \in \text{FixWP}(\lambda = 4, p)$.
- $\text{FixWP}(\lambda, p)$ and $\text{BndWP}(p) \Rightarrow \text{Parity}(p)$

Example



- $\rho \notin \text{FixWP}(\lambda = 3, p)$.
- $\rho \in \text{FixWP}(\lambda = 4, p)$.
- $\text{FixWP}(\lambda, p)$ and $\text{BndWP}(p) \Rightarrow \text{Parity}(p)$

Example



- $\rho \notin \text{FixWP}(\lambda = 3, p)$.
- $\rho \in \text{FixWP}(\lambda = 4, p)$.
- $\text{FixWP}(\lambda, p)$ and $\text{BndWP}(p) \Rightarrow \text{Parity}(p)$

(Some) results

- Bounded WP and Bounded PR coincide.
- Fixed WP games can be solved in polynomial time
  Idea: Keep track of the current minimum priority. If it is even, slide the window, otherwise go to next vertex if the end of the window is not reached.
- Fixed PR can be under and over approximated by Fixed WP.
- ...

	one-dimension		
	complexity	$\mathcal{P}_1$ mem.	$\mathcal{P}_2$ mem.
Fixed WP	**P-c.**	**polynomial**	
Fixed PR	PSPACE-c.	exponential	$\leq$ exponential
Bounded WP	P-c.	memoryless	infinite
Bounded PR			

	multi-dimension		
	complexity	$\mathcal{P}_1$ mem.	$\mathcal{P}_2$ mem.
Fixed WP	**EXPTIME**-c.	**exponential**	
Fixed PR			
Bounded WP		**exponential**	**infinite**
Bounded PR			

Paper available on ArXiV: `https://arxiv.org/pdf/1606.01831.pdf`

Paper available on ArXiV: `https://arxiv.org/pdf/1606.01831.pdf`

Thank you!

Krishnendu Chatterjee, Laurent Doyen, Mickael Randour, and Jean-François Raskin.
Looking at mean-payoff and total-payoff through windows.
Inf. Comput., 242:25–52, 2015.

K. Chatterjee, T.A. Henzinger, and F. Horn.
Finitary winning in omega-regular games.
ACM Trans. Comput. Log., 11(1), 2009.

Marcin Jurdzinski.
Deciding the winner in parity games is in UP n co-up.
Inf. Process. Lett., 68(3):119–124, 1998.

Alexander Weinert and Martin Zimmermann.
Easy to win, hard to master: Optimal strategies in parity games with costs.
CoRR, abs/1604.05543, 2016.