Decidability border for Petri nets with data: WQO Dichotomy Conjecture

Sławomir Lasota
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Highlights of Logic, Games and Automata,
Brussels, 2016.09.08

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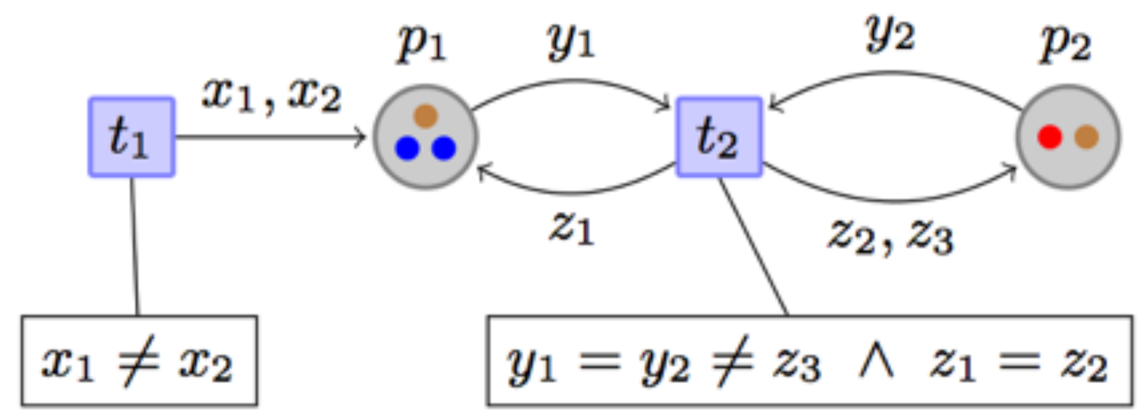

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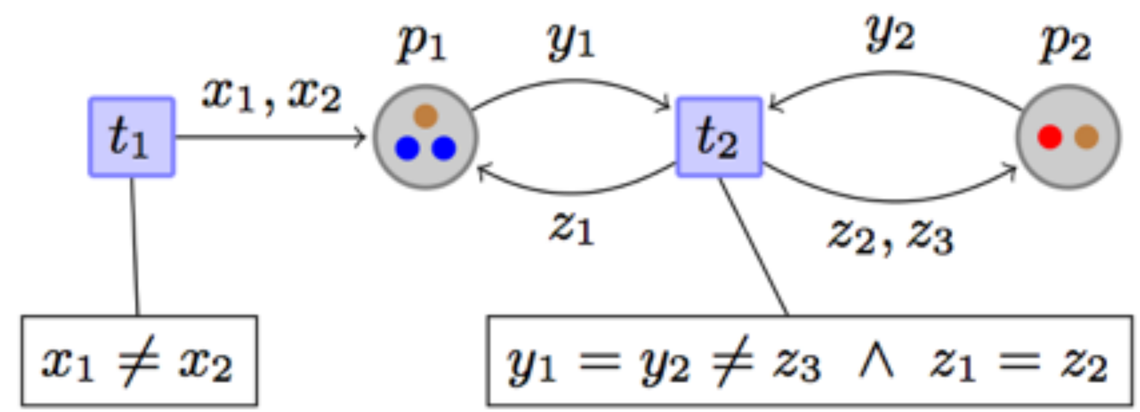

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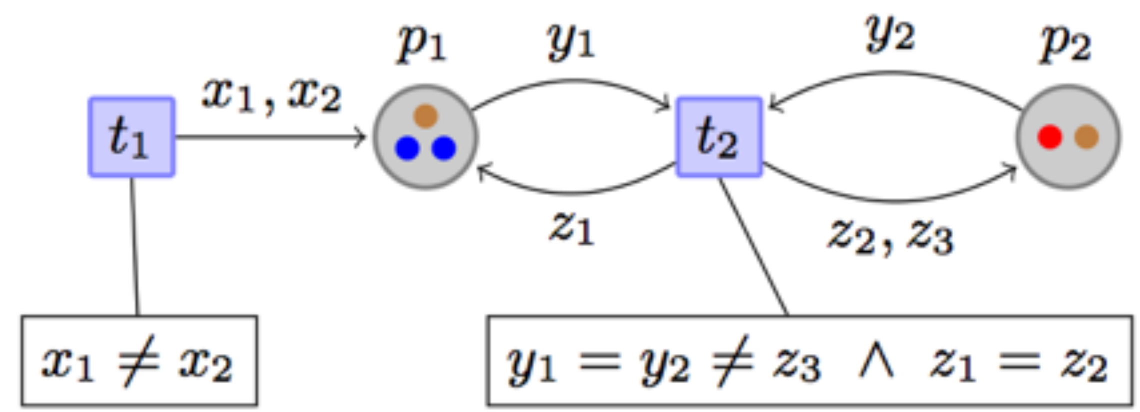

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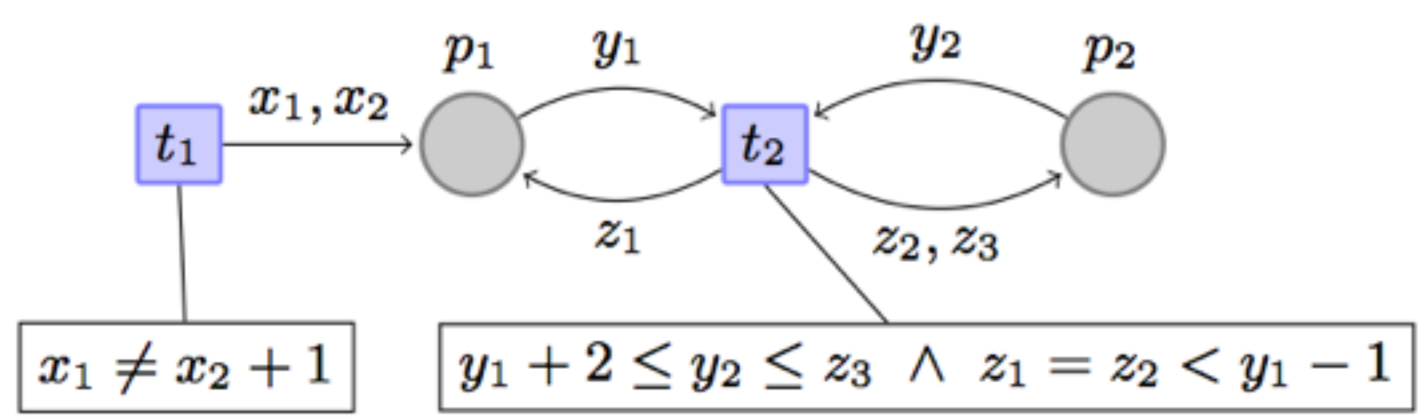

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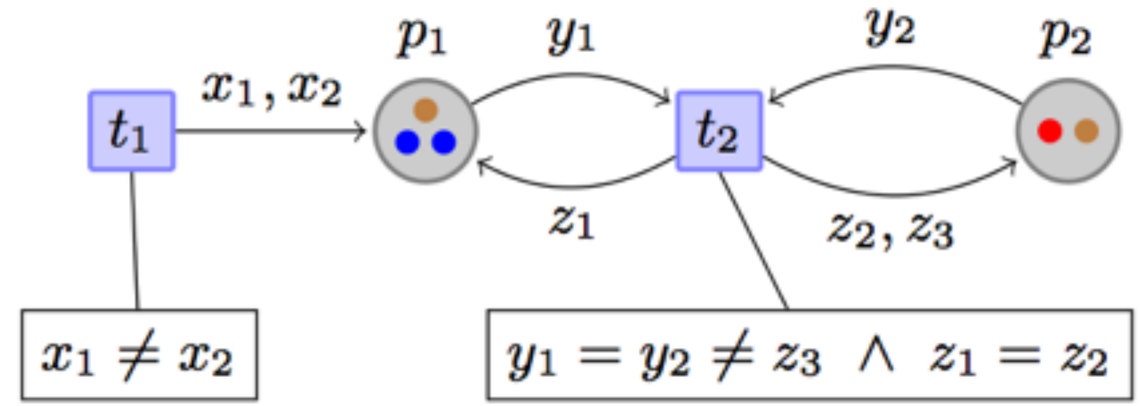

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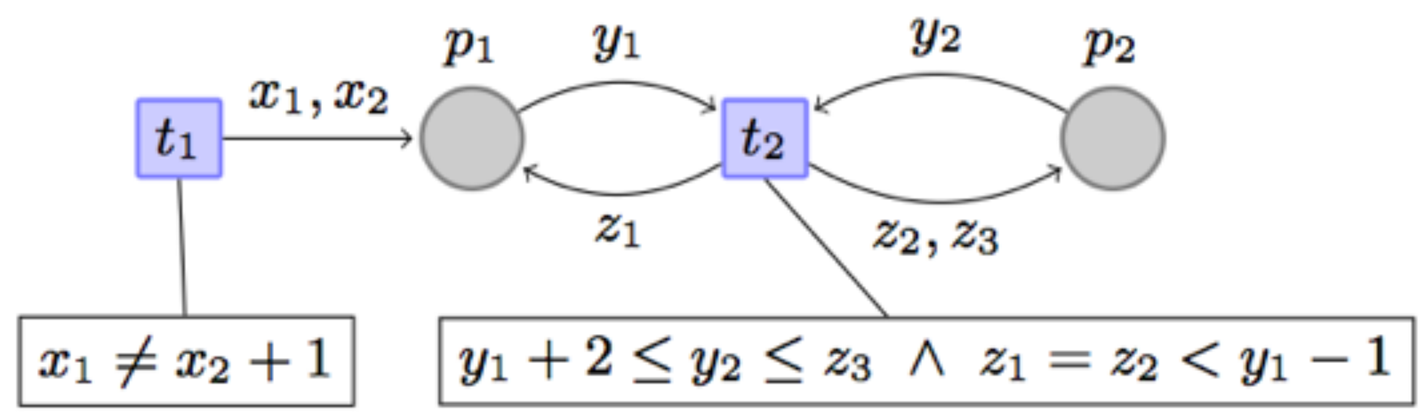

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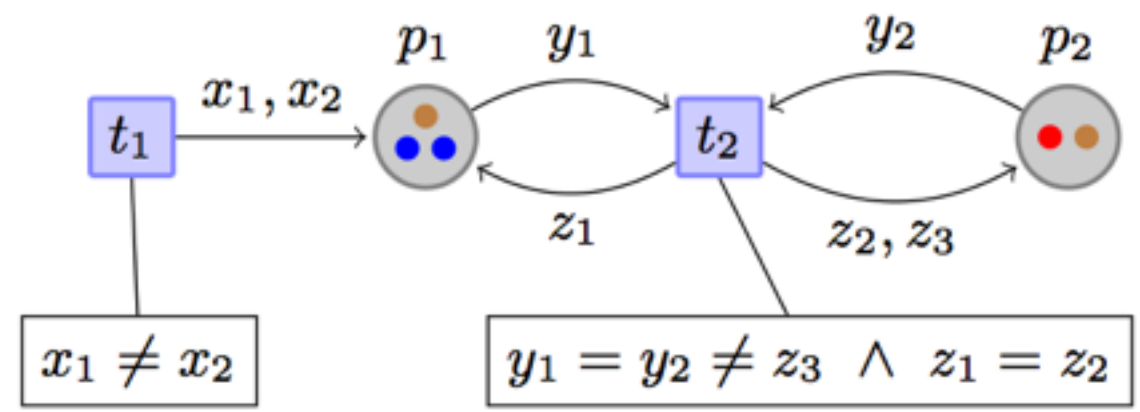

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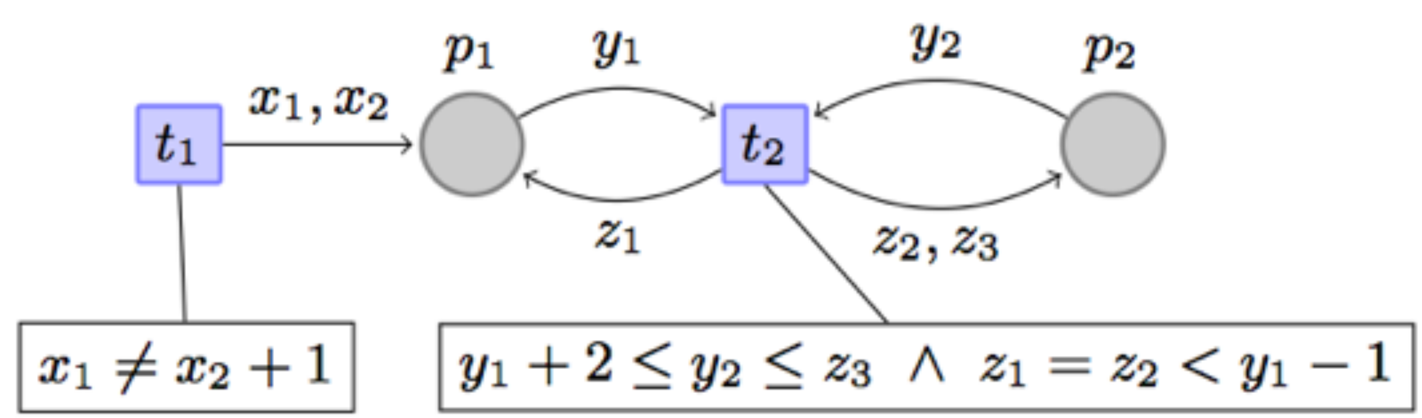

Configurations = $M(P \times \mathbb{A})$, for P the set of places.

In other words, a configuration is a finite induced structure of $\mathbb{A}$, labeled by elements of M(P).

standard decision problems

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- boundedness: is the set of reachable configurations finite, up to data automorphism?
- coverability: does some reachable configuration cover a given configuration, up to data automorphism?

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Petri nets, where sets of places and transitions are infinite but orbit-finite (definable) [Bojańczyk, Klin, L.'14]

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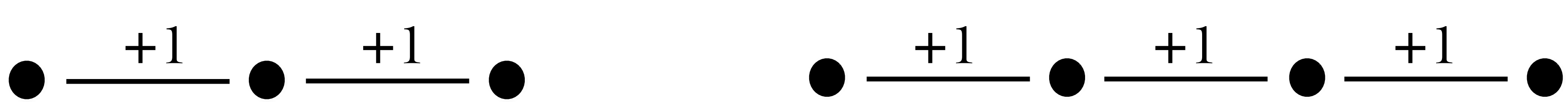

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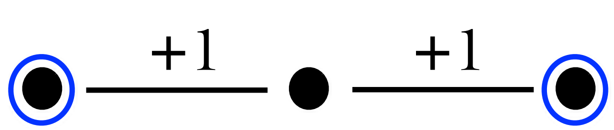

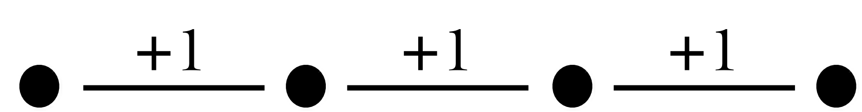

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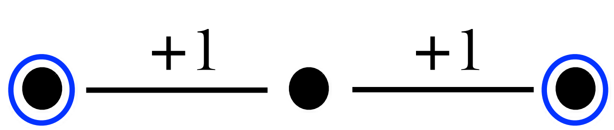

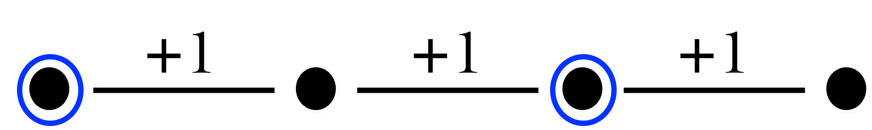

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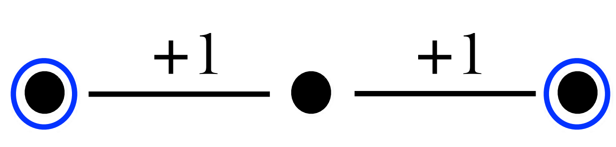

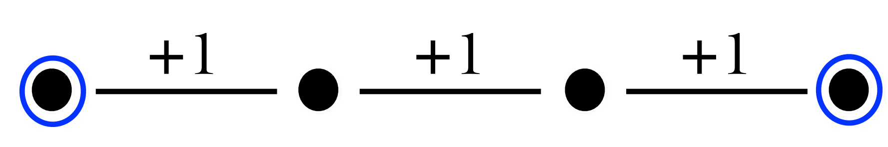

homogeneous data domains

homogeneous structure $\mathbb{A}$	amalgamation class
equality data $(\mathbb{N}, =)$	finite pure sets
total order data $(\mathbb{Q}, <)$	finite total orders
~~dense-time data $(\mathbb{Q}, <, +1)$~~	
~~discrete-time data $(\mathbb{Z}, <, +1)$~~	
universal (random) graph	finite graphs
universal equivalence relation	finite equivalence relations
universal partial order	finite partial orders
universal directed graph	finite directed graphs
universal tournament	finite tournaments
...	...

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Proof:

Using the framework of WSTS of [Finkel,Schnoebelen'01].

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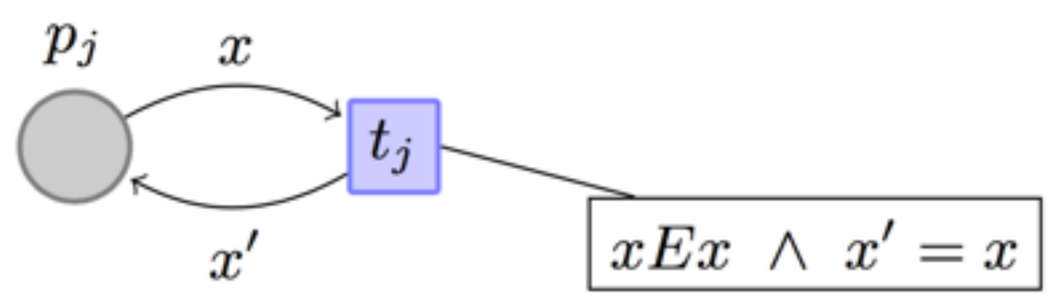

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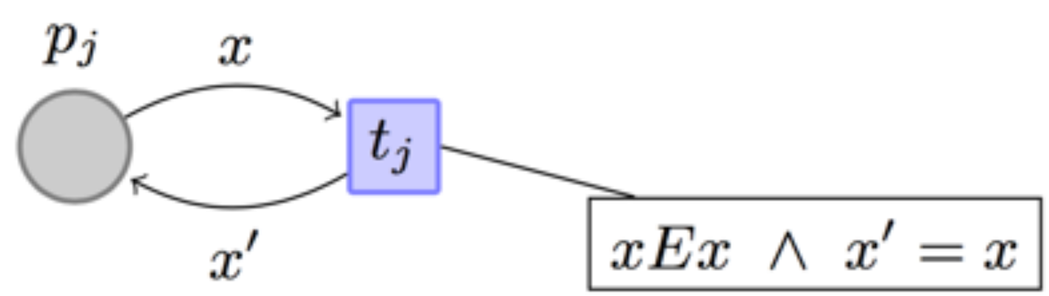

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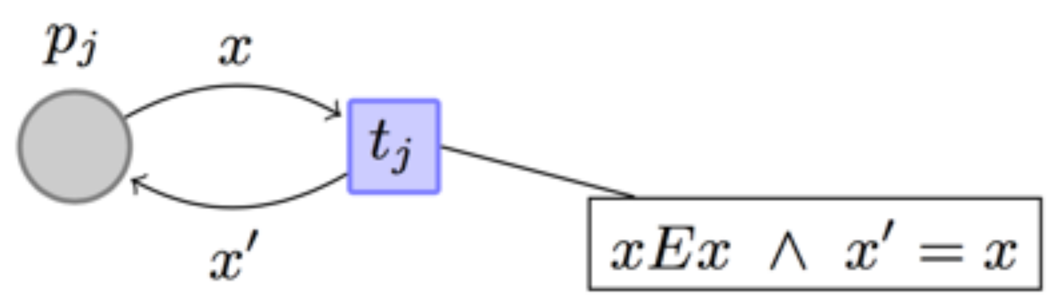

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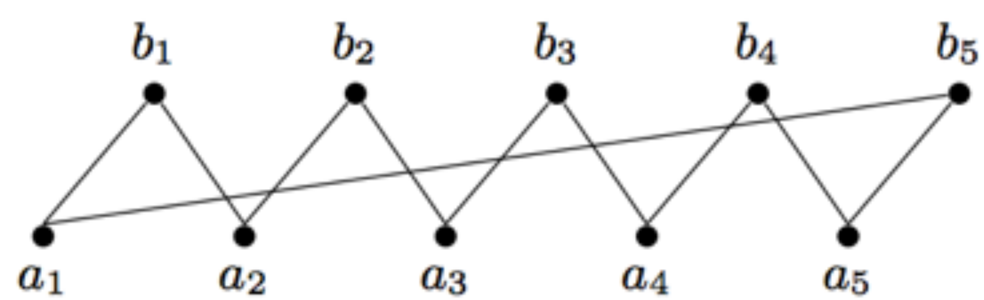

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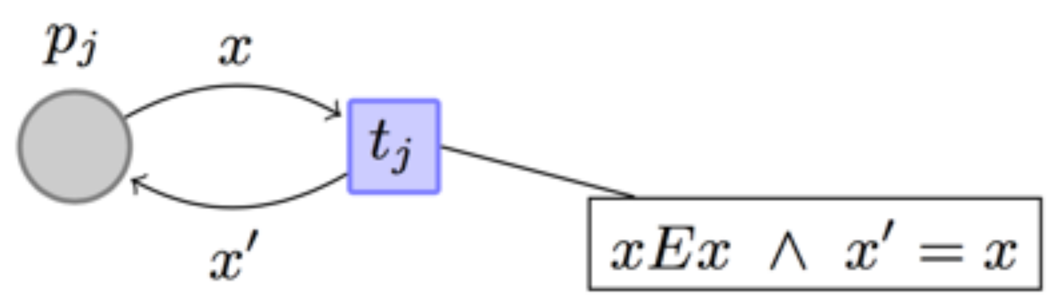

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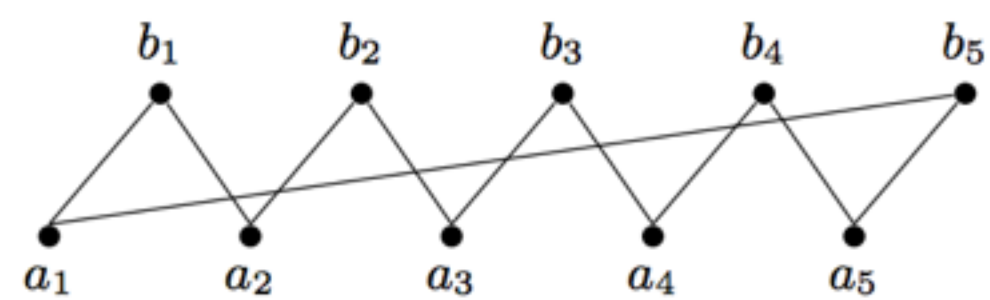

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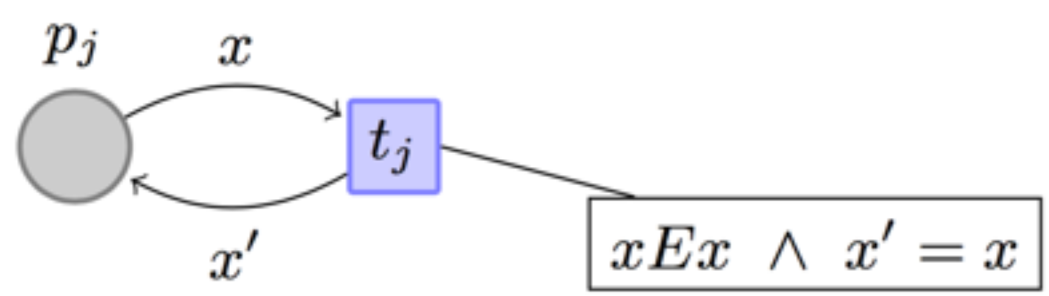

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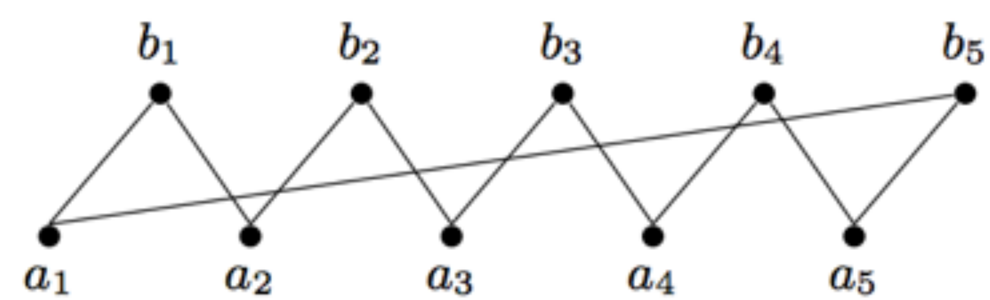

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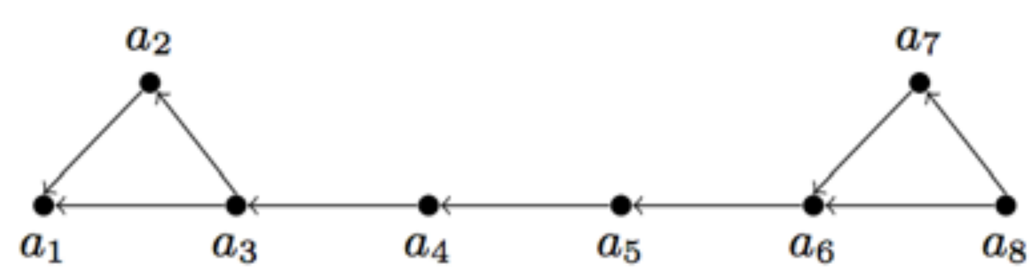

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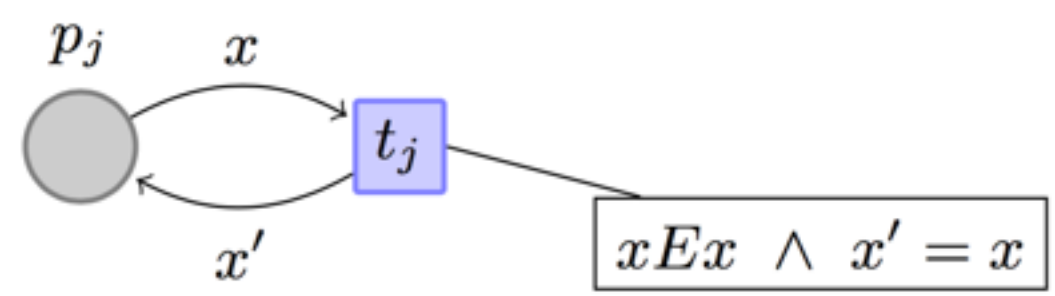

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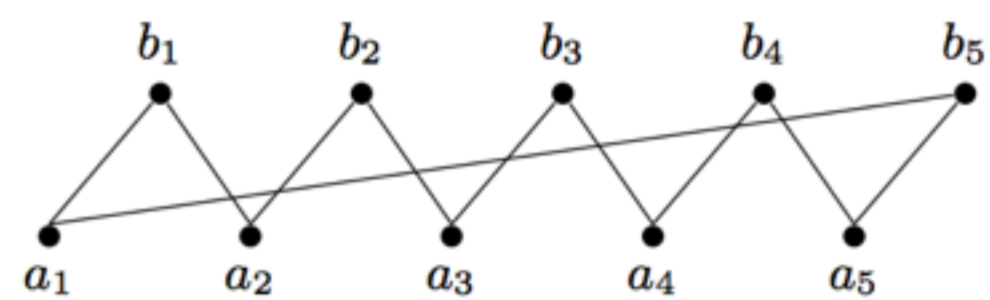

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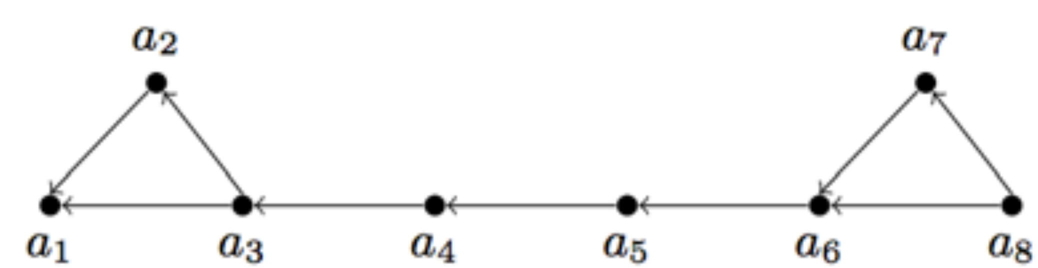

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