Decidability border for Petri nets with data: WQO Dichotomy Conjecture

Sławomir Lasota
University of Warsaw

Highlights of Logic, Games and Automata, Brussels, 2016.09.08
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Configurations $= M(P \times A)$, for $P$ the set of places.

In other words, a configuration is a finite induced structure of $A$, labeled by elements of $M(P)$.
standard decision problems
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input: a Petri net with data
and an initial configuration
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• **termination**: are all runs finite?
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Defined like classically
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• **coverability**: does some reachable configuration cover a given configuration, up to data automorphism?

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} defined up to automorphism
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restrict to **homogeneous** ones

Petri nets, where sets of places and transitions are infinite but orbit-finite (definable) [Bojańczyk, Klin, L.’14]
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A relational structure $\mathcal{A}$ is **homogeneous** if its finite induced substructures admit amalgamation.
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\[
\begin{array}{cccc}
\bullet & +1 & \bullet & +1 \\
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\[ +1 \bullet +1 \bullet +1 \]

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### Homogeneous Data Domains

<table>
<thead>
<tr>
<th>Homogeneous Structure A</th>
<th>Amalgamation Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>equality data ( (\mathbb{N}, =) )</td>
<td>finite pure sets</td>
</tr>
<tr>
<td>total order data ( (\mathbb{Q}, &lt;) )</td>
<td>finite total orders</td>
</tr>
<tr>
<td>dense-time data ( (\mathbb{Q}, &lt;, +1) )</td>
<td></td>
</tr>
<tr>
<td>discrete-time data ( (\mathbb{Z}, &lt;, +1) )</td>
<td></td>
</tr>
<tr>
<td>universal (random) graph</td>
<td>finite graphs</td>
</tr>
<tr>
<td>universal equivalence relation</td>
<td>finite equivalence relations</td>
</tr>
<tr>
<td>universal partial order</td>
<td>finite partial orders</td>
</tr>
<tr>
<td>universal directed graph</td>
<td>finite directed graphs</td>
</tr>
<tr>
<td>universal tournament</td>
<td>finite tournaments</td>
</tr>
<tr>
<td>...</td>
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decidability (uninteresting)
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**Theorem:**

Let $A$ be an effective homogeneous data domain such that configurations, ordered by embeddings, are a WQO.
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**Proof:**
Using the framework of WSTS of [Finkel, Schnoebelen’01].
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thank you!