Extending finite-memory determinacy by boolean combination of winning conditions

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Overview

Finite-memory determinacy

$\Downarrow$

Finite-memory multi-player multi-outcome Nash equilibrium

(Previous work with Arno Pauly)

Overview

In this talk:

Finite-memory determinacy with basic winning conditions
$\Downarrow$
Finite-memory determinacy with combined winning conditions

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Two players, win/lose.
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A winning strategy makes you win for sure.
Existence of a winning strategy = determinacy.

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Battery games simulate energy games. ([CD10] and Arno's talk)

Combining basic winning conditions: two known examples

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- in *n*-dimensional energy games, a player plays the same role on all components,
- in the energy-Muller games, only two conditions are combined,
- unbounded energy is a special case of battery.

NEW: Multi-dimension bounded energy Muller games

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Corollary (of a more general theorem)

These games are finite-memory determined.

The more general theorem

Definition

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If W *is determinacy-regular and ensures the existence of finite-memory optimal*[1] *strategies for all games, so do all boolean combinations of* W *and the* W_i*.*

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Exponential tower (height n) of bits suffice for the strategies.

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