Separability of Reachability Sets of Vector Addition Systems

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Wojciech Czerwiński

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Separability

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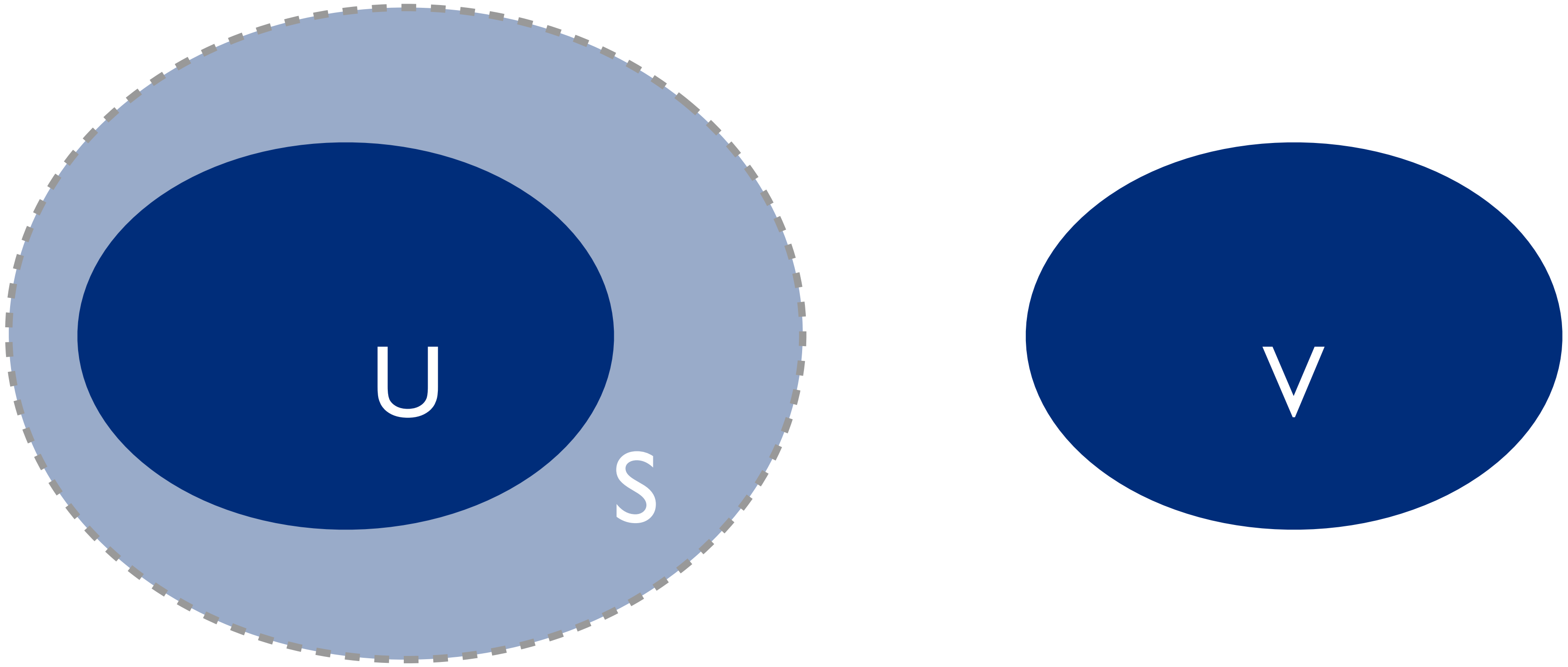

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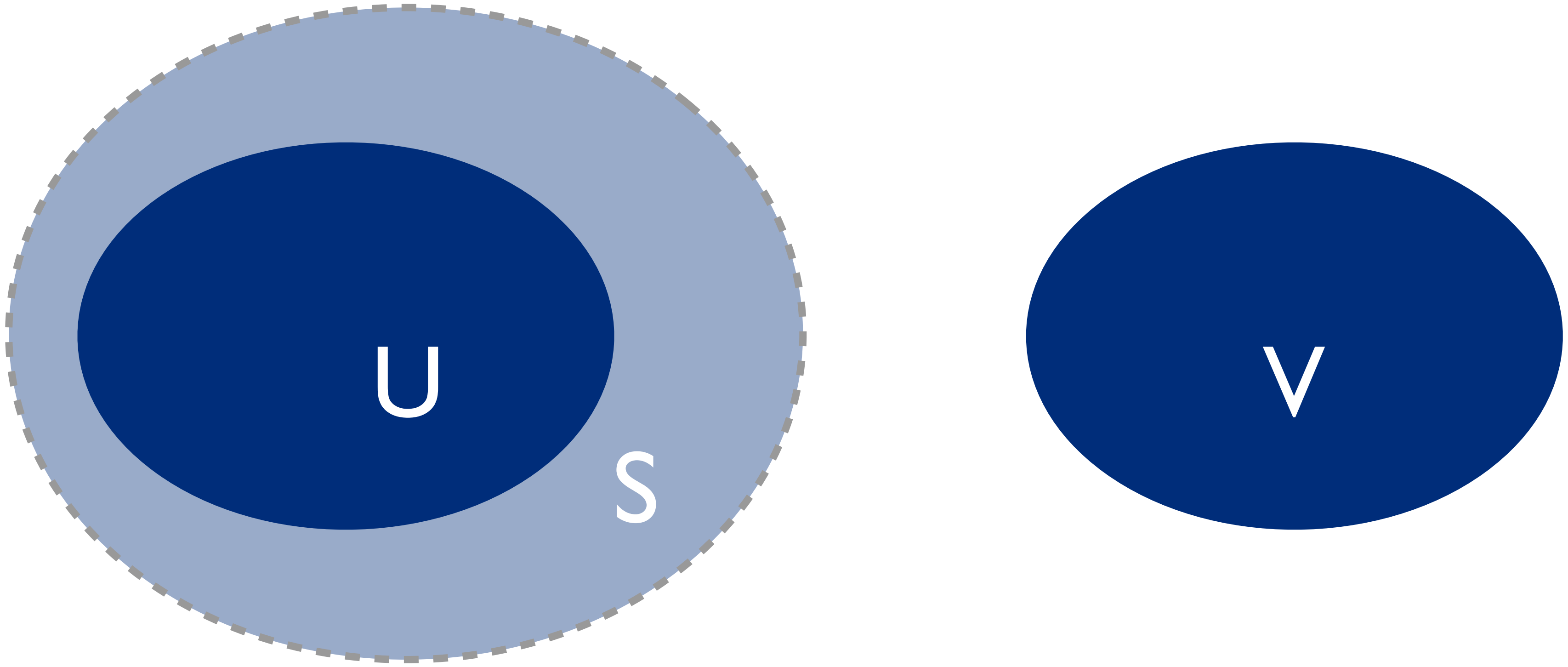

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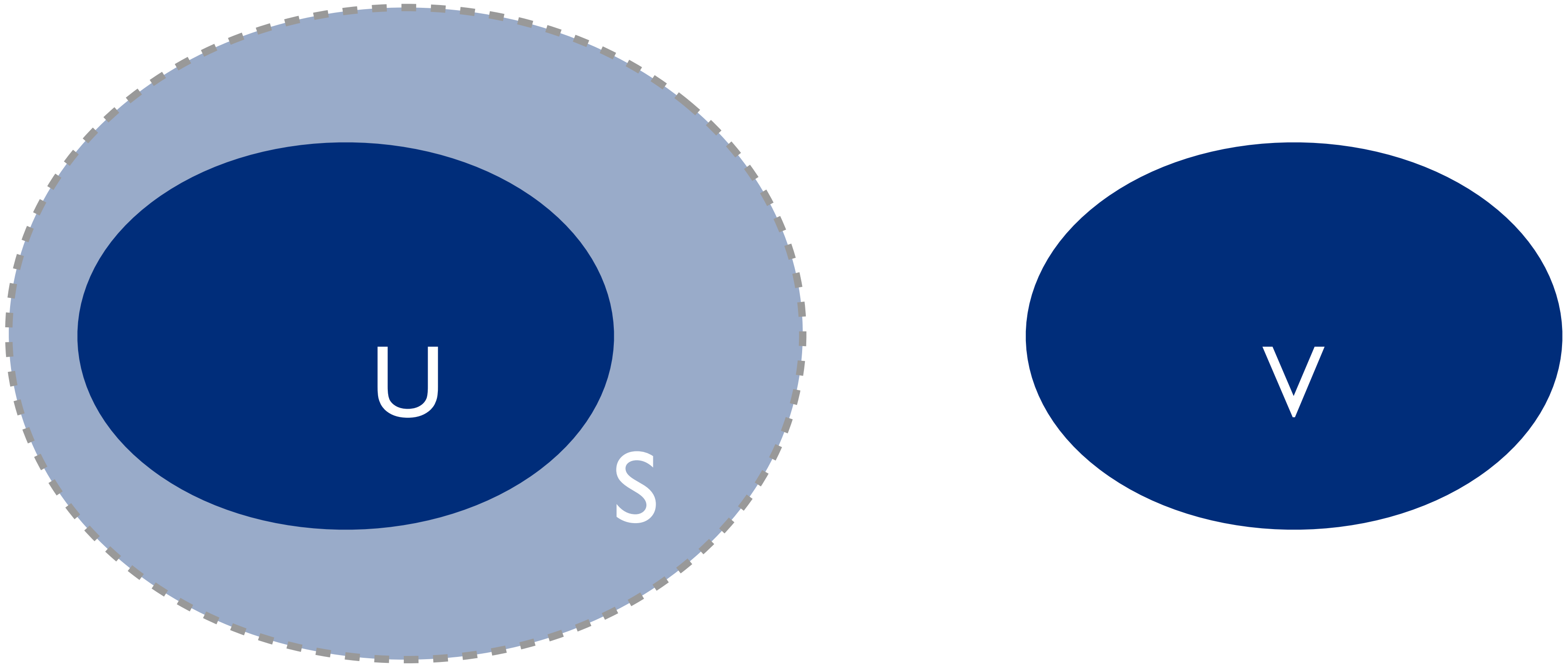
Separability



S *separates* U and V

U and V are *separable by* family F
if some S from F separates them

Vector Addition System

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initial vector v in N^n

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reachability set: vectors in N^n reachable
from v by a sequence of moves

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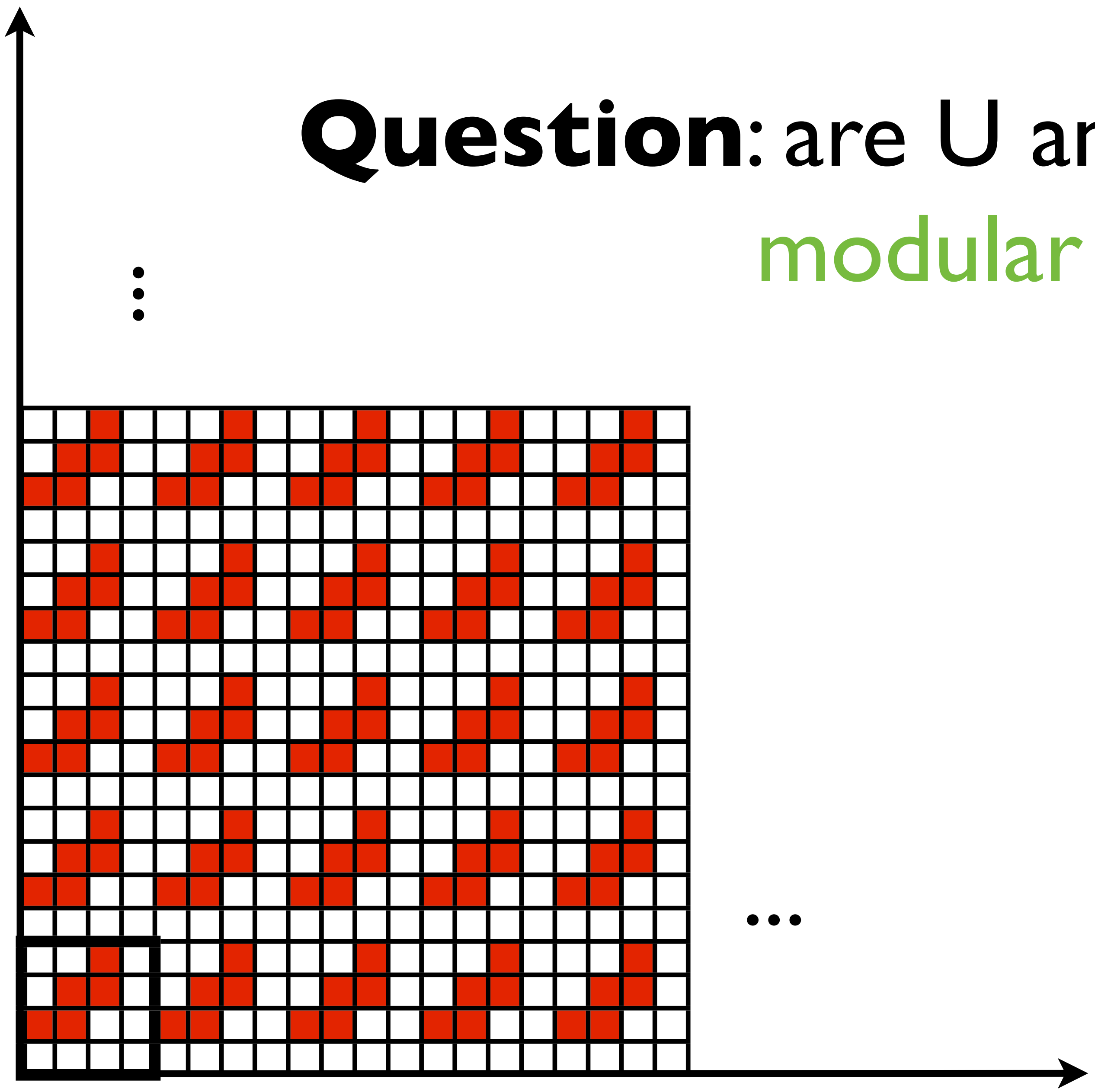

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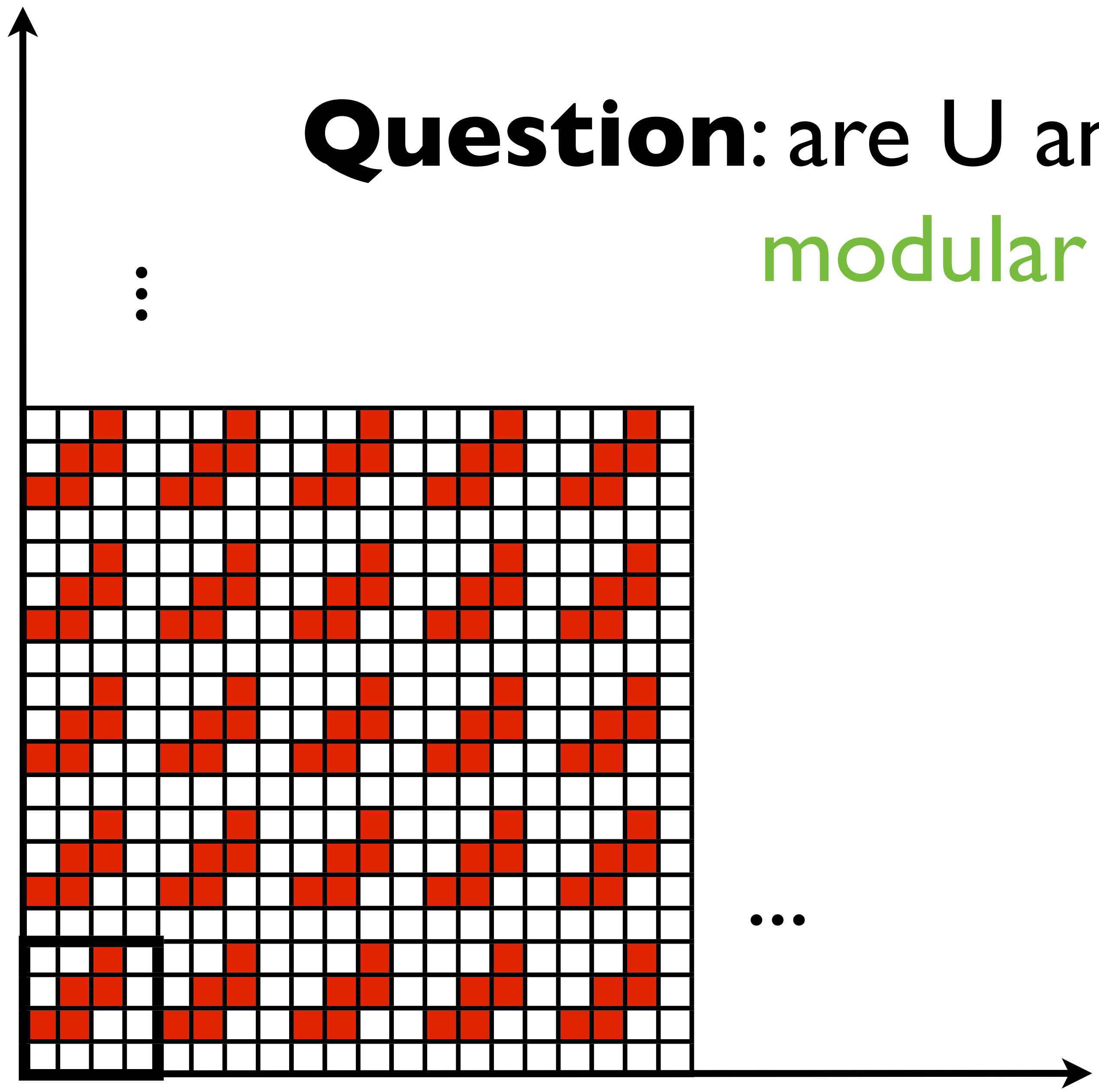

N = 4

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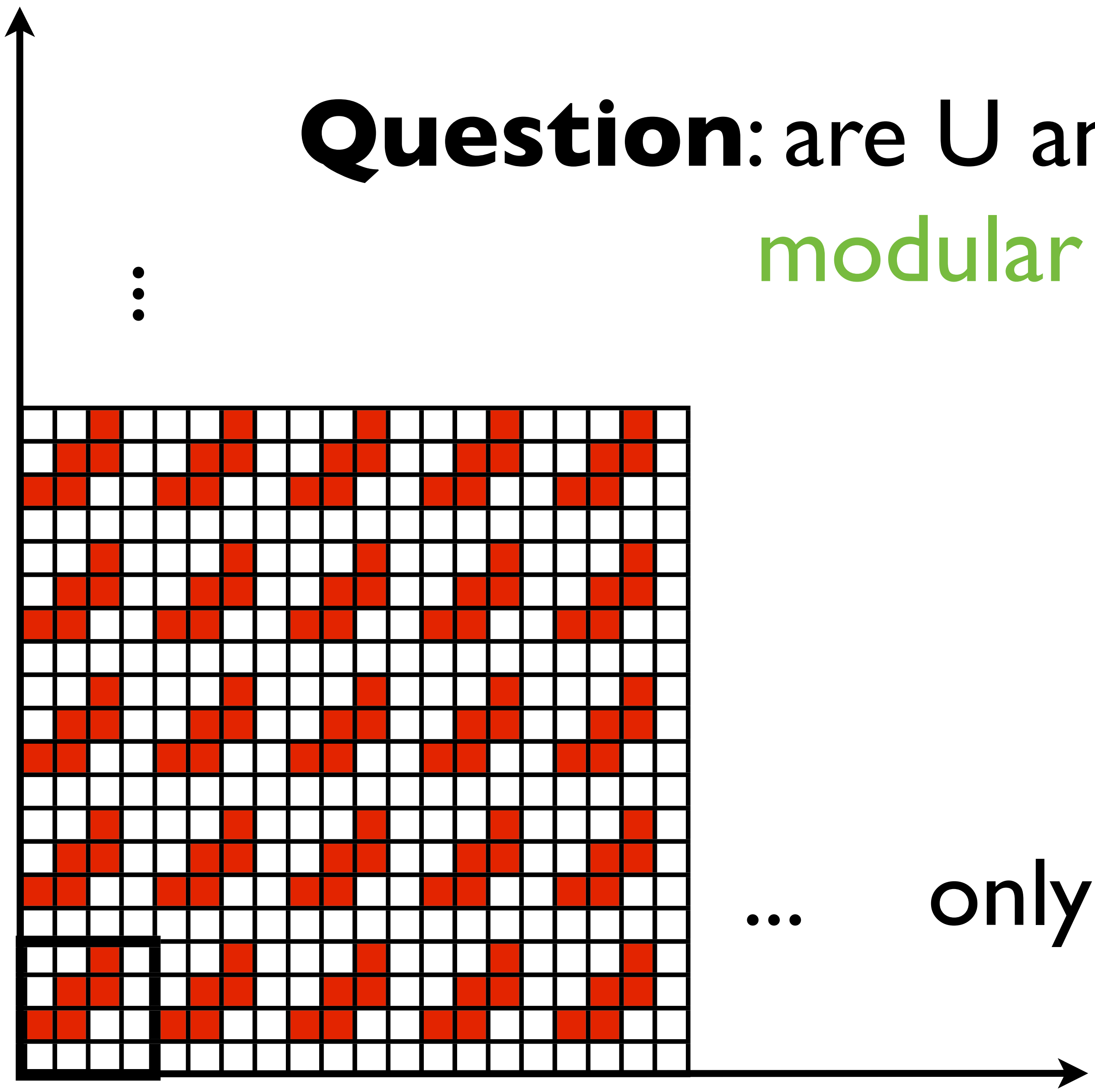

$N = 4$

only value modulo N matters

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- VAS reachability sets may not be semilinear
- Leroux `09: two VAS reachability sets are separable by a semilinear set iff they are disjoint
- so separability by semilinear sets is decidable

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First main result

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Theorem:
Separability of reachability sets of Vector Addition Systems by modular sets is decidable

Core idea

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two semiprocedures

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simple by VAS reachability

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enumerates special linear sets $L_U \subseteq U, L_V \subseteq V$ and checks whether they are modular separable

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simple by linear algebra

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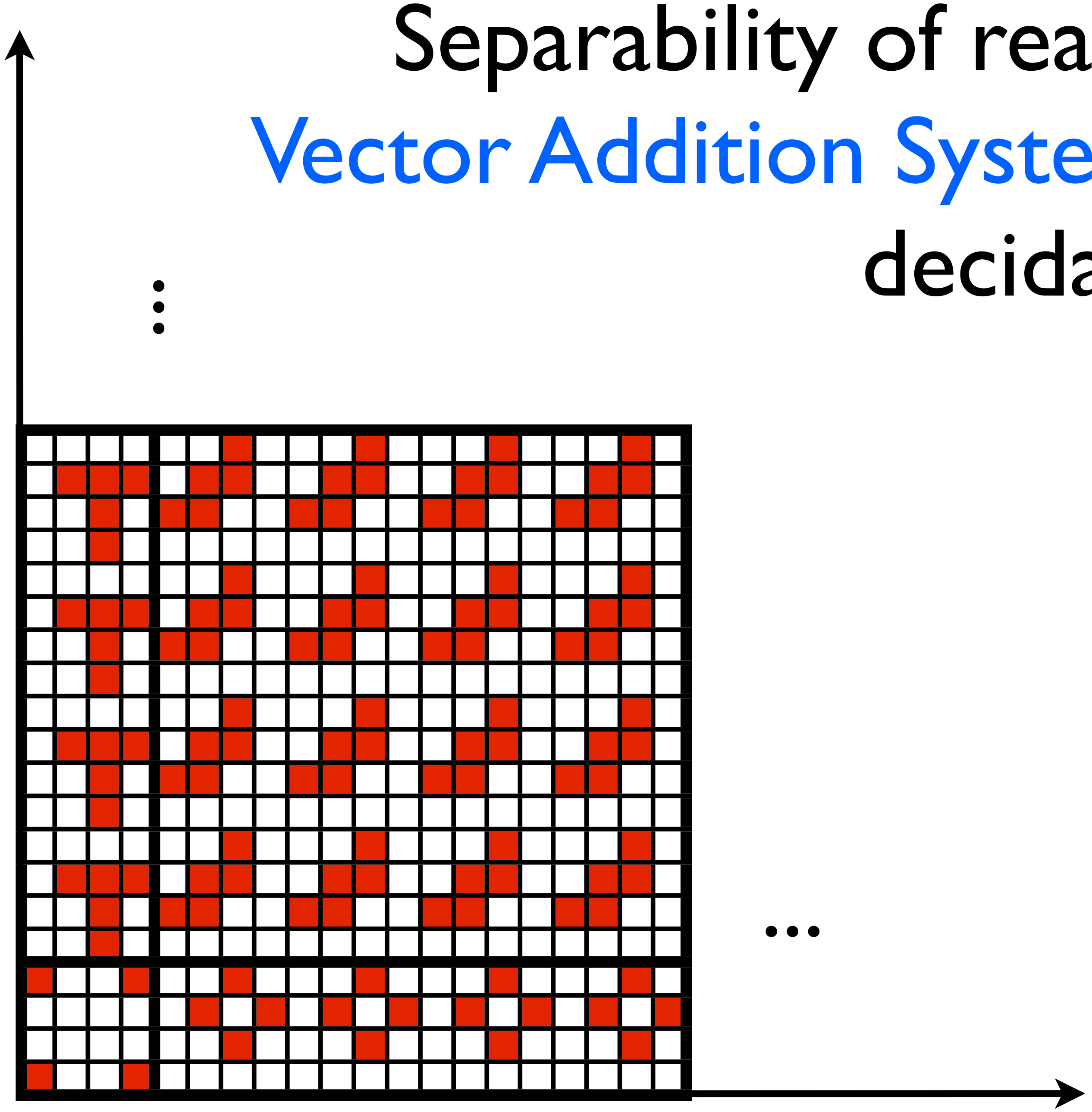

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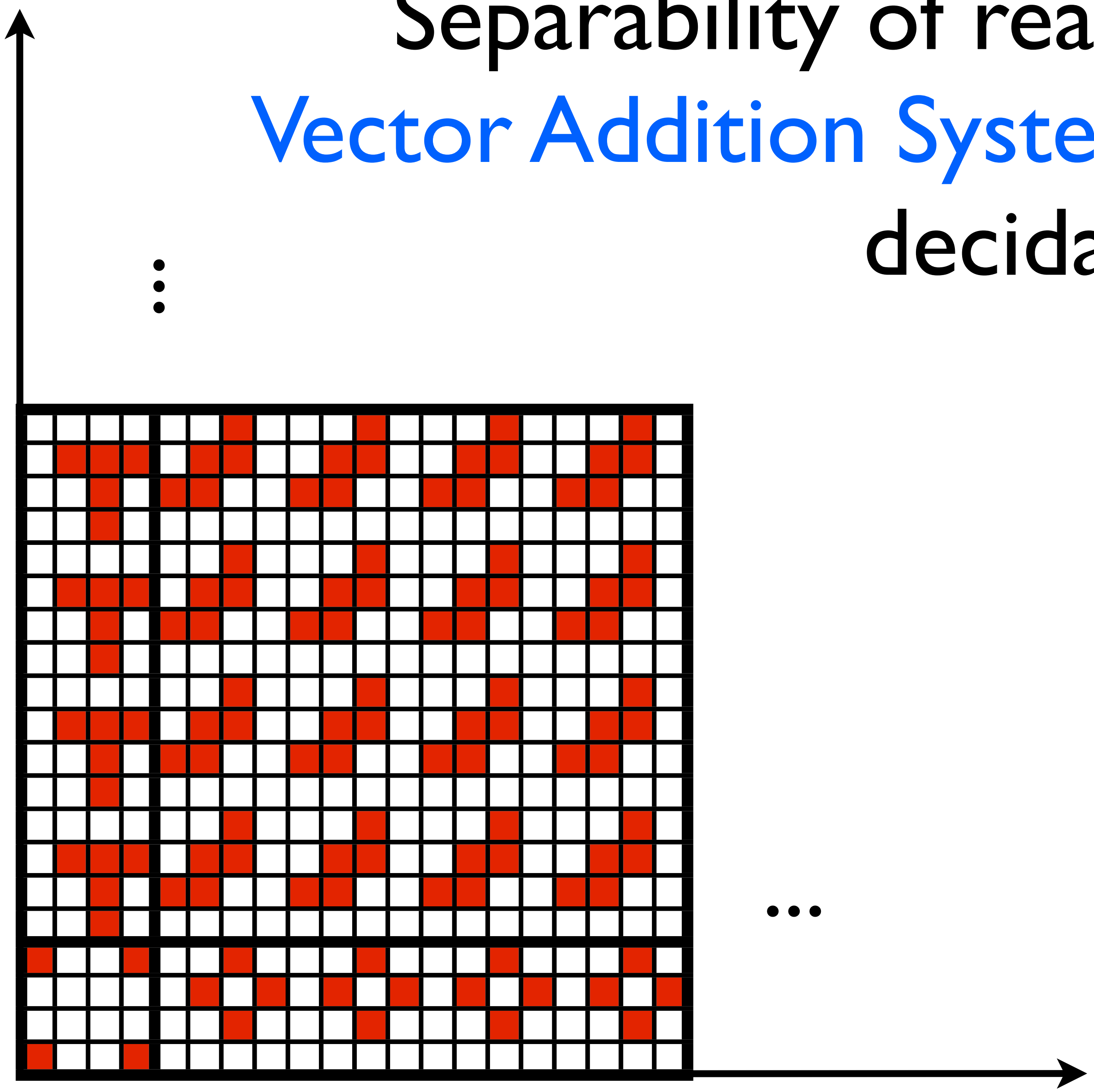

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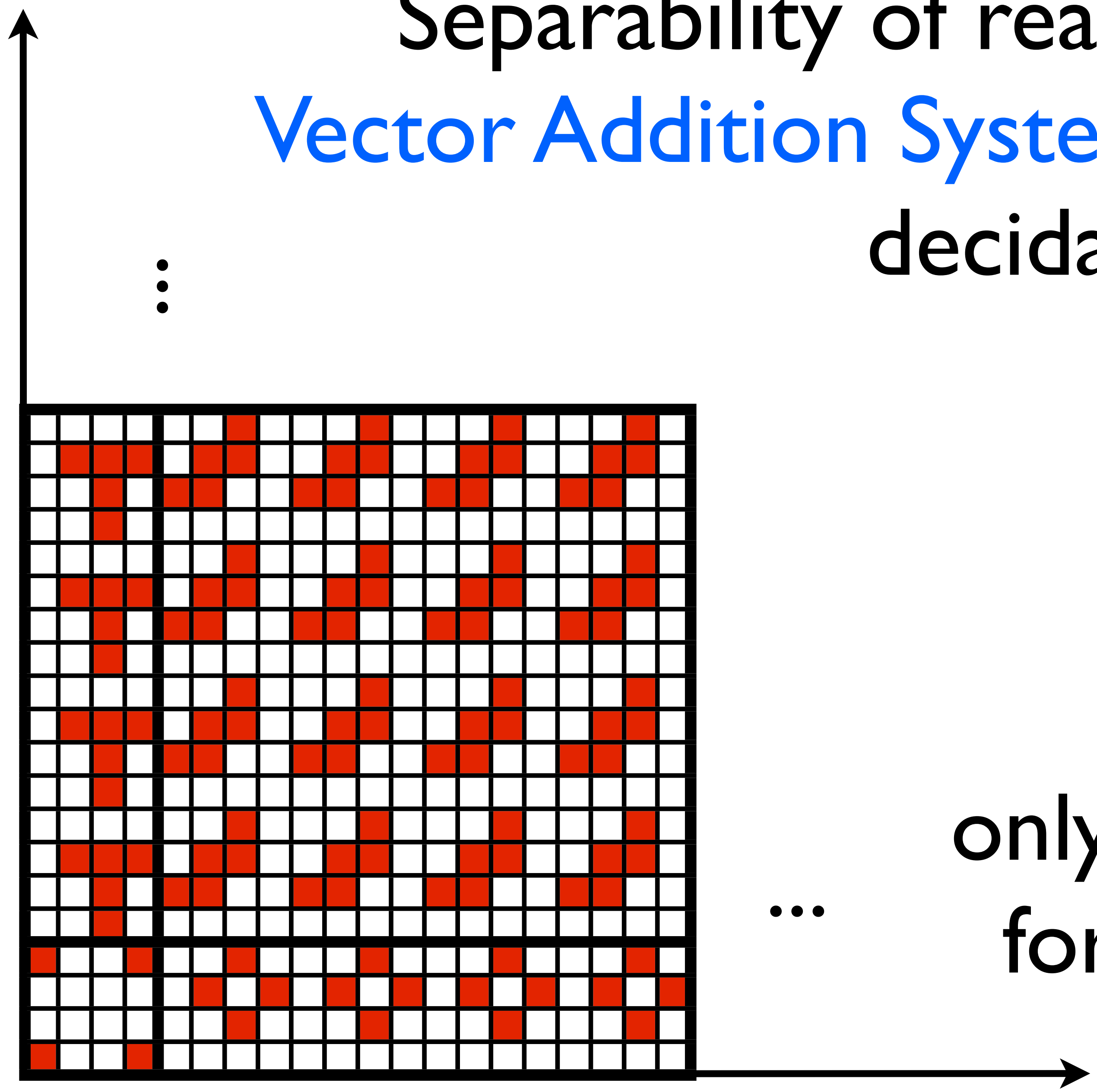

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only value modulo N matters for numbers bigger than N

Thank you!