Beyond One-Goal Strategy Logic

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A Language for Strategic Reasonings

Strategy Logic [CHP07]a[MMV10]b (SL)...  
- is a formalism that allows to explicitly reason about deterministic strategies as first order objects;  
- can express several solution concepts like dominance of strategies, Nash, resilient, immune, and secure equilibria, etc;  
- can be seen as a unifying framework for the specification of properties in the context of multi-agent systems.

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a[Chatterjee et al., 2007]  
b[M., Murano, and Vardi, 2010]
**Syntax of SL**

SL syntactically extends LTL by means of two *strategy quantifiers*, the existential $\langle x \rangle \varphi$ and the universal $[x] \varphi$, and an *agent binding* $(a, x) \varphi$.

**Definition**

*SL formulas* are built from sets of agents $\mathit{Ag}$ and variables $\mathit{Vr}$ according to the following grammar, where $a \in \mathit{Ag}$ and $x \in \mathit{Vr}$:

$$
\varphi ::= \text{LTL } (\varphi) \mid \langle x \rangle \varphi \mid [x] \varphi \mid (a, x) \varphi.
$$
## Semantics of SL

### Strategy quantifications

- $\langle x \rangle \varphi$: "there exists a strategy $x$ for which $\varphi$ is true".
- $[x] \varphi$: "for all strategies $x$, it holds that $\varphi$ is true".

### Agent binding

- $(a, x) \varphi$: "$\varphi$ holds, when the agent $a$ uses the strategy $x$".

### LTL sublogic

- LTL operators are classically interpreted on the resulting play.
**EXPRESSIVENESS AND MODEL CHECKING**

SL is strictly more expressive than ATL* [CHP07,MMV10].

SL model-checking problem has a PTIME-COMPLETE data complexity and a NONELEMENTARY-COMPLETE formula complexity [MMPV11][a].

[a][M., Murano, Perelli, and Vardi, 2011]
**Non-Behavioral Strategies**

SL semantics admits *non-behavioral strategies*, *i.e.*, a choice of an agent, at a given moment of a play, may depend on the choices another agent can make either in the future or in a counterfactual play.

Consequently, there are strategies that cannot be synthesized in practice, since some of the adversary moves may be unpredictable.

- **A**: Alice, **B**: Bob
- $\varphi = \langle x \rangle [y] [z] (\psi_1 \land \psi_2)$
- $\psi_1 = (A, x)(B, y)Xp$
- $\psi_2 = (A, x)(B, y)Fq \leftrightarrow (A, z)(B, y)Xp$
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Behavioral Semantics

A quantification of a strategy $\sigma_1$ is behavioral w.r.t. another strategy $\sigma_2$ iff the action selected by $\sigma_1$, for a given history, depends only on the action selected by $\sigma_2$ on the same history.

A formula is behaviorally satisfiable iff all strategy quantifications required to satisfy it are solved in a behavioral way.

There are strong connections between the behavioral semantics and good for games automata [HP06]a.

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a[Henzinger and Piterman, 2006]
The One-Goal Fragment of SL (SL[1G]) extends ATL* by allowing sharing of strategies and unrestricted alternation of strategy quantifiers.

SL[1G] enjoys the following properties [MMPV11]:

- it admits a behavioral semantics;
- it is strictly more expressive than ATL*;
- it has a model-checking problem with PTIME-COMPLETE data complexity and 2EXPTIME-COMPLETE formula complexity.

Unfortunately, SL[1G] cannot express the classic solution concepts.
Our Contributions

We introduce and study a hierarchy of fragments of Strategy Logic enjoying a behavioral semantics, i.e., all strategies involved in the reasonings are synthesizable.
The Alternating Hierarchy
**Quantification and Binding Prefixes**

A *quantification prefix* is a sequence of quantifiers $\varphi$ in which each variable occurs at most once (e.g., $\varphi = [x][y]⟨z⟩[w]$).

A *binding prefix* is a sequence of bindings $b$ in which each agent occurs exactly once (e.g., $b = (\alpha, x)(\beta, y)(\gamma, y)$).

A *goal* is the juxtaposition $b\psi$ of a binding $b$ and a temporal formula $\psi$. 
BINDING FRAGMENTS

Boolean-Goal SL ($SL_{[BG]}$) restricts SL by only allowing Boolean combinations of goals.

$$\varphi ::= \text{LTL} (\varphi) \mid \varphi \psi$$

$$\psi ::= \lnot \varphi \mid \varphi \land \psi \mid \varphi \lor \psi$$

$SL_{[BG]}$ can express almost all important solution concepts.

One-Goal SL ($SL_{[1G]}$) restricts $SL_{[BG]}$ by forcing the use of a single goal.

$$\varphi ::= \text{LTL} (\varphi) \mid \varphi \psi$$

$$\psi ::= \lnot \varphi$$

$SL_{[1G]}$ is well-suited for the description of zero-sum game properties.
Alternating-Goal SL (SL[AG]) only allows linear combinations of goals.

\[ \varphi ::= \operatorname{LTL}(\varphi) \mid \varphi \psi \]
\[ \psi ::= b \varphi \mid b \varphi \land \psi \mid b \varphi \lor \psi \]

SL[AG] can express nested conditionals over strategic properties with multiple antecedents and consequents.

**k-Conjunctive-Goal SL (SL[k-cG])**

\[ b_{1,1} \psi_{1,1} \land \cdots \land (b_{2,1} \psi_{2,1} \lor \cdots \lor (\cdots (b_{k,1} \psi_{k,1} \circ \cdots \circ b_{k,n_k} \psi_{k,n_k}) \cdots)) \]

**k-Disjunctive-Goal SL (SL[k-DG])**

\[ b_{1,1} \psi_{1,1} \lor \cdots \lor (b_{2,1} \psi_{2,1} \land \cdots \land (\cdots (b_{k,1} \psi_{k,1} \circ \cdots \circ b_{k,n_k} \psi_{k,n_k}) \cdots)) \]
Our Results

\(SL^{AG}\) enjoys the following properties:

- it admits a behavioral semantics;
- it has a model-checking problem with \(PSPACE\)-complete data complexity and \(2EXPSPACE\)-complete formula complexity.
Idea of Proof

By exploiting a game-semantic approach, we reduce the model checking for SL[AG] to the same problem of SL[1G]:

- we transform the original model $G$ into a more complex one $G'$, where all plays in $G$ identifiable by the goals are encoded into a unique play in $G'$;
- every conjunction/disjunction of goals is represented through a universal/existential strategy quantification of a fresh agent.

The reduction is possible thanks to the linear order between all Boolean connectives that links the goals (SL[BG] does not enjoy this property).
The alternating hierarchy may be not very interesting as a family of specification languages, but its study allows a deeper understanding of all the problems concerning the concept of behavioral semantics.
Thank you very much for your attention!

