Background

Bisimilarity

- **Bisimilarity**: equivalence of state-based systems
- **Proof method**: providing *bisimulations*
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Up-to techniques
- *Enhanced proof method(s)* (Milner, Sangiorgi): *bisimulation up-to* (bisimilarity, context, equivalence)
- Widely applied in concurrency theory, but also, e.g., automata.
Example: Bisimulation up to equivalence

The red states below are bisimilar. Let’s find a bisimulation containing this pair!
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Example: Bisimulation up to equivalence

A bisimulation up to equivalence containing the pair of red states.
Example: Bisimulation up to equivalence

A bisimulation up to equivalence containing the pair of red states.

Every bisimulation up to equivalence closure is contained in a bisimulation, hence up to equivalence is a sound up-to technique.
Coinduction, in general

- Bisimilarity is just one example of a **coinductive predicate**: it can be characterized as a **greatest fixpoint**.
- Coinductive predicates are everywhere:
  - weak bisimilarity, similarity,
  - language equality and inclusion, for (various kinds of) automata,
  - sequences being increasing, or always non-zero,
  - ...
- Up-to techniques are formulated at this general level!
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The goal

Prove the soundness of many up-to techniques for many types of state-based systems.
Coinduction in a fibration (Jacobs, Hermida, Hasuo, ...)

State-based systems can be modelled categorically as coalgebras, and coinductive predicates are final coalgebras in a fibre.

Our contribution


Coinduction up-to in a fibrational setting.

CSL-LICS 2014.

A coalgebraic understanding of up-to techniques using fibrations;

An abstract framework for proving soundness of several up-to techniques in a systematic and compositional way.

Examples and Applications

Up-to techniques for proving

▶ language inclusion for weighted automata
▶ language equivalence for nominal automata
▶ predicates on streams
▶ divergence of processes

Simulation up to, weak bisimulation up to

Jurriaan Rot
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