Boundedness games

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This talk is about our joint effort to understand boundedness games.
Motivation: expressing boundedness properties

A lot is known, and even more is not known about those two logics!
Definition of boundedness games

controlled by Eve
controlled by Adam
Definition of boundedness games

controlled by Eve
controlled by Adam
Definition of boundedness games

classified by Eve
controlled by Adam
Definition of boundedness games

controlled by Eve

controlled by Adam
Definition of boundedness games

controlled by Eve
controlled by Adam
Definition of boundedness games

controlled by Eve
controlled by Adam
Definition of boundedness games

boundedness condition:

parity and all counters are bounded
Definition of boundedness games

parity condition:
the minimal priority seen infinitely often is even
Definition of boundedness games

$c_1 = 0$
$c_2 = 0$

$\varepsilon :$ nothing
$i :$ increment
$r :$ reset
Definition of boundedness games

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$c_2 = 0$

$\varepsilon :$ nothing
$i :$ increment
$r :$ reset

\[ c_1 = 0 \]
\[ c_2 = 0 \]
Definition of boundedness games

\[ c_1 = 0 \]
\[ c_2 = 1 \]

\( \varepsilon : \text{nothing} \)
\( i : \text{increment} \)
\( r : \text{reset} \)
Definition of boundedness games

\[ \begin{align*}
    c_1 &= 0 \\
    c_2 &= 1 \\
    \varepsilon &: \text{nothing} \\
    i &: \text{increment} \\
    r &: \text{reset}
\end{align*} \]
Definition of boundedness games

\[ c_1 = 1 \]
\[ c_2 = 0 \]

\[ \varepsilon : \text{nothing} \]
\[ i : \text{increment} \]
\[ r : \text{reset} \]
Definition of boundedness games

\[ c_1 = 1 \]
\[ c_2 = 0 \]

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\[ r : \text{reset} \]
Definition of boundedness games

boundedness condition:
parity and all counters are bounded
Quantification

Eve wins means:

\[ \exists \sigma \text{ (strategy for Eve)}, \forall \pi \text{ (paths)}, \exists N \in \mathbb{N}, \exists \sigma \text{ (strategy for Eve)}, \exists N \in \mathbb{N}, \forall \pi \text{ (paths)}, \]

\pi \text{ satisfies parity and each counter is bounded by } N. \]
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\[ \text{non-uniform (MSO + } \mathcal{U}) \]

\[ \text{uniform (cost MSO)} \]
Research questions and some answers

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- When does Eve has finite-memory winning strategies?
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• When does Eve has finite-memory winning strategies?
  ↦ Uniform quantifications, the Büchi case over infinite chronological arenas [Vanden Boom, 2011].
  ↦ Uniform quantifications, the parity case over thin tree arenas [F., Horn, Kuperberg, Skrzypczak, unpublished].
Why finite-memory strategies?

Thomas Colcombet’s habilitation:

Conjecture 9.3. Les objectifs $hB \land \text{parité}$ et $\neg B \land \text{parité}$ sont à ≈-mémoire finie, sur toutes les arènes/sur les arènes chronologiques/sur les arènes «arborescentes».

Existence of finite-memory strategies in (some) boundedness games $\implies$ Decidability of cost MSO over infinite trees $\implies$ Decidability of the index of the non-deterministic Mostowski’s hierarchy (open for 40 years)!
Working with potato trees

Theorem (F., Horn, Kuperberg, Skrzypczak)

*The Colcombet’s conjecture holds for thin tree arenas!*

Corollary

*The cost MSO logic over thin trees is decidable.*