Compositional Verification and Optimization of Interactive Markov Chains

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Verification of Open Interactive Markov Chains

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Highlights, Paris
September 19, 2013

Specification formalism

We introduce modal continuous-time automata (MCA)

- may/must transitions as in modal transition systems [Larsen&Thomsen\'88]
- continuous time constraints extending timed automata [Alur,Courcoubetis&Dill\'91]

Example: after each outdated an update is ready within time $\sim \text{Exp}(3)$

\[
\begin{align*}
\text{up-to-date} & \xrightarrow{\text{update}} \text{wait} \xrightarrow{\text{Exp}(3)} \text{ready} \\
\text{wait} & \xrightarrow{\text{update}} \text{up-to-date} \\
\text{wait} & \xrightarrow{\text{Exp}(3)} \text{ready} \\
\end{align*}
\]
Symmetry? (in continuous-time stochastic games)
Symmetry? (in continuous-time stochastic games)
Game

- Idea
- Not good
- Chat over
- Notification
- Facebook
- Good
- Inspired
- Drawn
- Smile

Diagram: A cycle of idea, not good, chat over notification, Facebook, good, inspired, drawn, and a smile.
The Element of Surprise in Timed Games*

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Abstract. We consider concurrent two-person games played in real time, in which the players decide both which action to play, and when to play it. Such timed games differ from untimed games in two essential ways. First, players can take each other by surprise, because actions are played with delays that cannot be anticipated by the opponent. Second, a player should not be able to win the game by preventing time from diverging. We present a model of timed games that preserves the element of surprise and accounts for time divergence in a way that treats both players symmetrically and applies to all regular winning conditions. We prove that the ability to take each other by surprise adds extra power to the players. For the case that the game style of timed automata.
The Element of Surprise in Timed Games

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Abstract. We consider concurrent two-person games played in real time, in which the players decide both which action to play, and when to play it. Such timed games differ from untimed games in two essential ways. First, players can take each other by surprise, because actions are played with delays that cannot be satisfied by the opponent. Second, a player should not be able to win the game by preventing time from diverging. We present a model of timed games that preserves the element of surprise and accounts for time divergence in a way that treats both players symmetrically and applies to all regular winning conditions. We prove that the ability to take each other by surprise adds extra power to the players. For the case that the game has a unique winning strategy, we show that both players can win the game by using a strategy that is optimal in all possible future behaviors.

Continuous-Time Stochastic Games with Time-Bounded Reachability

T. Brázdil, V. Forejt, J. Krčál, J. Křetínský, A. Kučera
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Abstract. We study continuous-time stochastic games with time-bounded reachability objectives. We show that each vertex in such a game has a value (i.e., an equilibrium probability) and we present algorithms for computing these values. We also provide algorithms for computing optimal strategies in finite uniform games, and how to compute ε-optimal strategies in finitely branching games with bounded rates (for finite games, we provide detailed complexity estimates).

1 Introduction

Markov Decision Processes (MDPs) are widely used in many diverse areas such as economics, biology, and computer science. They have been used for performance and dependability analysis."
The Element of Surprise in Timed Games

Leca de Alfaro, Marco Faella, Thomas A. Henzinger, Rupak Majumdar, and Maritte Stoolings

Abstract. We consider concurrent two-person games played in real time, in which the players decide both which action to play and when to play it. Such timed games differ from untimed games in two essential ways. First, players can take each other by surprise, because actions can be played with delays that cannot be anticipated by the opponent. Second, a player should not be able to win the game by preventing time: an element of surprise can determine the outcome of the game.

Compositional Verification and Optimization of Interactive Markov Chains

Holger Hermanns, Jan Krčál, and Jan Křetínský

Abstract. Interactive Markov chains (IMC) are compositional behavioural models extending labelled transition systems and continuous-time Markov chains. We provide a framework and algorithms for compositional verification and optimization of IMC with respect to time-bounded properties. Firstly, we give a specification formalism for IMC. Secondly, given a time-bounded property, an IMC component and the assumption that its unknown environment satisfies a given specification, we synthesize a strategy for each player in order to satisfy the property up to a certain bound.
The Element of Surprise in Timed Games

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Abstract. We consider concurrent two-person games played over time, in which the players decide both which action to play, and to play it. Such timed games differ from untimed games in two essential ways. First, players can take each other by surprise, because actions played with delays that cannot be satisfied by the opponent. Second, a player should not be able to win the game by preventing time by winning the time of surprise. We present a model of timed games that preserves the main properties of untimed games in a way that they are both players symmetrically and applies to all kind of winning conditions. We prove that the ability to take each other by surprise admits of power to the players. For the case of the games, we give a generalization of the timed automata.

Compositional Verification and Optimization of Interactive Markov Chains

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Abstract. Interactive Markov chains (IMC) are compositional behavioural models extending labelled transition systems and continuous-time Markov chains. We provide a framework and algorithms for compositional verification and optimization of IMC with respect to time-bounded properties. Firstly, we give a specification formalism for IMC. Secondly, given a time-bounded property, an IMC component and the assumption that its unknown environment satisfies a given specification, we synthesize a controller that solves the problem.

Continuous-Time Stochastic Games with Time-Bounded Reachability

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Abstract. We study continuous-time stochastic games with time-bounded reachability objectives. We show that various time-bounded objectives have a winning set that is, an optimal strategy which guarantees that the game is a win for one of the players, and we provide an algorithm for computing the winning set. Finally, we provide a detailed complexity analysis for our algorithm and discuss some implications for the study of stochastic games with time-bounded objectives.
Application

Component $S$
Interactive Markov chain

$$\sup_{\sigma} \inf_{\mathcal{E}} \mathbb{P}_{\mathcal{S},\mathcal{E}}^{\sigma} [\mathcal{E}^c:]$$

Time bounded reachability
Application

Component S
Interactive Markov chain

\[ \sup_\sigma \inf_E P_{S^{11E}} [0^t:] \]

controller-environment game G

\[ \sup_\sigma \inf_\Pi P_{\sigma^\Pi} [0^t:] \]
Application

Component $S$ Interactive Markov chain

controller-environment game $G$

discrete game $A$

$\sup_{\sigma} \inf_{E} \Pr_{S^{\text{SILC}}}^{\sigma}[0^t : ] \Rightarrow \sup_{\sigma} \inf_{E} \Pr_{G}^{\sigma}[0^t : ] \approx \sup_{\sigma} \inf_{\pi} \Pr_{A}^{\sigma,\pi}[0^t : ]$

with error bound
Application

specification $\mathcal{Q}$
Modal continuous automaton

component $S$
Interactive Markov chain

$\sup_{\sigma \in \mathcal{Q}} \inf_{E \in E_\mathcal{Q}} \Pr_{S,\mathcal{E}}^\sigma [\Diamond^t : ]$

controller-environment game $G$

$\sup_{\sigma \in \mathcal{Q}} \inf_{\pi \in \Pi} \Pr_{G,\mathcal{E}}^\sigma [\Diamond^t : ]$

discrete game $A$

$\sup_{\sigma \in \mathcal{Q}} \inf_{\pi \in \Pi} \Pr_A^{\sigma,\pi} [\Diamond^t : ]$

with (asymmetric) partial observation

with error bound

time bounded reachability
Application

- specification \( \mathcal{Q} \)
- Modal continuous automaton

- component \( S \)
- Interactive Markov chain

\[
\sup_{\sigma} \inf_{E \in \mathcal{Q}} \Pr_{S|\mathcal{Q}}^{\sigma} [\cdot^t:] = \sup_{\sigma} \inf_{\Pi} \Pr_{G}^{\sigma} [\cdot^t:] \approx \sup_{\sigma} \inf_{\Pi} \Pr_{A}^{\sigma,\pi} [\cdot^t:] \]

with (asymmetric) partial observation

Theorem: For IMC \( S \) and MCA \( \mathcal{Q} \), the guarantee \( \sup_{\sigma} \inf_{E \in \mathcal{Q}} \Pr_{S|\mathcal{Q}}^{\sigma} [\cdot^t:] \) can be \( \varepsilon \)-approximated in exponential time.
Summary

1. First assume-guarantee reasoning on stochastic continuous-time systems
2. Crucial solution step: reduction to CE-game
   - a continuous-time stochastic game with asymmetric roles of players

Future Work

1. Lowering the theoretical/practical complexity
2. Other properties
Summary

- First assume-guarantee reasoning on stochastic continuous-time systems
  - Crucial solution step: reduction to CE-game
  - A continuous-time stochastic game with asymmetric roles of players

Future work

- Lowering the theoretical/practical complexity
- Other properties

Thank you for your attention!