Strong simulation for one counter nets is PSPACE-complete

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Plan

1. Model
2. Simulation
3. History
4. Results
5. Future work
One-Counter Nets

\((Q, \text{Act}, \delta)\)  
\(\delta \subseteq (Q \times \text{Act} \times \{-1, 0, +1\} \times Q)\)
One-Counter Nets

\( (Q, Act, \delta) \quad \delta \subseteq (Q \times Act \times \{-1, 0, +1\} \times Q) \)

Induced LTS over \( Q \times \mathbb{N} \)

\[
\begin{align*}
q0 & \xrightarrow{a} q1 \\
p0 & \xrightarrow{a} p1 \\
q1 & \xrightarrow{a} q2 \\
p1 & \xrightarrow{a} p2 \\
q2 & \xrightarrow{a} q3 \\
p2 & \xrightarrow{a} p3 \\
q3 & \ldots
\end{align*}
\]
One-Counter Nets

\((Q, Act, \delta) \quad \delta \subseteq (Q \times Act \times \{-1, 0, +1\} \times Q)\)

Induced LTS over \(Q \times \mathbb{N}\)

\[
\begin{align*}
q_0 & \xrightarrow{b} q_1 & q_1 & \xrightarrow{b} q_2 & q_2 & \xrightarrow{b} q_3 \\
p_0 & \xrightarrow{a} p_1 & p_1 & \xrightarrow{a} p_2 & p_2 & \xrightarrow{a} p_3
\end{align*}
\]
One-Counter Nets

\[(Q, \text{Act}, \delta) \quad \delta \subseteq (Q \times \text{Act} \times \{-1, 0, +1\} \times Q)\]

\[
\begin{array}{c}
\text{Induced LTS over } Q \times \mathbb{N} \\
q_0 \leftarrow b \quad q_1 \leftarrow b \quad q_2 \leftarrow b \quad q_3 \quad \ldots \\
\downarrow c & \downarrow c & \downarrow c & \downarrow c \\
p_0 & p_1 & p_2 & p_3 \quad \ldots
\end{array}
\]
Simulation Games

... are played in rounds between Spoiler and Duplicator on the pair of configurations:

\[
\begin{array}{cccccc}
  q_0 & \leftarrow & b & q_1 & \leftarrow & b \\
  \downarrow & & c & \downarrow & & c \\
  p_0 & & a & p_1 & & a \\
\end{array}
\]

\[
\begin{array}{cccccc}
  q_2 & \leftarrow & b & q_3 & \leftarrow & \ \\
  \downarrow & & c & \downarrow & & c \\
  p_2 & & a & p_3 & & a \\
\end{array}
\]

Round description on the next slide.

Winning conditions:

1. If a player cannot move the other wins.
2. Infinite plays are won by Duplicator.
Simulation Games

...are played in rounds between Spoiler and Duplicator.

In each round

\[ \alpha \text{ vs. } \beta \]
Simulation Games

... are played in rounds between Spoiler and Duplicator.

In each round

\[ \alpha \text{ vs. } \beta \]

1. Spoiler moves from \( \alpha \)

\[ a \]

\[ a' \]

\[ \alpha' \]
Simulation Games

...are played in rounds between Spoiler and Duplicator.

In each round

1. Spoiler moves from $\alpha$
2. Duplicator responds from $\beta$
Simulation Games

...are played in rounds between Spoiler and Duplicator.

In each round

\[ \alpha \quad \text{vs.} \quad \beta \]

1. Spoiler moves from $\alpha$
2. Duplicator responds from $\beta$
3. Game continues from $\alpha'$ vs. $\beta'$
History

Previous results
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- Simulation is Decidable for One-counter Nets
  by Parosh Aziz Abdulla, Karlis Cerans
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- Simulation is Decidable for One-counter Nets by *Parosh Aziz Abdulla, Karlis Cerans*
- Simulation and Bisimulation over One-Counter Processes by *Petr Jančar, Antonín Kučera, and Faron Moller*
### Previous results

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### Description of the relation

Duplicator

- Belt
- \( \alpha \)
- \( p, q \)

Spoiler
History

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Description of the relation

Slope $\alpha$ is $\text{poly}(|Net|)$.
Previous results

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Description of the relation

- Slope $\alpha$ is $\text{poly}(|\text{Net}|)$.
- Pattern inside the belt is regular.
Our contribution

Description of the relation

- New proof of the belt theorem.

Duplicator

Belt

p,q

Spoiler

P. Hofman, S. Lasota, R. Mayr and P. Totzke
Our contribution

Description of the relation

- New proof of the belt theorem.
- Width of the belt is $\text{poly}(|\text{Net}|)$.
Our contribution

Description of the relation

- New proof of the belt theorem.
- Width of the belt is $\text{poly}(\|\text{Net}\|)$.

One Counter Automata

OCA vs. OCN is decidable.
Thank You.
Questions?