NP Reasoning in the Monotone μ-Calculus
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Satisfiability Checking

Complexities of satisfiability checking for some modal logics:

K	PSPACE
modal μ-calculus / CTL	EXPTIME
monotone modal logic	NP [Vardi, 1989]
monotone μ-calculus	?

Satisfiability Checking

Complexities of satisfiability checking for some modal logics:

- **K** — PSPACE
- modal μ-calculus / CTL — EXPTIME
- monotone modal logic — NP [Vardi, 1989]
- alternation-free monotone μ-calculus — NP [here]

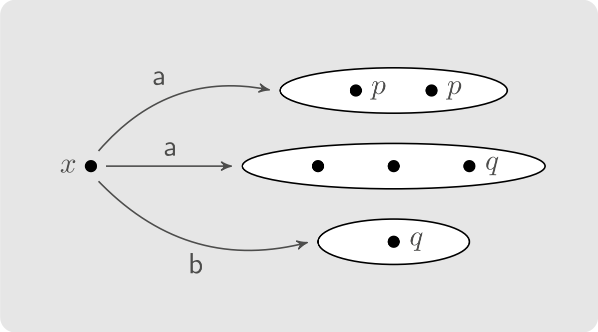
Monotone Modal Logic

Standard modal formulae, interpreted over *neighbourhood structures* $M = (W, N, I)$ where

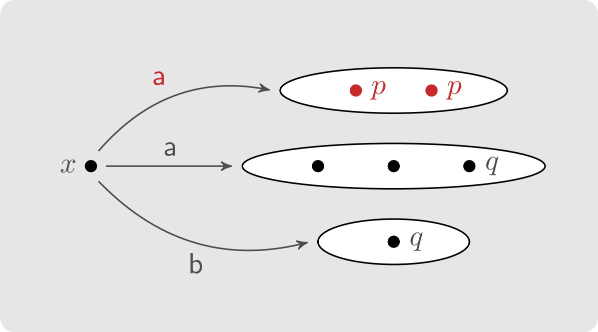
$$N : \mathsf{Act} \times W \to \mathcal{P}(\mathcal{P}(W)) \qquad I : \mathsf{At} \to \mathcal{P}(W)$$

$$\llbracket [a]\phi \rrbracket = \{w \in W \mid \forall S \in N(a, w).\, S \cap \llbracket \phi \rrbracket \neq \emptyset\}$$
$$\llbracket \langle a \rangle \phi \rrbracket = \{w \in W \mid \exists S \in N(a, w).\, S \subseteq \llbracket \phi \rrbracket\}$$

Monotone Modal Logic, example

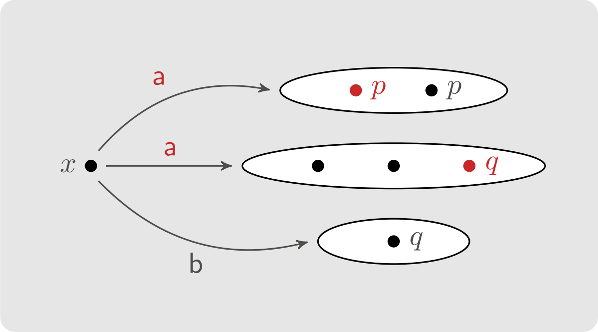


Monotone Modal Logic, example



$x \in [\![\langle a \rangle p]\!]$

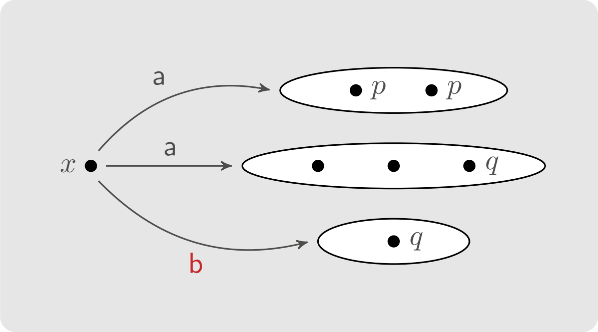
Monotone Modal Logic, example



$x \in [\![\langle a \rangle p]\!]$

$x \in [\![[a](p \vee q)]\!]$

Monotone Modal Logic, example

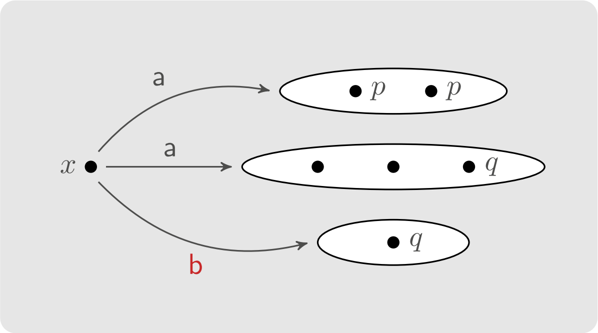


$x \in [\![\langle a \rangle p]\!]$

$x \in [\![[a](p \vee q)]\!]$

$x \notin [\![\langle b \rangle p]\!]$

Monotone Modal Logic, example



$x \in [\![\langle a \rangle p]\!]$

$x \in [\![[a](p \vee q)]\!]$

$x \notin [\![\langle b \rangle p]\!]$

Cannot express e.g. "p holds in every successor state"
"p holds in at least one successor state"

Main Result

Main Theorem

The satisfiability problem for the alternation-free monotone μ-calculus is NP-complete.

Proof sketch:

ϕ is satisfiable	$\Leftrightarrow$	there is tableau for ϕ	$\Leftrightarrow$	Eloise wins satisfiability game for ϕ

Satisfiability games: Two-player Büchi games with polynomial number of Eloise-nodes $\rightsquigarrow$ NP-algorithm for solving the games

Example Logics

Readings:

- Epistemic Logic

  $\langle a \rangle \phi$ – "Agent a knows ϕ"

- Concurrent PDL (CPDL), Peleg (1987)

  $\langle \alpha \rangle \phi$ – "There is execution of program α in parallel, nondeterministic system s.t. all end states satisfy ϕ"

- Game Logic, Parikh (1983)

  $\langle \alpha \rangle \phi$ – "Player Angel has strategy to achieve ϕ in game α"

Take-Away:

Results:

- Satisfiability checking for
  - CPDL
  - alternation-free Game Logic
  - alternation-free monotone μ-calculus (with global axioms)

  is only NP-complete!
- Polynomial bound on model size ($\mathcal{O}(n^2)$)

Future work:

- How about full monotone μ-calculus / Game Logic?