The Parameter-Synthesis Problem for One-Counter Automata

Ritam Raha

(joint work with Guillermo A. Pérez)

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Universiteit Antwerpen

One-Counter Automata

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n = 5
n = max(0, n - 5)
if n = 0:
  while n < 10:
    n = n + 1
  n = n + 100
  #make_progress
else:
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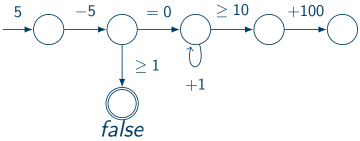

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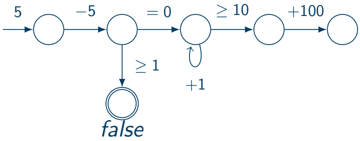

- Configurations: $(q, c), c \geq 0$;

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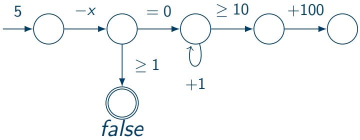
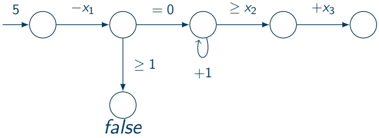


Parametric One-Counter Automata

$X = \{x_1, \dots, x_n\}$



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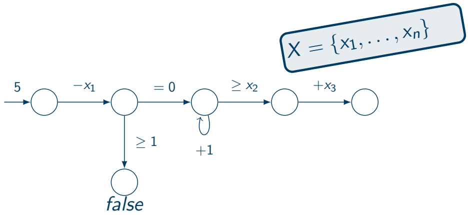

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$\mathcal{A} = (Q, q_{in}, T, \delta, X)$
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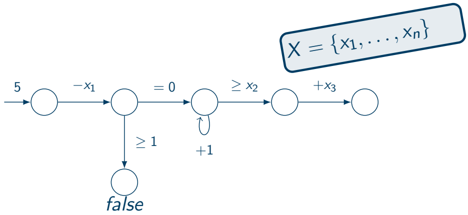

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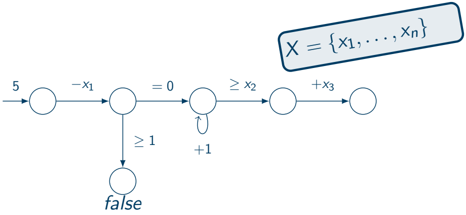

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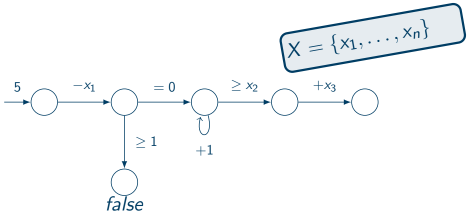

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Non-parametric: $X = \emptyset$

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(Hasse et al. '09)

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-in N3EXP (Lechner'15)

-Reduction to $\exists\forall_R PAD$(Bozga-Iosif'05)

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- We prove that synthesis problems are in N2EXP by reduction to BIL.
- We also consider OCAPT (only tests are parametric). Adapting and modifying ideas from Bollig et al.'19 we show that synthesis problem of OCAPT is in $\mathsf{NP}^{\mathsf{NP}}$.

  Idea. Reduction to non-emptiness of Alternating two-way automata.