The Parameter-Synthesis Problem for One-Counter Automata

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(joint work with Guillermo A. Pérez)

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One-Counter Automata

n = 5
n = max(0, n - 5)
if n = 0:
  while n < 10:
    n = n + 1
    n = n + 100
    #make_progress
else:
  assert(False)
Let $n = 5$. Then $n = \max(0, n - 5)$. If $n = 0$, while $n < 10$, $n = n + 1$, $n = n + 100$, and make_progress. Else, assert(False).
One-Counter Automata

```python
n = 5
n = max(0, n - 5)
if n == 0:
    while n < 10:
        n = n + 1
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else:
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```

- Configurations: \((q, c), c \geq 0;\)
```python
def foobar(x):
    n = 5
    n = max(0, n - x)
    if n == 0:
        while n < 10:
            n = n + 1
            n = n + 100
        #make_progress
    else:
        assert(False)
```
def foobar(x):
    n = 5
    n = max(0, n - x)
    if n == 0:
        while n < 10:
            n = n + 1
            n = n + 100
        # make_progress
    else:
        assert (False)
Parametric One-Counter Automata

\[ X = \{x_1, \ldots, x_n\} \]

\[ \text{false} \]

\[ \begin{align*}
5 & \rightarrow \neg x_1 \rightarrow = 0 \rightarrow \geq x_2 \rightarrow +x_3 \rightarrow \\
& \downarrow \geq 1 \quad \uparrow \text{+1} \\
\end{align*} \]
Parametric One-Counter Automata

$X = \{x_1, \ldots, x_n\}$

\[ 5 - x_1 = 0 \geq x_2 + x_3 \geq 1 \]

**Definition (Succinct OCA with Parameters)**

\[ \mathcal{A} = (Q, q_{in}, T, \delta, X) \]

\[ \delta : T \rightarrow Op \text{ with } Op \text{ the union of} \]

- \(CU := \{+a : a \in \mathbb{Z}\}\)
- \(PU := \{-x : x \in X\}\)
- \(CT := \{=0, \geq a, =a : a \in \mathbb{Z}\}\)
- \(PT := \{=x, \geq x : x \in X\}\)
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Non-parametric: \( X = \emptyset \)
Decision Problems

\[ \exists V : X \rightarrow \mathbb{N} \text{ s.t. } \exists \rho, (q_{in}, 0) \xrightarrow{\rho} \bigvee q_f \]
Decision Problems

Reach

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Non-parametric: \ NP-complete

Parametric: \text{in NEXP (reduction to EPAD)}

(Hasse et al. '09)
Decision Problems

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Decision Problems

**Reach**

\[ \exists V : X \rightarrow \mathbb{N} \text{ s.t. } \exists \rho, (q_{\text{in}}, 0) \xrightarrow{\rho} V q_f \]

**Synthesis**

\[ \exists V \text{ s.t. all infinite } \rho \text{ from } (q_{\text{in}}, 0) \text{ satisfy some } \omega\text{-regular property?} \]

Reachability-Safety-Büchi-coBüchi-LTL

**Non-parametric:** NP-complete

**Parametric:** in NEXP (reduction to EPAD)

(Hasse et al. ’09)

Non-parametric: coNP-complete

Parametric: ?-in N^3EXP (Lechner’15)

- Reduction to \(\exists \forall\) R\text{PAD} (Bozga-Iosif’05)
Decision Problems

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**Non-parametric:** coNP-complete

**Parametric:** ?

- in N3EXP (Lechner’15)
- Reduction to \( \exists \forall_R PAD \) (Bozga-Iosif’05)
Our Contributions

- We prove that $\exists \forall_R PAD$ is undecidable. The synthesis problems become open.
- We define BIL(Bozga-Iosif-Lechner) fragment and show that it is decidable.
- We prove that synthesis problems are in $\text{N}^2\text{EXP}$ by reduction to BIL.
- We also consider OCAPT (only tests are parametric). Adapting and modifying ideas from Bollig et al.’19 we show that synthesis problem of OCAPT is in $\text{NP}$.
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Idea. Reduction to non-emptiness of Alternating two-way automata.