Approximating Values of Generalized-Reachability Stochastic Games

Maximilian Weininger

joint work with Pranav Ashok, Krishnendu Chatterjee, Jan Kretínský, Tobias Winkler

HIGHLIGHTS 2020
(Paper appeared at LICS 2020)
Model
The problem
The problem
The problem
The problem
The problem

\[ T_2 \]

\[ T_1 \]
The problem

Want: Pareto frontier
The problem

Want: Pareto frontier
How: Value iteration from below [CFK+13]

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Solution: Convergent over-approximation
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Approximate values of generalized-reachability stochastic games for arbitrarily small precision.
Our contribution

Over-approximation need not converge
(multiple fixpoints)
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Over-approximation need not converge (multiple fixpoints)

- Consider single directions
- Apply single-dimensional solution
Our contribution

Over-approximation need not converge (multiple fixpoints)

- Consider single directions
- Apply single-dimensional solution
- Group directions into regions
## Context

<table>
<thead>
<tr>
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<td>$\mathsf{PSPACE}$-complete [RRS15]</td>
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