Regular Tree Algebras

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joint work with

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Algebraic Language Theory

Recognisability

φ : free algebra → finite algebra

$L = \varphi^{-1}[P]$

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Recognisability

φ : free algebra $\to$ finite algebra

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Which algebras?

finite words	monoids, semigroups
infinite words	ω-semigroups
finite trees	clones, preclones, term algebras, forest algebras,…
infinite trees	?

General Formalism

Algebras $\langle A, \pi \rangle$ where $\pi : \mathbb{T}A \to A$ and

finite words	$\mathbb{T}A = A^*$
infinite words	$\mathbb{T}A = A^\infty$
finite trees	$\mathbb{T}A$ finite A-labelled trees
infinite trees	$\mathbb{T}A$ finite and infinite A-labelled trees

(plus axioms for associativity:

$\mathbb{T}$ monad, $\pi : \mathbb{T}A \to A$ Eilenberg–Moore algebra)

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Tree Algebras

Regular Tree Algebras

Problem

Finitary tree algebras can recognise non-regular languages.

Regular algebras

$\langle A, \pi \rangle$ is **regular** if π can be evaluated by a tree automaton.

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Theorem

A finitary algebra $\langle A, \pi \rangle$ is regular if, and only if, every language recognised by $\langle A, \pi \rangle$ is regular.

Theorem

A language L is regular if, and only if, it is recognised by a regular tree algebra.

Syntactic algebras

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Every regular tree language L has a syntactic algebra $\mathrm{Syn}(L)$ which is a regular tree algebra.